J. Opt. B: Quantum Semiclass. Opt. **1** (1999) 662–667. Printed in the UK PII: S1464-4266(99)05041-7

Generation of mesoscopic quantum superpositions through Kerr-stimulated degenerate downconversion

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Received 8 June 1999, in final form 20 September 1999

Abstract. A two-step interaction scheme involving $\chi^{(2)}$ and $\chi^{(3)}$ nonlinear media is suggested for the generation of Schrödinger cat-like states of a single-mode optical field. In the first step, a weak coherent signal undergoes a self-Kerr phase modulation in a $\chi^{(3)}$ crystal, leading to a Kerr *kitten*, namely a microscopic superposition of two coherent states with opposite phases. In the second step, such a Kerr kitten enters a $\chi^{(2)}$ crystal and, in turn, plays the role of a quantum seed for stimulated phase-sensitive amplification. The output state in the above-threshold regime consists in a quantum superposition of mesoscopically distinguishable squeezed states, i.e. an optical cat-like state. The whole setup does not rely on conditional measurements, and is robust against decoherence, as only weak signals interact with the Kerr medium.

Keywords: Quantum superpositions, Kerr interactions

1. Introduction

One of the most striking features of quantum mechanics is perhaps the superposition principle. According to this principle, we cannot speak of an objective state of a physical system prior to a measurement. Rather, we should admit that a quantum system may be described as a superposition of classically (mesoscopically or macroscopically) distinguishable states. Such kind of states are often called Schrödinger cat states, following the original *gedanken* experiment to obtain a superposition of a live and a dead cat [1].

Many proposals have been made for the generation of Schrödinger cat-like states in different systems. Among these, we mention vibrating molecules or crystals [2, 3], trapped ions [4] and Bose condensates [5]. Experimental realization of cat-like states has been reported in the case of trapped ions [6].

A number of schemes have also been suggested with the aim of generating *optical* Schrödinger cats, namely superpositions of mesoscopically distinguishable states of the radiation field [7, 8]. Among these, we mention schemes based on conditional measurements performed on entangled states [10–13], nonlinear birefringence [14], or on self-Kerr phase modulation taking place in $\chi^{(3)}$ nonlinear crystals [15, 16]. In the framework of cavity QED, it has been shown that conditional measurement on atoms exiting a high-Q cavity may force, in a cat-like state, the radiation inside the cavity [10]. The state collapse due to photodetection has also been suggested as a source of cat-like states, both in a back-action evading scheme [11], and on a mode exiting a beam splitter fed by a squeezed vacuum state [13, 17].

Among these proposals, interaction schemes based on the Kerr effect have a specific advantage: they do not rely on conditional measurements. Actually, Kerr-based schemes were suggested earlier for the generation of cat states [15]. In a Kerr medium, the state evolution is governed by the interaction Hamiltonian

$$\hat{H}_K = \lambda (a^\dagger a)^2, \tag{1}$$

where λ is a coupling constant proportional to the nonlinear susceptibility of the medium. A coherent input signal $|\alpha\rangle$ evolves according to $|\psi_c\rangle = \exp\{-\mathrm{i}\hat{H}_K t\}|\alpha\rangle$, so that for a time interaction equal to $t = \pi/(2\lambda)$ the output state is given by

$$\begin{aligned}|\psi_c\rangle &= \frac{1}{\sqrt{2}}[\mathrm{e}^{-\mathrm{i}\pi/4}|\alpha\rangle + \mathrm{e}^{\mathrm{i}\pi/4}|-\alpha\rangle] \\ &= \frac{1}{\sqrt{2}}[\mathrm{e}^{-\mathrm{i}\pi/4}\hat{D}(\alpha) + \mathrm{e}^{\mathrm{i}\pi/4}\hat{D}(-\alpha)]|0\rangle,\end{aligned} \tag{2}$$

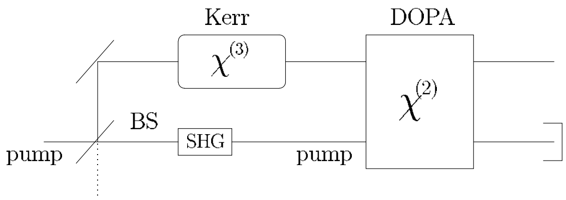

Figure 1. Schematic diagram of the scheme suggested for the generation of optical Schrödinger cat-like states. A strong coherent signal, provided by a stable laser source, impinges onto the nearly transparent beam splitter BS. As a result, the main part of the signal remains untouched, and continues, after frequency doubling (SHG), to form the pumping mode of the DOPA. The second output from the beam splitter BS consists of a very weak coherent signal, which interacts with the Kerr medium and evolves to a superposition of coherent states. Such a microscopic superposition enters the DOPA, and it is amplified to a superposition of mesoscopically distinguishable states.

$\hat{D}(\alpha) = \exp\{\alpha a^\dagger - \bar{\alpha} a\}$ being the displacement operator. The state in equation (2) describes a superposition of two coherent states with opposite phases. As far as $|\alpha|$ becomes a large quantity the two components become mesoscopically distinguishable states of the radiation field. Unfortunately, realistic values of Kerr nonlinear susceptibilities are quite small, thus requiring a long interaction time, or equivalently a large interaction length. As a consequence, losses become significant, and the resulting decoherence [18–21] may destroy the quantum superposition in equation (2). For these reasons, Kerr-based schemes are not generally considered to be realistic [8]. However, as will be shown in section 2, in the case of weak input signals the effects of losses are not so critical, and the output state approaches the ideal superposition of equation (2). In this case, the superposition is only microscopic (Kerr *kitten*), as the two components cannot be considered classically distinguishable. However, such a Kerr kitten may serve as a quantum seed [9], which leads to a Schrödinger cat-like state after suitable amplification.

In this paper, we pursue this idea and consider the setup depicted in figure 1: a nearly transparent beam splitter is fed by a strong coherent signal provided by a stable laser source. After the beam splitter the main part of the signal remains untouched, and continues, after doubling of frequency, to form the pumping mode of a degenerate optical parametric amplifier (DOPA). The other output from the beam splitter consists of a very weak coherent signal, which is allowed to interact with the Kerr medium, thus evolving to a superposition of coherent states. Such a microscopic superposition enters the DOPA, and it is amplified to a superposition of mesoscopically distinguishable states, namely a Schrödinger cat-like state. As we will show in the following, the whole scheme is robust against decoherence. This is due mainly to the fact that only a weak signal interacts with the Kerr medium, and, correspondingly, only a microscopic superposition stimulates the amplification process.

The paper is structured as follows: in the next section the dynamics of a Kerr medium in presence of losses is analysed, and its effectiveness in producing a Kerr kitten is demonstrated. In section 3 we study the phase-sensitive amplification of a Kerr kitten by DOPA. Effects of losses are taken into account, and the appearance of cat-like states at the output is demonstrated. In section 4 we consider the amplification of the actual Kerr output (which slightly differs from an ideal kitten), and we prove that the treatment of section 3 provides a reliable description of the amplification process. Section 5 closes the paper by summarizing the results.

2. Kerr interaction with weak signal

The dynamics of a Kerr medium in the presence of losses is governed by the master equation

$$\frac{\mathrm{d}\hat{\varrho}}{\mathrm{d}t} = -\mathrm{i}[\hat{H}_K, \hat{\varrho}] + \Gamma[a\hat{\varrho}a^\dagger - \tfrac{1}{2}a^\dagger a\hat{\varrho} - \tfrac{1}{2}\hat{\varrho}a^\dagger a], \tag{3}$$

where $\hat{H}_K$ is the interaction Hamiltonian of equation (1), and Γ denotes the damping rate of the optical cavity. In terms of the Fock matrix elements $\varrho_{p,q} = \langle p|\hat{\varrho}|q\rangle$ equation (3) can be re-expressed as follows:

$$\dot{\varrho}_{p,q} = -\mathrm{i}\frac{\lambda}{\Gamma}(p^2 - q^2)\varrho_{p,q} + \sqrt{(1+p)(1+q)}\varrho_{1+p,1+q} - \tfrac{1}{2}(p+q)\varrho_{p,q}, \tag{4}$$

where the overdot denotes the derivative with respect to the rescaled time $\tau = \Gamma t$. We solve equation (4) with the signal initially in a coherent state $\varrho_0 = |\alpha\rangle\langle\alpha|$, and look for a solution of the form

$$\varrho_{p,q}(\tau) = \frac{\alpha^p\bar{\alpha}^q}{\sqrt{p!q!}}A_{p,q}(\tau). \tag{5}$$

Inserting equation (5) in (4) we arrive at the following equation:

$$\dot{A}_{p,q}(\tau) = -\tfrac{1}{2}(p+q)\Delta A_{p,q}(\tau) + |\alpha|^2 A_{1+p,1+q}(\tau), \tag{6}$$

which should be solved with the initial condition $A_{p,q}(0) = \exp\{-|\alpha|^2\}$, and where Δ is given by

$$\Delta = 1 + \frac{2\mathrm{i}\lambda}{\Gamma}(p-q). \tag{7}$$

Equation (6) suggests a solution of the form

$$A_{p,q}(\tau) = \exp\{-\tfrac{1}{2}(p+q)\Delta\tau - |\alpha|^2 B_{p,q}(\tau)\}, \tag{8}$$

where $B_{p,q}(\tau)$ only depends on the difference $p - q$. By substituting equation (8) into (6) we arrive at the simple relation $\dot{B}_{p,q}(\tau) = -\exp\{-\Delta\tau\}$, which, with the initial condition $B_{p,q}(0) = 1$, leads to the solution

$$B_{p,q}(\tau) = \frac{1}{\Delta}(\mathrm{e}^{-\Delta\tau} - 1) + 1, \tag{9}$$

and thus

$$A_{p,q}(\tau) = \exp\left\{-\frac{1}{2}(p+q)\Delta\tau - |\alpha|^2\left[1 - \frac{1-\mathrm{e}^{-\Delta\tau}}{\Delta}\right]\right\}. \tag{10}$$

Finally, the matrix elements are obtained by substitution of equation (10) into equation (5).

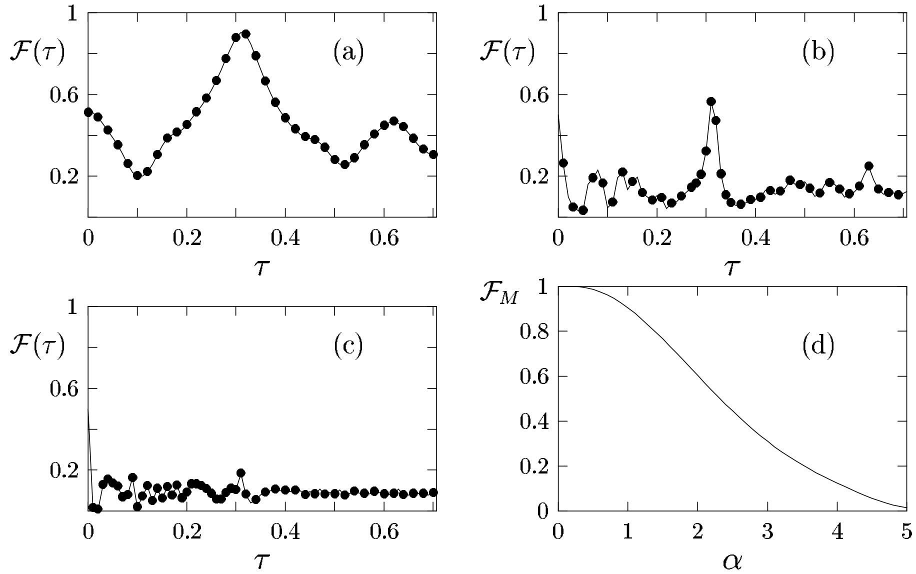

Figure 2. The fidelity $\mathcal{F}(\tau)$ as a function of the rescaled time τ and for a ratio between nonlinear coupling and loss parameter equal to $\lambda/\Gamma = 10$. In (*a*), (*b*) and (*c*) we show the fidelity for the initial coherent amplitude equal to $|\alpha| = 1, 2, 3$, respectively. In (*d*) we show the maximum fidelity as a function of the amplitude of the input signal.

This solution should be compared with the ideal cat state $\hat{\nu} = |\psi_c\rangle\langle\psi_c|$ of equation (2), whose density matrix elements are given by

$$\nu_{n,m} = \frac{1}{2}\mathrm{e}^{-|\alpha|^2}\frac{\alpha^n\bar{\alpha}^m}{\sqrt{n!m!}}[1 + \mathrm{i}(-1)^n][1 - \mathrm{i}(-1)^m]. \qquad (11)$$

In order to make such a comparison, we consider the fidelity

$$\mathcal{F}(\tau) = \mathrm{Tr}\{\hat{\varrho}(\tau)\hat{\nu}\} = \langle\psi_c|\hat{\varrho}(\tau)|\psi_c\rangle, \qquad (12)$$

between the ideal 'cat' density matrix and the state coming from realistic evolution. The fidelity ranges from zero to unity as $\hat{\nu}$ actually describes a pure state.

In figure 2 we show the behaviour of $\mathcal{F}(\tau)$ as a function of the rescaled time τ for realistic values of the loss parameter and nonlinear coupling, and for different values of the initial coherent amplitude. The fidelity oscillates as a function of τ, and the maximum $\mathcal{F}_M$ individuates the optimal working regime for the generation of superpositions. The value of $\mathcal{F}_M$ decreases as a function of the input amplitude α, approximately as $\mathcal{F}_M \simeq \exp\{-|\alpha|^2/8\}$, so that $\mathcal{F}_M \geqslant 0.95$ only for $|\alpha| \leqslant 1$. This means that cat states of the form (2) can be reliably generated only for weak signals at the input, namely that the Kerr interaction can be successfully employed to produce the optical kitten, i.e. superposition of weakly excited coherent states. It should also be mentioned that the Kerr effect is inherently a weak process. Therefore, in order to make it appreciable in the quantum limit of weak coherent signals, short pulses should be used.

3. Phase-sensitive amplification of a Kerr kitten

A DOPA consists of a $\chi^{(2)}$ nonlinear optical crystal cut for type-I phase matching. The crystal couples a signal mode a, at frequency ω_a, with a pump mode c at double frequency $\omega_c = 2\omega_a$. Each photon in the pump mode produces a photon pair in the signal mode, thus leading to phase-sensitive amplification, sub-Poissonian statistics, and squeezing [22]. In the rotating wave approximation, and under phase-matching conditions, the effective Hamiltonian of a DOPA can be written as

$$\hat{H}_S = \tfrac{1}{2}\lambda(\bar{z}a^{\dagger 2} + za^2), \qquad (13)$$

λ being a coupling constant proportional to the nonlinear susceptibility, and z the amplitude of the pump mode. The evolution operator $\hat{U}_S = \exp\{-\mathrm{i}\hat{H}_S t\}$ coincides with the squeezing operator $\hat{S}(\zeta) = \exp\{\frac{1}{2}(\zeta^2 a^{\dagger 2}) - \bar{\zeta}^2 a^2\}$, where $\zeta = \sqrt{\lambda|z|\tau}\,\mathrm{e}^{\mathrm{i}(\pi-\phi_z)/2}$; ϕ_z being the phase of the pumping mode.

Let us consider the DOPA fed by the Kerr kitten exiting the $\chi^{(3)}$ medium. The output state from the DOPA can be easily evaluated by means of equation (2) and using the following formula for squeezing and displacement operators:

$$\begin{gathered}\hat{S}(\zeta)\hat{D}(\alpha) = \hat{D}(\mu\alpha + \nu\bar{\alpha})\hat{S}(\zeta)\\ \mu = \cosh|\zeta|, \qquad \nu = \sinh|\zeta|\mathrm{e}^{2\mathrm{i}\arg\zeta}.\end{gathered} \qquad (14)$$

In particular, as both signal and pump mode originated from the same source, we can take both $\zeta = r$ and $\alpha = x_0$ as real numbers. In this case the output state is given by

$$\begin{aligned}|\psi_{\mathrm{out}}\rangle &= \frac{1}{\sqrt{2}}\hat{S}(r)[\mathrm{e}^{-\mathrm{i}\pi/4}\hat{D}(x_0) + \mathrm{e}^{\mathrm{i}\pi/4}\hat{D}(-x_0)]|0\rangle\\ &= \frac{1}{\sqrt{2}}[\mathrm{e}^{-\mathrm{i}\pi/4}|x_0\mathrm{e}^r, r\rangle + \mathrm{e}^{\mathrm{i}\pi/4}|-x_0\mathrm{e}^r, r\rangle],\end{aligned} \qquad (15)$$

$|\alpha, r\rangle = \hat{D}(\alpha)\hat{S}(r)|0\rangle$ being a squeezed coherent state of amplitude α and squeezing parameter r. Besides squeezing, the DOPA amplifies the coherent amplitude of the two components by the relevant factor e^r, making them mesoscopically distinguishable. Therefore, the state in equation (15) approaches a cat-like state of a single-mode radiation field in the limit of large amplification gain.

In order to also prove the effectiveness of the process in realistic situations, we proceed by taking into account the effect of losses. In this case, the dynamics of the DOPA is governed by a master equation of the form (3), with $\hat{H}_K$ replaced by $\hat{H}_S$. Such an equation can be converted into a Fokker–Planck differential equation for the Wigner function of the signal mode, which is defined as follows:

$$W(x, y) = \int_{\mathbb{R}} d\mu \int_{\mathbb{R}} d\nu \exp\{-2i(\nu x - \mu y)\} \operatorname{Tr}\{\hat{\varrho}\hat{D}(\mu + i\nu)\}. \tag{16}$$

Using the differential representation of all the operators in (3), the Fokker–Planck equation for $W(x, y)$ reads as follows:

$$\partial_\tau W_\tau(x, y) = [\tfrac{1}{8}\partial^2_{xx} + \tfrac{1}{8}\partial^2_{yy} + \gamma_x \partial_x x + \gamma_y \partial_y y] W_\tau(x, y), \tag{17}$$

where differentiation is with respect to the rescaled time $\tau = \Gamma t$, and the quantities γ_x and γ_y are given by ($\kappa = \lambda|z|$)

$$\gamma_x = \frac{1}{2}\left(1 - \frac{2\kappa}{\Gamma}\right), \qquad \gamma_y = \frac{1}{2}\left(1 + \frac{2\kappa}{\Gamma}\right). \tag{18}$$

The solution of equation (17) can be written as

$$W_\tau(x, y) = \int_{\mathbb{R}} dx' \int_{\mathbb{R}} dy' \, W_0(x', y') G_\tau(x|x') G_\tau(y|y'), \tag{19}$$

where $W_0(x, y)$ is the Wigner function of the input signal, and the Green functions $G_\tau(x_j|x'_j)$, $j = x, y$ are given by

$$G_\tau(x_j|x'_j) = \frac{1}{\sqrt{2\pi\sigma_j^2}} \exp\left[-\frac{(x_j - x'_j e^{-\gamma_j\tau})^2}{2\sigma_j^2}\right] \tag{20}$$

$$\sigma_j^2 = \frac{1}{8\gamma_j}[1 - \exp(-2\gamma_j\tau)]. \tag{21}$$

The initial Wigner function, corresponding to the Kerr kitten, is given by

$$W_0(x, y) = \frac{1}{\pi} e^{-2y^2}[e^{-2(x-x_0)^2} + e^{-2(x+x_0)^2} - 2e^{-2x^2}\sin(4x_0 y)], \tag{22}$$

whereas the evolute Wigner function at time τ is obtained through equation (19). After a tedious but straightforward integration one has

$$W_\tau(x, y) = \frac{1}{2} \frac{\exp\left\{-\frac{y^2}{2\Sigma_y^2}\right\}}{\sqrt{2\pi\Sigma_y^2}} \times \left[\frac{\exp\left\{-\frac{(x - x_0 e^{-\gamma_x\tau})^2}{2\Sigma_x^2}\right\}}{\sqrt{2\pi\Sigma_x^2}} + \frac{\exp\left\{-\frac{(x + x_0 e^{-\gamma_x\tau})^2}{2\Sigma_x^2}\right\}}{\sqrt{2\pi\Sigma_x^2}} - 2\frac{\exp\left\{-\frac{x^2}{2\Sigma_x^2}\right\}}{\sqrt{2\pi\Sigma_x^2}} \exp\left\{-2x_0^2 \frac{\sigma_y^2}{\Sigma_y^2}\right\} \sin\left(\frac{x_0 y}{\Sigma_y^2} e^{-\gamma_y\tau}\right)\right], \tag{23}$$

where the Σ_j^2, $j = x, y$ are given by

$$\Sigma_x^2 = \sigma_x^2 + \frac{1}{4}e^{-2\gamma_x\tau} = \frac{1}{8\gamma_x}\left[\frac{2\kappa}{\Gamma}e^{-2\gamma_x\tau} - 1\right]$$
$$\Sigma_y^2 = \sigma_y^2 + \frac{1}{4}e^{-2\gamma_y\tau} = \frac{1}{8\gamma_y}\left[\frac{2\kappa}{\Gamma}e^{-2\gamma_y\tau} + 1\right]. \tag{24}$$

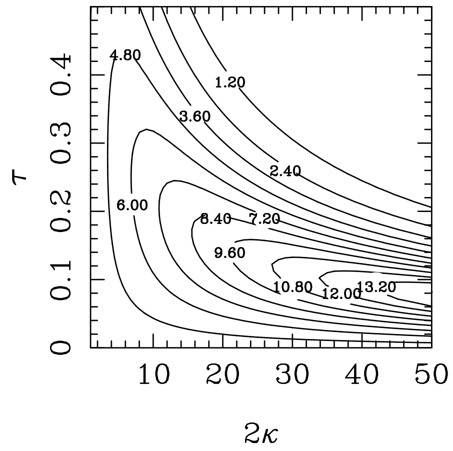

Figure 3. Contour plot of the factor $\mathcal{O} = \exp(-\gamma_y\tau)/\Sigma_y^2$ in the oscillatory term of the Wigner function as a function of the interaction time τ and the amplifier gain 2κ. Both τ and κ are in units of cavity damping γ. The factor $\mathcal{O}$ increases with κ and decreases with τ, whereas the region of maximum values is approximately located around $\tau \sim (2\kappa)^{-1}$.

The evolute Wigner function in equation (23) should be compared with the ideal one $W_0(xe^{-r}, ye^r)$, corresponding to amplification without losses, i.e. to the quantum superposition $|\psi_{out}\rangle$ in equation (15). Such a comparison deserves some comments. Formally, in the limit $2\kappa/\Gamma \gg 1$, when the DOPA works well above threshold, $W_\tau(x, y)$ approaches $W_0(xe^{-r}, ye^r)$. However, such a requirement is not sufficient from the physical point of view, as the further requirement of a short interaction time should also be fulfilled. This is in order for two reasons. On the one hand, one should avoid saturation effects and, on the other hand, the interaction time should match the duration of the input pulse. As already mentioned, the kitten pulses coming from the $\chi^{(3)}$ medium are unavoidably short, in order to make Kerr interaction appreciable in the quantum limit of weak signals. We estimate an effective value for the interaction time on the basis of the following argument. The quantum features that make the output signal distinguishable from a statistical mixture are governed by the oscillatory term in the Wigner function (23). In turn, such a term consists of two factors. The damping factor $\exp(-2x_0^2\sigma_y^2/\Sigma_y^2)$ varies monotonically with κ and τ, rapidly approaching the saturation value $\exp(-4x_0^2)$. On the other hand, the oscillations depend on the factor $\mathcal{O} = \exp(-\gamma_y\tau)/\Sigma_y^2$ which modulates the argument of the sine function. The behaviour of such a factor as a function of κ and τ is shown in figure 3: the maximum value at fixed gain is obtained for $\tau \sim (2\kappa)^{-1}$, which represents an optimized choice for the time interaction. The same condition qualitatively states how far the state can be amplified without destroying its quantum features. Remarkably, the dependence of $\mathcal{O}$ on the interaction time is not dramatic. Therefore, a slightly larger interaction time does not substantially modify the picture, while allowing for amplification of the two components to mesoscopic scale.

4. Phase-sensitive amplification of the Kerr output

In the previous section we demonstrated the appearance of cat-like states at the output of a DOPA fed by a Kerr kitten. However, one should take into account that the output from the Kerr medium, and thus the input for the DOPA, is actually described by the density matrix of equations (5) and (10), which may slightly differ from the ideal Kerr kitten of equation (2). Although for small amplitude the two states are very close, as proved in section 2, one may still argue that the amplification via the DOPA could make this small difference significant. The purpose of this section is to prove that this is not actually the case, and that the solution of the previous section reliably describes the dynamics of the proposed setup. Indeed, we will prove that the system is not hyper-sensitive to the initial condition, i.e. it does not exhibit any chaotic-like behaviour.

For sake of convenience, let us slightly change our notation and denote by $W_\tau^{\rm c}(x,y)$ the Wigner function of equation (23), namely the Wigner function of the output state from the DOPA, as obtained by evolving the ideal Kerr kitten of equation (2). Analogously, we denote by $W_\tau^{\rm k}(x,y)$ the Wigner function of the output state obtained by evolving the real output from the Kerr medium, which may be expressed as

$$W_\tau^{\rm k}(x,y) = \int_{\mathbb{R}} {\rm d}x' \int_{\mathbb{R}} {\rm d}y'\, W_0^{\rm k}(x',y') G_\tau(x|x') G_\tau(y|y'), \tag{25}$$

where $W_0^{\rm k}(x',y')$ is the Wigner function of the state exiting the Kerr medium. We compare the two output in term of their fidelity $\mathcal{F}_\tau$, which, in turn, may be expressed in terms of their Wigner functions

$$\mathcal{F}_\tau = \int_{\mathbb{R}} {\rm d}x \int_{\mathbb{R}} {\rm d}y\, W_\tau^{\rm c}(x,y) W_\tau^{\rm k}(x,y). \tag{26}$$

Inserting equations (19) and (25) into equation (26) we have

$$\begin{aligned}\mathcal{F}_\tau = &\int_{\mathbb{R}} {\rm d}x_1 \int_{\mathbb{R}} {\rm d}y_1 \int_{\mathbb{R}} {\rm d}x_2 \int_{\mathbb{R}} {\rm d}y_2\, W_0^{\rm c}(x_1,y_1) \\ &\times W_0^{\rm k}(x_2,y_2) K_\tau(x_1,x_2) K_\tau(y_1,y_2),\end{aligned} \tag{27}$$

where

$$\begin{aligned}K_\tau(x_1,x_2) &= \int_{\mathbb{R}} {\rm d}x\, G_\tau(x|x_1) G_\tau(x|x_2) \\ &= \frac{1}{\sqrt{4\pi\sigma_x^2}} \exp\left\{-\frac{(x_1-x_2)^2}{4\sigma_x^2 {\rm e}^{\gamma_x \tau}}\right\}\end{aligned} \tag{28}$$

$$\begin{aligned}K_\tau(y_1,y_2) &= \int_{\mathbb{R}} {\rm d}y\, G_\tau(y|y_1) G_\tau(y|y_2) \\ &= \frac{1}{\sqrt{4\pi\sigma_y^2}} \exp\left\{-\frac{(y_1-y_2)^2}{4\sigma_y^2 {\rm e}^{\gamma_y \tau}}\right\}.\end{aligned} \tag{29}$$

By separating the integral over the x_1, y_1 variables we may write

$$\mathcal{F}_\tau = \int_{\mathbb{R}} {\rm d}x_2 \int_{\mathbb{R}} {\rm d}y_2\, W_0^{\rm k}(x_2,y_2) H_\tau(x_1,x_2), \tag{30}$$

where

$$\begin{aligned}H_\tau(x_2,y_2) = &\int_{\mathbb{R}} {\rm d}x_1 \int_{\mathbb{R}} {\rm d}y_1\, W_0^{\rm c}(x_1,y_1) \\ &\times K_\tau(x_1,x_2) K_\tau(y_1,y_2).\end{aligned} \tag{31}$$

In the regime of short interaction time, we have

$$\begin{aligned}K_\tau(x_1,x_2) &\simeq \frac{1}{\pi\tau} \exp\left\{-\frac{(x_1-x2)^2}{\tau}\right\} \\ K_\tau(y_1,y_2) &\simeq \frac{1}{\pi\tau} \exp\left\{-\frac{(y_1-y2)^2}{\tau}\right\},\end{aligned} \tag{32}$$

such that, using equations (22) and (32), we have

$$\begin{aligned}H_\tau(x_2,y_2) = &\frac{1}{\pi(2\tau+1)} {\rm e}^{-\frac{2y_2^2}{2\tau+1}} \\ &\times \left[\exp\left(-\frac{2(x_2-x_0)^2}{2\tau+1}\right) + \exp\left(-\frac{2(x_2+x_0)^2}{2\tau+1}\right)\right. \\ &\left. - 2\exp\left(\frac{-4x_0^2\tau}{2\tau+1}\right) \exp\left(-\frac{2x_2^2}{2\tau+1}\right) \sin\left(\frac{4x_0 y_2}{2\tau+1}\right)\right] \\ = &\, W_0^{\rm c}(x_2,y_2) + \left.\frac{\partial H_\tau(x_2,y_2)}{\partial\tau}\right|_{\tau=0} \tau + {\rm O}[\tau^2].\end{aligned} \tag{33}$$

Finally, upon inserting equation (33) into (30) we arrive at

$$\mathcal{F}_\tau = \mathcal{F}_0 + \tau\mathcal{K} + {\rm O}[\tau^2], \tag{34}$$

where $\mathcal{F}_0$ is the fidelity between the two inputs (very close to unity, as proved in section 2) and $\mathcal{K}$ is a bounded constant†. Equation (34) shows that the departure from the initial fidelity is linear in the interaction time, and thus negligible in the relevant working regime pertaining to the scheme.

5. Summary

In this paper we have suggested a two-step scheme, involving self-Kerr phase modulation and phase-sensitive parametric amplification, in order to generate cat-like states of a single-mode optical field. In the first step a weak coherent signal passes through a Kerr medium, leading to a superposition of weakly excited coherent states. The small amplitude of the signal assures that decoherence does not play a significant role. On the other hand, as the Kerr effect is inherently a weak process, this also requires one to work with short pulses. The Kerr kitten is then used to stimulate phase sensitive amplification in a DOPA, such that the output state approaches a superposition of mesoscopically distinguishable squeezed coherent states, i.e. a cat-like state of radiation field. Also, this second step is robust against decoherence, provided the amplifier operates in the above-threshold regime with a short interaction time. An effective value for the interaction time has been obtained by maximizing the oscillatory term in the Wigner function.

Acknowledgments

The author would thank G M D'Ariano, F De Martini, L Maccone, O Haderka, M Sacchi and V De Renzi for interesting discussions. This work is cosponsored by CNR and NATO through the Advanced Fellowship Programme 1998.

† By expanding $H_\tau(x_2,y_2)$ for small interaction time we have

$$\mathcal{K} = \int_{\mathbb{R}} {\rm d}x_2 \int_{\mathbb{R}} {\rm d}y_2\, W_0^{\rm k}(x_2,y_2) \left.\frac{\partial H_\tau(x_2,y_2)}{\partial\tau}\right|_{\tau=0}.$$

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