Hot-Cavity Loading: A Heisenberg-Langevin Analysis

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Abstract: The photon-absorption dynamics of neutral-atom quantum memories are analyzed using Heisenberg-Langevin equations. Two-level and V-level configurations are considered, as is the case of two memories illuminated by an entangled state.

OCIS codes: (270.5580) Quantum electrodynamics; (020.1670) Coherent optical effects; (270.5290) Photon statistics.

Neutral-atom quantum memories (QMs), connected by photon transmission, are essential elements of several proposals for long-distance quantum communication, see, e.g., [1, 2]. Efficient coupling between the photon field and a single-atom QM can be achieved by trapping the atom in a high-finesse microcavity [3]. Most previous studies of such cavity quantum electrodynamics have addressed the intracavity (atom-field) system's time evolution when the surrounding field reservoir is in thermal equilibrium, or when the cavity is empty (cold-cavity dynamics) [4]. Whereas the master-equation approach combined with Monte Carlo simulation can be used to numerically evaluate the photon-absorption behavior of an intracavity atom (hot-cavity loading dynamics) in response to a specific external illumination, we prefer an input-output representation for the external field [5], using the Heisenberg-Langevin (HL) equations of motion [4], so that we can explore *analytically* the pulse-shape dependence of memory loading. We have used HL equations to analyze two neutral-atom QMs: a two-level atom [shown in Fig. 1(a)], and a V-level atom [shown in Fig. 1(b)]. In the two-level case we consider single-photon illumination—in the appropriate polarization and spatial mode—and derive the loading probability as a function of time, pulse shape, and cavity parameters. For the V-level atom, we perform a similar calculation but for an arbitrary polarization state. It turns out that the V-level atom's loading statistics are isomorphic to those of an equivalent two-level system. Moreover, we have also shown that illuminating a pair of two-level atom QMs with a photon-number-entangled state, as shown in Fig. 1(c), also reduces to single-photon excitation of an equivalent two-level system. The common and simplifying feature of these three examples is the presence of at most one excitation in the entire system. In the following, we outline our methodology and our results for hot-cavity loading of a two-level atom.

Figure 1(a) shows a two-level atom, with excited state $|e\rangle$ and ground state $|g\rangle$, inside a single-ended high-Q cavity. [Here, as in the rest of Fig. 1, we neglect any auxiliary (shelving) levels that may be needed for long-term storage, see, e.g., [2].] We assume that the frequency-ω_a atomic transition between $|e\rangle$ and $|g\rangle$ is coupled, at coupling rate g, to the cavity field operator b of frequency ω_0. The intracavity Hamiltonian, using the dipole and rotating-wave approximations, is then $H_c = \hbar\omega_0 b^\dagger b + \hbar\omega_a\sigma_{ee} + \hbar g(b^\dagger\sigma_{ge} + \sigma_{eg} b)$, where $\sigma_{ij} = |i\rangle\langle j|$, for $i, j \in \{g, e\}$. In the strong-coupling regime, the intracavity atom-field system is connected to the outside world at decay rate κ through the cavity's output-coupler mirror. Through this coupler the cavity is illuminated by a single photon, with a pulse shape $\Phi(t)$, whose spatial mode is appropriately matched to that of the cavity mode. This intracavity system is governed by the following HL equations [4]:

$$\dot{b}(t) = -(i\omega_0 + \kappa)b(t) - ig\sigma_{ge}(t) + \sqrt{2\kappa}A_{\rm in}(t)$$

$$\dot{\sigma}_{ee}(t) = ig[\sigma_{ge}(t)b^\dagger(t) - \sigma_{eg}(t)b(t)] \quad \text{and} \quad \dot{\sigma}_{eg}(t) = i\omega_a\sigma_{eg}(t) + ig[\sigma_{gg}(t) - \sigma_{ee}(t)]b^\dagger(t)$$

where $A_{\rm in}(t) = \int d\omega\, e^{-i\omega t}a(\omega)/\sqrt{2\pi}$ is the Langevin operator representing the external input field. The frequency-dependent annihilation operators $a(\omega)$ characterize the temporal modes outside the cavity for the spatial mode of interest, and they satisfy $[a(\omega), a^\dagger(\omega')] = \delta(\omega - \omega')$. These operators are assumed to be initially in a superposition of single photon states in the form $\int d\omega\, \phi(\omega)|1\rangle_\omega$, where $a(\omega')\,|1\rangle_\omega = \delta(\omega-\omega')|0\rangle_{\rm in}$, $|0\rangle_{\rm in}$ is the vacuum state of $A_{\rm in}$, and $\phi(\omega)$ is the frequency-domain representation of the input field's pulse shape, i.e., $\Phi(t) = \int d\omega\, e^{-i\omega t}\phi(\omega)/\sqrt{2\pi}$. Before the illumination begins, the atom is initially in its ground state and the intracavity field is in its vacuum state.

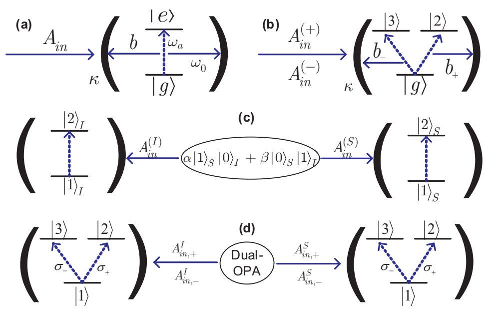

Fig. 1. Trapped-atom quantum memories: (a) a two-level atom illuminated by a single photon; (b) a V-level atom illuminated by a single photon with an arbitrary polarization; (c) a pair of two-level atoms suitable for storing photon-number entanglement; and (d) a pair of V-level atoms suitable for storing polarization-entanglement. All cavities are single ended.

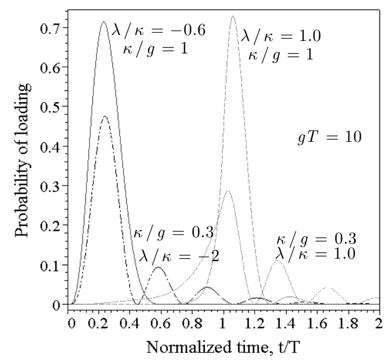

Fig. 2. Time evolution of the photon-absorption probability for a two-level atom trapped in a single-ended high-Q cavity undergoing single-photon illumination by various exponential pulse shapes.

We measure the success of QM loading by $\langle\sigma_{ee}(t)\rangle$, which is the probability that the atom is in its excited state at time t. We find this probability by repeatedly using the HL equations until we obtain a closed set of moment equations that can be solved by means of Laplace transforms. For the on-resonance case, $\omega_a = \omega_0$, we obtain

$$\begin{aligned}\langle\sigma_{ee}(t)\rangle &= -\frac{g}{2\xi}\int_0^t \mathrm{d}\tau\, \mathrm{Im}\{F_1(\tau,\tau)\} e^{-\kappa(t-\tau)}\left(e^{-\xi(t-\tau)} - e^{\xi(t-\tau)}\right) \\ &\quad + \frac{1}{2\xi^2}\int_0^t \mathrm{d}\tau\, \left(2g^2\mathrm{Re}\{F_2(\tau,\tau)\} - g\kappa \mathrm{Im}\{F_1(\tau,\tau)\}\right) e^{-\kappa(t-\tau)}\left(2 - e^{-\xi(t-\tau)} - e^{\xi(t-\tau)}\right)\end{aligned}$$

with the definitions $F_1(t,t') = -i4g\kappa\Phi(t')\int_0^t \mathrm{d}\tau\, \Phi^*(\tau)\left(e^{\alpha_+(\tau-t)} - e^{\alpha_-(\tau-t)}\right)/\xi$, $\alpha_\pm = -i\omega_a + (\kappa \pm \xi)/2$, $\xi = \sqrt{\kappa^2 - 4g^2}$, $F_2(t,t') = -4\kappa\Phi(t')\int_0^t \mathrm{d}\tau\, \Phi^*(\tau)\left(\beta_+ e^{\alpha_+(\tau-t)} + \beta_- e^{\alpha_-(\tau-t)}\right)$, and $\beta_\pm = 1/2 \pm \kappa/2\xi$.

In Fig. 2, we have plotted the loading probability for the on-resonance case, $\omega_0 = \omega_a$, with an exponential pulse shape, $\Phi(\tau) = \sqrt{2\lambda/(e^{2\lambda T} - 1)}\, e^{(-i\omega_0+\lambda)\tau}$, for $0 \le \tau \le T$. Here, λ may be either positive (an exponentially-rising pulse) or negative (an exponentially-decaying pulse). From our knowledge of cold-cavity dynamics, we might expect that only an exponentially-rising input pulse with rate $\lambda \approx \kappa$ can load the cavity with a high probability. [This follows from time-reversing the external field emanating from the output coupler when a single photon is initially present inside the cold cavity.] Figure 2 shows that fine tuning the system parameters permits a high loading probability to be achieved with exponentially-decaying pulses, cf. the $\lambda/\kappa = -0.6$ and $\lambda/\kappa = 1$ curves. Figure 2 also shows the oscillatory behavior of QM loading that occurs as κ/g decreases. Here, the intracavity field's decay time exceeds the excited state's lifetime. Thus the atom and the intracavity field can exchange their excitation several times before that excitation leaks out of the cavity. We also observe some overshoot in the loading process, i.e., the loading probability may still be increasing after the input pulse has ended. This overshoot results from the time it takes for the intracavity field to exchange an excitation with the atom and vice versa. Extending our work to the Fig. 1(d) QM architecture, which was proposed in [2], will be discussed in the presentation. Here, finding the loading probability for a polarization-entangled state involves a more complicated two-excitation analysis, but the results obtained can help choose operating parameters for the QM from [2].

This research was supported in part by the MIT-HP Alliance.

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