OPTICAL QUBIT USING LINEAR ELEMENTS

Matteo G. A. Paris (paris@unipv.it)
Quantum Optics Group, Unità INFM and Dipartimento di Fisica "A. Volta"
Università di Pavia, via Bassi 6, I-27100 Pavia, ITALY

Abstract A conditional scheme to prepare optical superposition of the vacuum and one-photon states using linear elements (beam splitters and phase-shifters) and avalanche photodetectors is suggested.

In recent years, the quantum engineering of light have received much attention, which is mainly motivated by the potential improvement offered by quantum mechanics to the manipulation and the transmission of information. In particular, some conditional schemes have been suggested to prepare superpositions. Among these we mention the Fock filtering by an active Fabry-Perot cavity [1], the displacing/photon-adding scheme of Ref. [2], and the so-called optical state truncation [3]. In this paper we describe a partially interferometric conditional scheme to prepare any desired superposition $a_0|0\rangle + a_1|1\rangle$ of the vacuum and one-photon states using only linear optical components and avalanche photodetectors. For a fully interferometric setup and for more details we refer the readers to Ref. [4].

The present scheme (see Fig. 1) is built by a balanced beam splitter, fed by one-photon state in the mode a, followed by a Mach-Zehnder interferometer, with inputs consisting of one of the output from the beam splitter (mode b) and of an additional mode c excited in a weak coherent state. The two modes b and c exiting the interferometer are detected, and the situation in which a click is observed in one mode, and no clicks are seen in the other one, corresponds to the (conditional) preparation of a superposition of the vacuum and one-photon states in the mode a. The amplitudes in the superposition may be tuned by varying the internal phase-shift of the interferometer and the amplitude of the coherent state.

After the first beam splitter, the joint state of the modes a and b is given by the superposition

$$|\psi\rangle_{ab} = \frac{1}{\sqrt{2}} \left[|0\rangle_a |1\rangle_b + |1\rangle_a |0\rangle_b \right] \ .$$

Mode b then enters the interferometer where it is mixed with mode c excited in a weak coherent state $|\gamma\rangle$. The evolution operator of the interferometer is given by [5] $\hat{U}(\phi) = \exp\left\{i\phi\left(b^\dagger c + c^\dagger b\right)\right\}$, where $\phi = \theta/2$. The overall output state is thus given by

$$\begin{aligned} |\psi\rangle_{out} = \hat{U}(\phi)\,|\psi\rangle_{ab}|\gamma\rangle_c &= \frac{1}{\sqrt{2}}\Big[|1\rangle_a|\gamma\cos\phi\rangle_b|\gamma\sin\phi\rangle_c \\ &+ \sin\phi\,|0\rangle_a\, b^\dagger|\gamma\cos\phi\rangle_b|\gamma\sin\phi\rangle_c \\ &- \cos\phi\,|0\rangle_a|\gamma\cos\phi\rangle_b\, c^\dagger|\gamma\sin\phi\rangle_c\Big] \end{aligned} \quad (1.1)$$

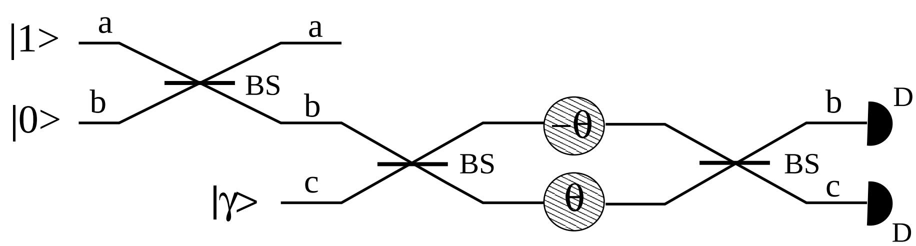

Figure 1 Schematic diagram of the setup for the generation of optical superpositions. The BS's are balanced beam splitters whereas the D's are avalanche photodetectors.

Let us now analyze the effects of the photodetection of modes b and c. The outcomes from an avalanche detector may be either YES, which means a "click", corresponding to any number of photons, or NO, which means that no photons have been recorded. This kind of measurement is described by a two-value POM

$$\hat{\Pi}_N = \sum_{p=0}^{\infty}(1-\eta)^p\,|p\rangle\langle p| \qquad \hat{\Pi}_Y = \hat{\mathbf{I}} - \hat{\Pi}_N\,, \quad (1.2)$$

where η is the quantum efficiency, and $\hat{\mathbf{I}}$ denotes the identity operator. For high quantum efficiency $\hat{\Pi}_N$ and $\hat{\Pi}_Y$ approach the projection operator onto the vacuum state and the orthogonal subspace respectively. In this case the event of "no clicks" corresponds exactly to the absence of photons. In general, the event of observing a click at the detector surveying the output mode b, and no photons at c, is characterized by the probability

$$P_{YN}[\eta,\gamma,\phi] = \mathrm{Tr}_{abc}\left[|\Psi_{out}\rangle\langle\Psi_{out}|\;\hat{\Pi}_Y\;\otimes\hat{\Pi}_N\right] = e^{-\eta|\gamma|^2\sin^2\phi}\times$$
$$\left\{1 - e^{-\eta|\gamma|^2\cos^2\phi} + \frac{\eta}{2}\left[e^{-\eta|\gamma|^2\cos^2\phi} + \cos^2\phi(\eta|\gamma|^2\sin^2\phi - 1)\right]\right\}. \quad (1.3)$$

The corresponding conditional output state is

$$\begin{aligned}\hat{\varrho}_{YN} &= \frac{1}{P_{YN}}\mathrm{Tr}_{bc}\left[|\Psi_{out}\rangle\langle\Psi_{out}|\ \hat{\Pi}_Y\ \otimes\hat{\Pi}_N\right] = \\ &= \frac{1}{P_{YN}}\Big[d_{00}\ |0\rangle\langle 0| + d_{11}\ |1\rangle\langle 1| + d_{01}\ |0\rangle\langle 1| + d_{01}^*\ |1\rangle\langle 0|\Big]\end{aligned} \quad (1.4)$$

where the coefficients are given by

$$d_{11} = \frac{1}{2}e^{-\eta|\gamma|^2\sin^2\phi}\Big[1 - e^{-\eta|\gamma|^2\cos^2\phi}\Big] \quad d_{01} = e^{-\eta|\gamma|^2\sin^2\phi}\,\frac{\eta\gamma}{2}\,\sin\phi\cos\phi$$

$$d_{00} = \frac{1}{2}e^{-\eta|\gamma|^2\sin^2\phi}\Big[1-(1-\eta)e^{-\eta|\gamma|^2\cos^2\phi} + \eta\cos^2\phi(\eta|\gamma|^2\sin^2\phi - 1)\Big]\,.$$

The symmetric case of a click observed in the mode c and no clicks in the mode b leads to an equivalent result, up to the replacement $\phi\to\phi+\pi/2$.

Due to non unit quantum efficiency of photodetectors, the conditional output state $\hat{\varrho}_{YN}$ is not a pure state. However, as we will see, there are regimes in which $\hat{\varrho}_{YN}$ approaches the desired superposition. In order to compare $\hat{\varrho}_{YN}$ with the ideal conditional output $|\psi_{10}\rangle$ we consider the fidelity $F = \langle\psi_{10}|\hat{\varrho}_{yn}|\psi_{10}\rangle$. Notice that ideal output corresponds to a conditional photodetection performed by fully efficient detectors, which are also able to discriminate among the number of photons. From previous equations we have

$$\begin{aligned}F[\eta,\gamma,\phi] &= \frac{1}{2P_{YN}}\,\frac{e^{-\eta|\gamma|^2\sin^2\phi}}{\sin^2\phi + |\gamma|^2\cos^2\phi}\Big\{|\gamma|^2\sin^2\phi\Big(1-e^{-\eta|\gamma|^2\cos\phi^2}\Big) \\ &+2\eta|\gamma|^2\sin^2\phi\cos^2\phi + \sin^2\phi\,\Big[1-(1-\eta)e^{-\eta|\gamma|^2\cos^2\phi} \\ &+\eta\cos^2\phi(\eta|\gamma|^2\sin^2\phi-1)\Big]\Big\}\,.\end{aligned} \quad (1.5)$$

Our aim is to find regimes in which the fidelity of the conditional output state is close to unit and, at the same time, the corresponding detection probability P_{YN} does not vanish.

In Fig. 2 we show $P_{YN}[\eta,\gamma,\phi]$ and $F[\eta,\gamma,\phi]$ as a function of γ and ϕ for quantum efficiency equal to $\eta = 80\%$. As it is apparent from the plot (darker regions correspond to lower values of detection probability and fidelity) for a weak coherent signal $\gamma\le 1$ there is, even for non unit quantum efficiency of the conditional photodetectors, a large range of values of ϕ corresponding to high fidelity and detection probability ($P_{YN}\simeq 20\%$ for the plotted case). We conclude that the present method is a reliable source of optical superpositions (qubit) employing only linear components and avalanche photodetectors.

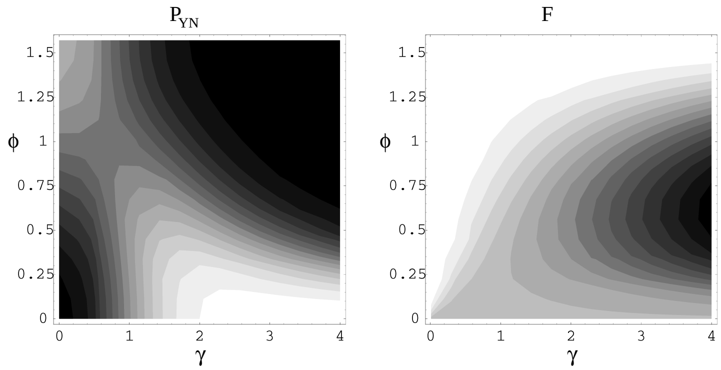

Figure 2 Detection probability and fidelity as a function of the coherent amplitude γ and the interferometric shift ϕ for quantum efficiency equal to $\eta = 80\%$.

Acknowledgments

This work has been cosponsored by C.N.R. and N.A.T.O under the 1999 Advanced Fellowship Program 215.31. The author thanks R. F. Antoni for valuable hints.

References

[1] G. M. D'Ariano, L. Maccone, M. G. A. Paris, M. F. Sacchi, Phys. Rev. A **61** 053817 (2000); Acta Phys. Slov. **49**, 659 (1999).

[2] M. Dakna, J. Clausen, L. Knöll, and D. -G. Welsch, Phys. Rev. A **59**, 1658 (1999); here the effects of non unit quantum efficiency of photodetectors are not taken into account.

[3] D. T. Pegg, L. S. Philips, S. M. Barnett, Phys. Rev. Lett. **81**, 1604 (1998); the authors employ a linear model (not avalanche) for photodetectors, thus overestimating the achievable fidelity.

[4] M. G. A. Paris, Phys. Rev. A **62**, 033815 (2000).

[5] M. G. A. Paris, Phys. Rev. A **59**, 1615 (1999).