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Verification of parametric sources of entanglement

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Abstract
We suggest a novel scheme for the verification of parametric entanglement sources. The scheme is based on the simple ON/OFF measurement of one of the two parties sharing the entangled state, and on the subsequent verification of the nonclassical character of the conditional state of the other party. The setup is suitable for verifying parametric sources using both photodetection and homodyne detection, depending on the amplifier gain. An experimental setup to implement the verification in the low-gain regime by photodetection is analysed in some detail.

Keywords: Entanglement, nonclassical light, parametric amplifiers

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Over the last five years we have witnessed an escalation in experimental quantum information, showing that entanglement is the key ingredient of the novel field of quantum technology. Entanglement is the essential resource for quantum computing, quantum teleportation, and secure cryptographic protocols. Entanglement improves optical spectroscopy [1], quantum lithography [2], and has been applied to the tomography of a quantum device [3], and the discrimination of unitary operations [4]. As a matter of fact, most of the successful experiments in quantum information have been performed in the quantum optical domain, e.g. both the teleportation of a single qubit and of a quantum harmonic oscillator state, which are so far the only known feasible schemes, teleport radiation states.

Optical sources of entanglement consist of parametric amplifiers. In the low-gain regime they are used to produce polarization entangled pairs, whereas in the high-gain regime the resulting output approaches the so-called twin-beam state of radiation. In this paper we present a conditional scheme to verify optical parametric sources of entanglement. The scheme can be applied both in the low- and high-gain regimes, using photodetection or homodyne detection in the final stage. Here, we will analyse in some detail the experimental setup, based on direct noise detection, to implement verification in the low-gain regime.

Our scheme is based on the fact that any measurement performed by one of the two parties sharing an entangled state *projects* the state at the disposal of the other party. In particular, we show that a simple ON/OFF photodetection of one of the beams exiting a parametric amplifier leaves the other beam in a highly nonclassical state. This nonclassicality is present only when the beams are entangled, such that its verification corresponds to the verification of the parametric source. The interest in the present scheme stems from the fact that the verification of nonclassicality, and in turn entanglement, is robust against any imperfections in the measurement scheme, such as the quantum efficiency of photodetectors.

The paper is structured as follows: in section 2 we describe the conditional measurement and illustrate, also in the presence of nonunit quantum efficiency, the properties of the projected state. In section 3 we analyse the measurement schemes aimed at verifying the nonclassicality of the projected state, whereas in section 4 the experimental implementation by photodetection is described in detail. Section 5 closes the paper with some concluding remarks.

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2. ON/OFF measurements and conditional dynamics

Let us consider an active $\chi^{(2)}$ crystal operating as a nondegenerate parametric amplifier (NOPA). The NOPA, pumped at a frequency of $\omega_P = \omega_a + \omega_b$, couples two modes a and b (idler and signal modes) via the medium nonlinearity. In the rotating-wave approximation, the evolution operator of the NOPA under phase-matching conditions can be written as $U_\lambda = \exp[\lambda(a^\dagger b^\dagger - ab)]$ where the 'gain' λ is proportional to the interaction time, the nonlinear susceptibility, and the pump intensity. For vacuum input, we have spontaneous parametric down conversion and the output state is given by the two-mode state

$$|\psi\rangle\rangle = \sqrt{1-x^2}\sum_{n=0}^{\infty} x^n |n\rangle_a \otimes |n\rangle_b \qquad x = \tanh\lambda. \tag{1}$$

The mean photon number of $|\psi\rangle\rangle$ is given by $N = 2\sinh^2\lambda = 2x^2/(1-x^2)$. We now consider the situation in which one of the two beams (say, mode a) is revealed by an ideal avalanche ON/OFF photodetector, i.e. a detector which has no output when no photon is detected and an output exceeding some threshold when one or more photons are detected. The action of an ON/OFF detector is described by the two-value positive operator-valued measure (POVM)

$$\Pi_0 \doteq \sum_{k=0}^{\infty}(1-\eta)^k|k\rangle\langle k| \qquad \Pi_1 \doteq \boldsymbol{I} - \Pi_0 \tag{2}$$

with η being the quantum efficiency of the avalanche photodetector. The outcome '1' (i.e. registering a 'click', corresponding to one or more incoming photons) occurs with probability

$$P_1 = \langle\langle\psi|\boldsymbol{I}\otimes\Pi_1|\psi\rangle\rangle = \frac{\eta x^2}{1-x^2(1-\eta)} = \frac{\eta N}{2+N\eta} \tag{3}$$

and correspondingly, the conditional output states for the mode a is given by

$$\varrho_1 = 1/P_1 \,\mathrm{Tr}_b[|\psi\rangle\rangle\langle\langle\psi|\boldsymbol{I}\otimes\Pi_1].$$

In the Fock basis we have

$$\varrho_1 = \frac{1-x^2}{P_1}\sum_{k=1}^{\infty} x^{2k}[1-(1-\eta)^k]|k\rangle\langle k|. \tag{4}$$

The density matrix in (4) describes a *pseudo*-thermal state, where the vacuum component has been removed by the conditional measurement. This means that the state is highly nonclassical [5], and that the nonclassicality is present only when the state exiting the amplifier is entangled. In the limit of very low gain $\lambda \ll 1$ the conditional state ϱ_1 approaches the number state $|1\rangle\langle 1|$ with one photon.

The nonclassicality of ϱ_1 is confirmed by the fact that the corresponding Wigner function is negative for any value of the gain λ and of the quantum efficiency η, and by the subPoissonian character of the number distribution, at least in the low-gain regime. In particular, the Wigner function of ϱ_1 in the origin of the phase space is given by

$$W(0) = -\frac{2}{\pi}\frac{1}{1+N}\frac{2+\eta N}{2+N(2-\eta)}, \tag{5}$$

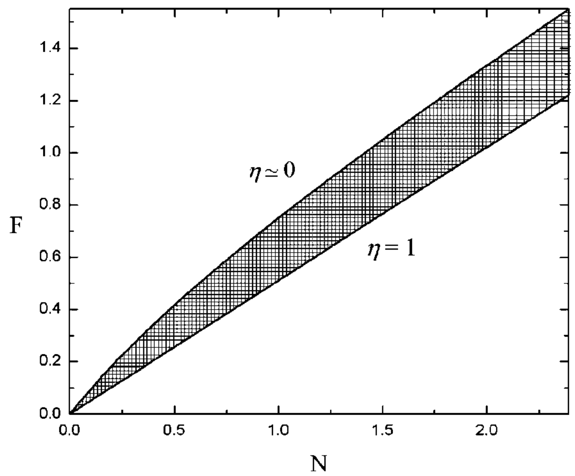

Figure 1. The Fano factor, F, as a function of the mean photon number, N, at the NOPA output and of the quantum efficiency of the ON/OFF detector. The two limiting curves correspond to $\eta \simeq 0$ and $\eta = 1$, respectively.

which is negative for any η and N. One can also see that the generalized Wigner function for s-ordering $W_s(\alpha) = -2/(\pi s)\int \mathrm{d}^2\gamma\, W_0(\gamma)\exp[2/s|\alpha-\gamma|^2]$ shows negative values for $s \in (-1, 0)$. In particular one has

$$W_s(0) = -\frac{2(1+s)}{\pi}\frac{2}{(1-s)(N+2)+N(1+s)} \times \frac{2+N\eta}{(1-s)(N+2)+N(1+s)(1-\eta)}. \tag{6}$$

If we take as a measure of nonclassicality the lowest index $s^\star$ for which W_s is a well behaved probability (regular, positive definite) [6] (6) says that for ϱ_1 we have $s^\star = -1$, i.e. ϱ_1 describes a nonclassical state as a number state.

The Fano factor of ϱ_1 is given by

$$F = \frac{1}{2}(2+N)\left(1+\frac{2}{2+N\,\eta} - \frac{4(2+N)}{4+N(4+N\eta)}\right). \tag{7}$$

This means roughly, that if η is not too low, then we have $F = N/2 + N(N-2)/(N+2)^2(1-\eta)$. In terms of the gain, we have that the beam b is subPoissonian for $\lambda \lesssim 0.9$. In figure 1 we show the Fano factor (the shaded region) as a function of the mean photon number N of $|\psi\rangle\rangle$ and of the quantum efficiency of the ON/OFF detector. The two limiting curves correspond to $\eta \simeq 0$ (upper curve) and $\eta = 1$, respectively.

2.1. Dark counts

In addition to quantum efficiency, i.e. lost photons, the performance of a realistic photodetector is also degraded by the presence of dark counts, i.e. by 'clicks' that do not correspond to any incoming photon. In order to take into account both these effects a real photodetector can be modelled as an ideal photodetector (unit quantum efficiency, no dark count) preceded by a beam splitter (of transmissivity equal to the quantum efficiency η) whose second port is in an auxiliary excited state ν, that can either be a thermal state or a phase-averaged coherent state, depending on the kind of background noise (thermal or Poissonian, respectively). If the second port of the beam splitter is the vacuum $\nu = |0\rangle\langle 0|$ we have no dark

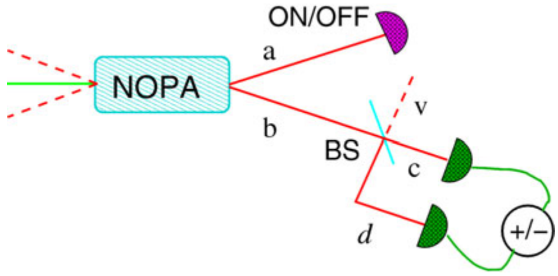

Figure 2. Schematic diagram of the setup for the entanglement verification of the low-gain NOPA by conditional ON/OFF photodetection on one of the output beams, and direct noise measurement of the Fano factor of the other beam.

count and the POVM of the photodetector is reduced to that of (2). For the second port of the BS excited by a thermal state or a phase-averaged coherent state (with $M = D/(1-\eta)$ mean photons, in order to reproduce a background noise with mean photon number D)

$$\nu_t = \frac{1}{M+1}\sum_s \left(\frac{M}{M+1}\right)^s |s\rangle\langle s| \qquad \nu_p = \mathrm{e}^{-M}\sum_s \frac{M^s}{s!}|s\rangle\langle s|, \tag{8}$$

we have $\Pi_0 = \sum_n F(D,\eta,n)|n\rangle\langle n|$ with

$$F_t(D,\eta,n) = \left(1 - \frac{\eta}{1+D}\right)^n \qquad F_p(D,\eta,n) = \mathrm{e}^{-D}(1-\eta)^n L_n\left(-\frac{D\eta}{1-\eta}\right), \tag{9}$$

where t and p denote thermal and Poissonian, respectively, and $L_n(x)$ is the Laguerre polynomial of order n. For small values of D the two models lead to the same detection probability at first order in $NP_1^t \simeq P_1^p = P_1 + \mathrm{O}[D^2]$ with

$$P_1 = \frac{\eta N}{2+N\eta} + \frac{4}{(2+N\eta)^2}D. \tag{10}$$

At optical frequencies the number of dark counts is negligible. Therefore, in the rest of this paper, we do not take into account the second-order term in (10).

3. Verification of the conditional state

The verification of nonclassicality can be performed, for any value of the gain, by checking the negativity of the Wigner function and, in the low-gain regime, also by verifying the subPoissonian character of ϱ_1. In this section, we first describe how to detect a negative Wigner function by homodyne detection, and then how to measure the Fano factor by direct noise detection. The experimental implementation of the latter scheme will be discussed in more detail in the next section.

The negativity of the Wigner function in the origin of the phase space can be revealed through quantum homodyne tomography. In fact, one has

$$W_s(0) = \mathrm{Tr}\,[\varrho_1 W_s] \qquad W_s = \frac{2}{\pi}\frac{1}{1-s}\left(\frac{s+1}{s-1}\right)^{a^\dagger a}, \tag{11}$$

and therefore [7]

$$W_s(0) = \int \mathrm{d}x\, p_{\eta_h}(x) R_{\eta_h}[W_s](x), \tag{12}$$

where $p_{\eta_h}(x)$ is the probability distribution of a random-phase homodyne detection (with quantum efficiency η_h) and $R_{\eta_h}[W_s](x)$ is the tomographic kernel for the operator W_s, which is given by

$$R_{\eta_h}[W_s](x) = \frac{2\eta_h}{\pi[(1-s)\eta_h - 1]}\Phi\left(1;\,1/2;\,-\frac{2\eta_h x^2}{(1-s)\eta_h - 1}\right), \tag{13}$$

where $\Phi(a,b;z)$ is the confluent hypergeometric function. $R_{\eta_h}[W_s](x)$ is a bounded function for $s < 1 - \eta_h^{-1}$ which represents the maximum index of the Wigner function $W_s(0)$ that can be reconstructed by homodyne tomography with efficiency η_h.

The measurement of the Fano factor on the second beam, conditional to a click in the ON/OFF photodetector on the first one, can be achieved following two different strategies. The first strategy is to perform the direct count of the photons and reconstruct the photon-number distribution, while the second one is to make a direct measurement of the Fano factor. In order to implement the first strategy we need a phototube endowed with a response proportional to the number of detected photons or a fast single-photon detector that is able to reveal the photons within the single pulse. Of course, the pulse must be of long enough duration to allow the use of such a detector: for instance if we use a detector with a time response of about 1 ns, the pulse must have a time duration of about 10 ns to discriminate the presence of two to three photons in it. If the pulse is shorter, this method is difficult to implement.

The second way is the so-called 'direct noise measurement' [8] and is based on the mixing of the signal to be measured with a vacuum field through a balanced beam splitter. A schematic diagram of the overall setup is depicted in figure 2. The mode b is mixed with the vacuum (mode v) in a balanced beam splitter and, at the output, the two modes are photodetected. A click in the ON/OFF detector works as trigger for the measurement stage, where both difference and sum photocurrents are electronically formed. After sufficient statistics the ratio between the noise in the sum and difference photocurrents yields the Fano factor of the beam under investigation. Let us see this in detail: after the beam splitter the modes b and v are transformed as follows:

$$c = \frac{b+v}{\sqrt{2}} \qquad d = \frac{b-v}{\sqrt{2}}. \tag{14}$$

Using two photodetectors we measure the intensities $I_c = \eta_o c^\dagger c$ and $I_d = \eta_o d^\dagger d$ and take the sum and the difference of these intensities $I_\pm = I_c \pm I_d$, where η_o is the quantum efficiency of the two detectors (which we suppose to be equal). By collecting a sufficiently large ensemble of measurements, we can build the variance of the measured quantities, i.e. ΔI_+ and ΔI_-, which represent the intrinsic noises of the photocurrents. The ratio between the noises reads

$$\frac{\Delta I_+^2}{\Delta I_-^2} = \frac{\langle[(c^\dagger c + d^\dagger d) - \langle c^\dagger c + d^\dagger d\rangle]^2\rangle}{\langle[(c^\dagger c - d^\dagger d) - \langle c^\dagger c - d^\dagger d\rangle]^2\rangle} = \frac{\langle[b^\dagger b - \langle b^\dagger b\rangle]^2\rangle}{\langle b^\dagger b\rangle} \equiv F, \tag{15}$$

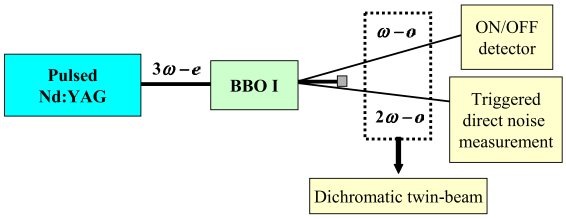

Figure 3. Block diagram of the experimental setup.

where $\langle\cdots\rangle$ denotes the ensemble average Tr $[\varrho\cdots]$ and, in the second equality, we have used the fact that the mode v is in the vacuum. We notice that the ratio in (15) gives the Fano factor of the mode b independently of the quantum efficiency η_o of the output photodetectors. Moreover, the subPoissonian character is independent of the quantum efficiency η of the conditional ON/OFF detector, and only depends on the amplifier gain. We conclude that the present scheme is robust against losses.

4. Experimental implementation

The experimental implementation of the scheme proposed in the previous section can be obtained by pumping a BBO crystal with the second or third harmonics of a Nd:YAG laser to produce entangled beams (either frequency degenerate or frequency nondegenerate) via spontaneous down conversion. If the crystal is cut for type I phase-matching interaction, the pump must be extraordinarily polarized, so that both the outgoing beams are ordinarily polarized. We proceed by tuning the crystal to optimize the noncollinear interaction between the three modes involved in the process. This can be obtained by maximizing the production of down-converted photons in one of the two desired modes by injecting a seed in the other with a given angle from the pump. We then can easily position two pinholes in the direction of the seed and of the down-converted signal so that, when the seed is switched-off, the photons trespassing the pinholes constitute the beams we want to use. A block diagram of the setup is reported in figure 3.

In order to evaluate the quantum regime in which the NOPA is operating, we calculate the levels of the gain (i.e. the number of generated photons), for noncollinear interaction at the end of the crystal [9]

$$2\Gamma = 2\sinh^2\left(\frac{|\gamma_P|}{2}z\right) = N, \qquad (16)$$

where $I_P \propto |\gamma_P|^2$ and $\gamma_P z/2 = \lambda$. In figure 4 we show the gain calculated according to (16) as a function of the BBO crystal thickness z for different input intensities.

The ON/OFF photodetector used to trigger the conditional measurement can be a single-photon avalanche photodiode, whose quantum efficiency does not influence the nonclassicality of the projected state. To minimize the noise in the measurement of this detector we suggest a gated measurement synchronized with the laser pulse. The direct noise measurement requires the use of two detectors with the same quantum efficiency endowed with a very high amplification factor, in order to produce a significant current output even in the presence of a few incident photons. Such detectors can be either amplified photodiodes or photomultipliers. Finally, we do not need to perform the analysis of the noise giving the Fano factor of the conditional state (see (15)) in the whole spectral range of the noise, but we can choose a restricted interval so as to minimize the influence of other noise sources and to keep only the meaningful intrinsic noise. This can be obtained with a spectrum analyser.

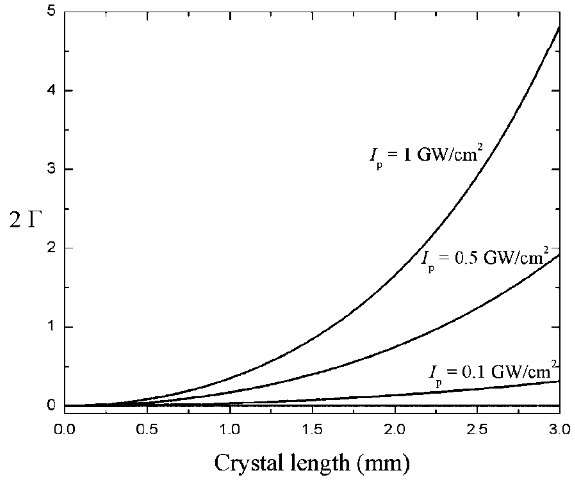

Figure 4. Gain of the parametric process in the BBO I crystal as a function of the crystal length for different input intensities ($I_p \propto |\gamma_p|^2$) for $\omega_P = 355$ nm, $\omega_a = 532$ nm and $\omega_b = 1.064$ nm. The interaction geometry is noncollinear and the angle between the pump and signal is $\theta_s \simeq 2.5°$ while the one between the pump and idler is $\theta_i \simeq 5°$.

5. Conclusions

We analysed a robust scheme for the verification of the entangled beams exiting a NOPA in the low-gain regime. The scheme consists of a conditional ON/OFF measurement performed by an avalanche photodiode on one of the beams, and of the subsequent verification of the nonclassical character of the other beam by direct noise detection of the Fano factor. The implementation of the scheme by a BBO crystal pumped by the second or third harmonic of a Nd:YAG laser is suggested.

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