Detection of mesoscopic entanglement

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Abstract

We suggest a feasible method, based on a homodyne-like detection scheme, to detect the degree of entanglement obtainable by mixing a couple of excited squeezed coherent states in a balanced beam splitter.

In the novel field of quantum technology, entanglement is a resource that can be exploited for the transmission and the manipulation of information. Actually, the entanglement between two photons has been widely investigated, both theoretically and experimentally. Entangled photon pairs have been used to test nonlocality of quantum mechanics, and to explore potential applications such as secure quantum key distribution and teleportation [1]. More recently, the experimental realization of continuous teleportation [2] raised attention to mesoscopic entanglement, namely the quantum correlations that can be established between two radiation beams containing many photons. Indeed, the continuous teleportation protocol relies on the mesoscopic entanglement obtained by mixing a couple of squeezed states in a balanced beam splitter. For these reasons, it is a matter of interest to quantify the entanglement between excited beams, and to devise detection schemes capable of revealing their degree of entanglement.

In experiments involving correlated photon pairs, entanglement is revealed by measuring the coincidence counting rate at the output, namely the fourth-order correlation function $K(\phi) = \langle\psi_{\rm OUT}|a^\dagger a\, b^\dagger b|\psi_{\rm OUT}\rangle$, where ϕ is a phase-shift between the two photon paths. However, when many photons are present, namely when we are dealing with mesoscopic entanglement, this corresponds to low fringes visibility, and thus we need a more sensitive kind of measurement. The homodyne-like detection of the output difference photocurrent $\langle\psi_{\rm OUT}|a^\dagger a - b^\dagger b|\psi_{\rm OUT}\rangle$ is widely used in interferometry and generally results in a very sensitive measurement scheme [3]. Starting from this consideration, here we suggest the squared difference photocurrent $H(\phi) = \langle\psi_{\rm OUT}|(a^\dagger a - b^\dagger b)^2|\psi_{\rm OUT}\rangle$ as a suitable fourth-order quantity to be measured at the output. Apart from the very low signals regime (the photon-pair regime) homodyne-like detection shows very high fringes visibility, thus providing a reliable detection scheme to reveal mesoscopic entanglement.

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In order to present explicit calculations, and to compare the two different measurement schemes, we focus our attention on the specific setup depicted in Fig. 1. First, a couple of degenerate optical parametric amplifiers (DOPAs) is employed to generate a couple of uncorrelated identical squeezed coherent state [4]

$$|\Psi_{in}\rangle = |\alpha, r\rangle_a |\alpha, r\rangle_b = \hat{D}_a(\alpha)\hat{D}_b(\alpha)\hat{S}_a(r)\hat{S}_b(r)|0\rangle$$

where $\hat{D}(\alpha) = \exp\{\alpha a^\dagger - \bar{\alpha} a\}$ is the displacement operator, and $\hat{S}(r) = \exp\{1/2 r^2 (a^{\dagger 2} - a^2)\}$ the squeezing operator.

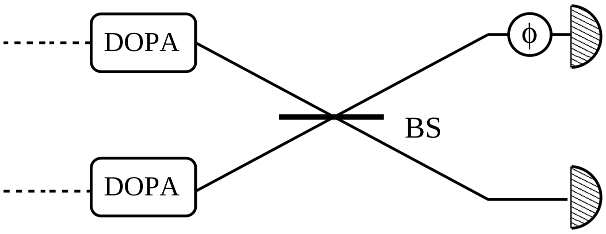

FIG. 1. Schematic diagram of the setup to generate and reveal states with different degree of mesoscopic entanglement.

The two squeezed states are then mixed in a balanced beam splitter, and the resulting output state, $\hat{U} = \exp\{i\pi/4(a^\dagger b + a b^\dagger)\}$ being the evolution operator of the balanced beam splitter,

$$|\Psi_{out}\rangle = \hat{U}|\Psi_{in}\rangle$$

ranges from a totally disentangled state to a maximally entangled state, depending on the squeezing fraction γ of each of the input state. This is defined as the fraction of the total number of photons engaged in squeezing $\gamma = \sinh^2 r/N$, $N = |\alpha|^2 + \sinh^2 r$ being the total number of photons of the state. In particular, for a couple of squeezed vacuum at the input $(\gamma = 1)$ the output state is given by the so-called twin-beam state

$$|\chi\rangle = \sqrt{1 - |\chi|^2} \sum_{n=0}^{\infty} \chi^n \, |n, n\rangle \tag{1}$$

where $\chi = \tanh r$. Twin-beam $|\chi\rangle$ represents a maximally entangled state that may contain a mesoscopic number of photons, we have

$$N_\chi = \langle\chi|a^\dagger a + b^\dagger b|\chi\rangle = 2|\chi|^2/(1 + |\chi|^2)^2 \, .$$

In general, a measure of the entanglement for pure state in provided by the normalized entropy of entanglement [5,6]

$$\epsilon = \frac{S[\hat{\varrho}_a]}{S[\hat{\varrho}_{th}]}$$

where $S[\hat{\varrho}] = -\mathrm{Tr}\{\hat{\varrho}\log\hat{\varrho}\}$ is the Von-Neumann entropy of the quantum state $\hat{\varrho}$, $\hat{\varrho}_a = \mathrm{Tr}_b\{|\Psi_{out}\rangle\langle\Psi_{out}|\}$ is the partial trace of the output state, and $\hat{\varrho}_{th}$ describes a thermal state (a maximum entropy state) with the same number of photon of the partial trace $\hat{\varrho}_a$. The degree of entanglement ϵ ranges from zero for uncorrelated states to unit for maximally entangled states. After some algebra we obtain for the degree of entanglement of $|\Psi_{out}\rangle$

$$\epsilon = \frac{\log(1+\gamma N) + \gamma N \log(1+\frac{1}{\gamma N})}{\log(1+N) + N\log(1+\frac{1}{N})} \ . \tag{2}$$

From Eq. (2) it is apparent that the degree of entanglement is an increasing function of the squeezing fraction, and that a maximum entangled state ($\epsilon = 1$) at the output is reached for a couple of squeezed vacuum ($\gamma = 1$) at the input. For highly excited states the entanglement is given by the asymptotic formula

$$\epsilon \overset{N\gg 1}{\simeq} 1 + \frac{\log\gamma}{\log N} \ . \tag{3}$$

We now study the visibility of the interference fringes that are observed, by varying the phase-shift ϕ between the two signals, in intensity measurements at the output. Besides being originated by interference effects, the variations in the quantities measured at the output also reflect the variations in the quantum correlations between the two output signals. In analogy with experiments involving correlated photon pairs, we may consider the detection of the coincidence counting rate at the output, namely of the fourth-order correlation function $K(\phi)$. However, as we will show in the following, this corresponds to low fringes visibility, and thus we sought for a more sensitive kind of measurement. Here, we consider the squared difference photocurrent $H(\phi) = \langle\Psi_{out}|(a^\dagger a - b^\dagger b)^2|\Psi_{out}\rangle$ as a suitable fourth-order quantity to be measured at the output of the interferometer. The fringes visibilities of both detection schemes are given by

$$V_K = \frac{K_{max} - K_{min}}{K_{max} + K_{min}} \qquad V_H = \frac{H_{max} - H_{min}}{H_{max} + H_{min}} \ . \tag{4}$$

In Fig. 2 we report V_K and V_H as a function of the intensity N for different values of the input squeezing fraction γ. The H-measurement visibility V_H is larger than V_K in almost all situations, with the exception of the very low signals regime, where very few photons are present. The behavior of fringes visibility versus intensity N also confirms that V_H represents a good measure of the entanglement at the output. As it happens for the degree of entanglement, in fact, a couple of squeezed vacuum at the input corresponds to maximum visibility $V_H = 1$ independently on the intensity. On the other hand, the coincidence counting rate shows a visibility V_K that rapidly decreases versus N, and saturates to a value well below 1/2. For non unit squeezing fraction, and moderate input intensities ($N < 10$), the behavior of V_H looks qualitatively similar to that of the degree of entanglement, whereas again V_K rapidly decreases. Remarkably, for highly excited states $N > 10$, the visibility V_H has the same asymptotic dependence of the degree of entanglement ϵ, in formula

$$\epsilon \overset{N\gg 1}{\simeq} 1 + \frac{A(\gamma)}{\log N} \ , \tag{5}$$

where the proportionality constant $A(\gamma) \simeq 1/5 \log\gamma$ is roughly proportional to that appearing in Eq. (3).

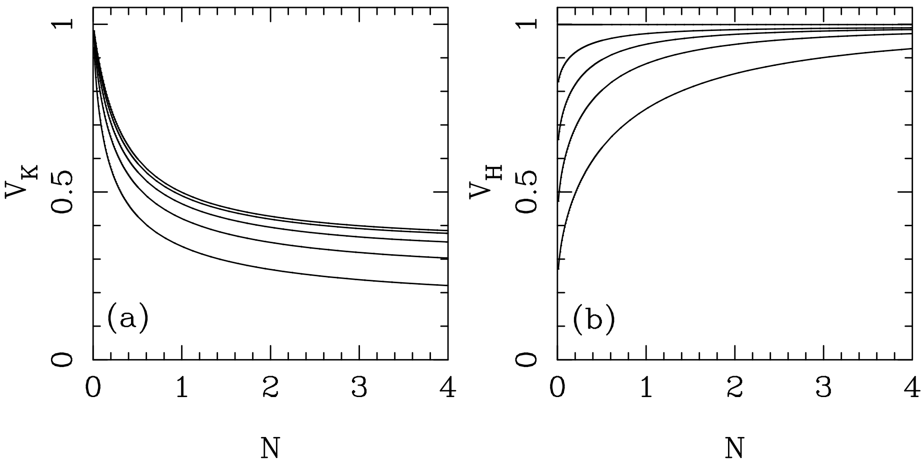

FIG. 2. Fringes visibility as a function of the intensity N for different values of the input squeezing fraction γ. In (a) the visibility of K-measurement V_K, and in (b) the visibility of H-measurement V_H . In both plots we report the visibility versus N for five values of the input squeezing fraction. From bottom to top we have the curves for $\gamma = 0.2, 0.4, 0.6, 0.8,$ and 1.0. As it is apparent, V_H is larger than V_K in almost all situations, with the exception of the very low signals regime.

In conclusion, we have analytically evaluated the degree of entanglement at the output of a balanced beam splitter fed by a couple of squeezed coherent states. By varying the input energy, we can produce entangled states of arbitrary large intensity, whereas the degree of entanglement can be tuned by varying the input squeezing fraction. We have suggested an effective experimental characterization of the output entanglement through the measurement of the squared difference photocurrent between the output modes. The interference fringes that are observed by varying the phase-shift between the signals show, in fact, high visibility for the whole range of input squeezing parameter.

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