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Increasing the Visibility of Multiphoton Entanglement

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Abstract

The entanglement between the two beams exiting a Mach-Zehnder interferometer fed by two squeezed states involves many photons. We evaluate the degree of this multiphoton entanglement in terms of the excess entropy of the pure state at the output. A novel kind of fourth-order correlation function based on homodyne detection is suggested to reveal entanglement. Apart from the very low signals regime homodyne-like detection shows a higher fringes visibility than the measurement of the coincidence counting rate.

Entanglement is at the basis of modern applications of quantum mechanics to the transmission and the manipulation of information. In the last decade, the entanglement between two photons has been widely investigated, both theoretically and experimentally. Entangled photon pairs have been used to test nonlocality of quantum mechanics, and to explore potential applications such as secure quantum key distribution and teleportation [1]. More recently, the experimental realization of continuous teleportation [2] raised attention to multiphoton quantum correlations, namely to the mesoscopic entanglement that can be established between two radiation beams containing many photons. The idler and the signal beams exiting a parametric down-converter, or the two beams exiting a Mach-Zehnder interferometer fed by squeezed states, are examples of systems where such an entanglement may appear.

In this paper, we address the problem of the quantification and the measurement of multiphoton entanglement. In order to present explicit calculations, and to compare different measurement schemes, we focus our attention on a specific setup: a Mach-Zehnder interferometer fed by a couple of squeezed states [3]. As we will see, the state of the two beams exiting the interferometer ranges from a factorized state to a maximally entangled state. Therefore, the setup is suitable for comparing different measurement schemes designed to reveal the output entanglement.

The interferometric scheme we have in mind is depicted in Fig. 1. The input modes are denoted by a and b, whereas BS_1 and BS_2 are symmetric beam splitters. Equal and opposite phase-shifts ϕ are imposed in each arm of the interferometer. The evolution operator of the whole setup can be written as $\hat{V}_{\rm MZ}(\phi) = \hat{U} \exp\left\{i\phi(a^\dagger a - b^\dagger b)\right\} \hat{U}^\dagger$ where $\hat{U} = \exp\left\{i\,\frac{\pi}{4}\,(a^\dagger b + b^\dagger a)\right\}$ is the evolution operator of a symmetric beam splitter. After straightforward calculations one rewrites $\hat{V}_{\rm MZ}(\phi)$ as

$$\hat{V}_{\rm MZ}(\phi) = \exp\left\{i\,\frac{\pi}{2}\,b^\dagger b\right\} \exp\left\{-i\,\frac{\phi}{2}\,(a^\dagger b + b^\dagger a)\right\} \exp\left\{-i\,\frac{\pi}{2}\,b^\dagger b\right\}, \tag{1}$$

which shows that a Mach-Zehnder interferometer is equivalent to a single beam splitter BS_ϕ of transmissivity $\tau = \cos^2\frac{\phi}{2}$, plus rotations of $\pi/2$ performed on one of the two modes.

The interferometer is fed by a couple of uncorrelated squeezed-coherent states, which are described by the pure state $|\psi_{\rm in}\rangle = \hat{D}_a(\alpha)\,\hat{D}_b(\alpha)\,\hat{S}_a(\zeta)\,\hat{S}_b(\zeta)\,|\mathbf{0}\rangle$ where $\hat{D}(\alpha) = \exp\left(\alpha a^\dagger - \bar{\alpha} a\right)$ is the displacement operator and $\hat{S}(\zeta) = \exp\left[1/2(\zeta^2 a^{\dagger 2} - \bar{\zeta}^2 a^2)\right]$ is

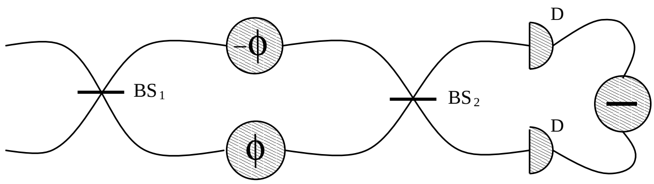

Fig. 1: Schematic diagram of the setup for creating and measuring multiphoton entanglement. The two input ports are fed by a squeezed-coherent state, and equal and opposite phase-shifts are imposed in each arm. At the output the two beams are detected and correlated: the squared difference photocurrent provides higher visibility than the coincidence counting rate.

the squeezing operator, $|\mathbf{0}\rangle$ being the vacuum state. In the following, we will consider a complex field amplitude $\alpha \in \boldsymbol{C}$ and a real squeezing parameter $\zeta \equiv r \in \boldsymbol{R}$. Indeed, any shift between the input signals can be reabsorbed into the internal phase shift ϕ.

As a matter of fact, also the output state $|\psi_{\rm out}\rangle = \hat{V}_{\rm MZ}(\phi)|\psi_{\rm in}\rangle$ is a pure state. Therefore, a good measure of entanglement is provided by the excess information entropy [4, 5] which, in general, can be written as $I_e = S[\hat{\varrho}_a] + S[\hat{\varrho}_b] - S[\hat{\varrho}]$ where $S[\hat{\varrho}] = -\mathrm{Tr}_{ab}\,[\hat{\varrho}\log\hat{\varrho}]$ is the Von-Neumann entropy of the global state of the two modes, and $S[\hat{\varrho}_i] = -\mathrm{Tr}_i\,[\hat{\varrho}_i\log\hat{\varrho}_i]$, $i = a, b$ are the Von-Neumann entropies of the two partial traces. In our case the output state is pure, this implies that $S[\hat{\varrho}] = 0$ and that the two partial traces are equal, i.e. $I_e = 2S[\hat{\varrho}_a]$. Therefore, we introduce the degree of entanglement ϵ of the two-mode state $|\psi_{\rm out}\rangle$ as the normalized excess entropy

$$\epsilon = \frac{S[\hat{\varrho}_a]}{T[N]}\,, \tag{2}$$

where $T[N] \equiv S[\hat{\nu}_{\rm N}] = (1+N)\log(1+N) - N\log N$ is the entropy of a thermal state $\hat{\nu}_{\rm N}$ with the same number of photons $N = \mathrm{Tr}_a\,[\hat{\varrho}_a a^\dagger a]$ of the partial trace. The degree of entanglement ϵ ranges from zero to unit, with $\epsilon = 0$ for a factorized state and $\epsilon = 1$ for a maximally entangled state.

In order to evaluate the entanglement, it is convenient to evaluate the output state of the interferomter by evolving the two-mode Wigner function $W(x_a, y_a; x_b, y_b)$. In fact, the $\pm\pi/2$ rotations of mode b correspond to simple rotations in the sole b-variables, that is $W'(x_a, y_a; x_b, y_b) = W(x_a, y_a; \pm y_b, \mp x_b)$ if $\hat{\varrho}' = e^{\pm i\frac{\pi}{2}b^\dagger b}\,\hat{\varrho}e^{\mp i\frac{\pi}{2}b^\dagger b}$, whereas the action of the beam splitter BS_ϕ, i.e. $\hat{\varrho}' = \hat{U}_\phi\hat{\varrho}\hat{U}_\phi^\dagger$ corresponds to a mixing of variables of the two modes, in formula $W'(x_a, y_a; x_b, y_b) = W(x_a\cos\delta - x_b\sin\delta, y_a\cos\delta - y_b\sin\delta; x_a\sin\delta + x_b\cos\delta, y_a\sin\delta + y_b\cos\delta)$ where we use the notation $\delta = \phi/2$. The Wigner function at the output results

$$\begin{aligned} W_{\rm out}(x_a, y_a; x_b, y_b) = \; & W_{\rm in}(x_a\cos\delta - y_b\sin\delta, y_a\cos\delta + x_b\sin\delta; \\ & x_b\cos\delta - y_b\sin\delta, x_a\sin\delta + y_b\cos\delta)\,, \end{aligned} \tag{3}$$

where $W_{\rm in}(x_a, y_a; x_b, y_b)$ is the initial Wigner function, i.e. the product of two identical single-mode Gaussian Wigner functions $W_{\rm in}(x_a, y_a; x_b, y_b) = 4/\pi^2\exp\{-2e^{-2r}(x_a - \mathrm{Re}[\alpha])^2 - 2e^{2r}(y_a - \mathrm{Im}\,[\alpha])^2 - 2e^{-2r}(x_b - \mathrm{Re}\,[\alpha])^2 - 2e^{2r}(y_b - \mathrm{Im}\,[\alpha])^2\}$. By integration over the b-variables we obtain

$$W_{\rm out}(x_a, y_a) = \frac{1}{\pi\Sigma_x\Sigma_y}\exp\left\{-\frac{(x_a - \mathrm{Re}\,[\alpha_\phi])^2}{\Sigma_x^2} - \frac{(y_a - \mathrm{Im}\,[\alpha_\phi])^2}{\Sigma_y^2}\right\}, \tag{4}$$

which represents the Wigner function of the sole mode a after partial trace over the mode b. The quantities Σ_x and Σ_y in Eq. (4) are given by

$$\Sigma_x^2 = e^{2r}\cos^2\delta + e^{-2r}\sin^2\delta\,, \qquad \Sigma_y^2 = e^{-2r}\cos^2\delta + e^{2r}\sin^2\delta\,, \tag{5}$$

and $\alpha_\phi = \alpha\sqrt{1+\frac{1}{2}\sin^2\phi}$. In order to evaluate entanglement, we note that any unitary transformation $\hat{T}$ acting on the single mode a does not change the value of the entropy, i.e. $S[\hat{\varrho}_a] = S[\hat{T}\hat{\varrho}_a\hat{T}^\dagger]$. Using this property, we displace with amplitude α_ϕ, and then squeeze with parameter $r^* = \log\sqrt{\Sigma_y/\Sigma_x}$ the Wigner function in Eq. (4), thus arriving at the following entropy-equivalent state

$$W'_{\text{out}}(x_a, y_a) = \frac{1}{\pi\Sigma_x\Sigma_y}\exp\left\{-\frac{x_a^2+y_a^2}{\Sigma_y\Sigma_x}\right\}. \tag{6}$$

Remarkably, the Wigner function in Eq. (6) coincides with the Wigner function of a thermal states with thermal photons given by

$$N_\phi = \frac{1}{2}\left[\Sigma_y\Sigma_x - 1\right] = \frac{1}{2}\left[\sqrt{1+\sin^2\phi\,\sinh^2 2r} - 1\right]. \tag{7}$$

The corresponding entropy can be easily computed, and thus the entanglement at the output is given by

$$\epsilon = \frac{\log(1+N_\phi) + N_\phi\log\left(1+\dfrac{1}{N_\phi}\right)}{\log(1+N) + N\log\left(1+\dfrac{1}{N}\right)}. \tag{8}$$

One has $N_\phi = 0$ for $\phi = 0$, and $N_\phi = \gamma N$ for $\phi = \pi/2$, where γ is the squeezing fraction of the input signals (the fraction of the total energy engaged in squeezing). At fixed intensity N, the degree of entanglement is an increasing function of the squeezing fraction γ, with the condition $\phi = \frac{\pi}{2}$ corresponding to the maximum value. Different values of the intensity N does not substantially modify the behavior of ϵ versus γ and ϕ. At fixed $\phi = \pi/2$, for $\gamma = 1$ one has $\epsilon = 1$ independently on N, whereas for $\gamma < 1$ the degree of entanglement becomes a slightly increasing function of N. For highly excited states the entanglement is given by the asymptotic formula

$$\epsilon \overset{N\gg 1}{\simeq} 1 + \frac{\log\gamma}{\log N}. \tag{9}$$

The above discussion holds for the two input states having the same degree of squeezing. However, a pair of input states with different squeezing fractions does not substantially modify the picture. In this case, the entanglement stil oscillates from $\epsilon = 0$ to a maximum value as a function of the internal phase-shift of the interferometer, however maximally entangled states cannot be achieved.

We now study the visibility of the interference fringes that are observed, by varying the internal phase-shift ϕ, in intensity measurements at the output of the interferometer. Besides being originated by interference effects, the variations in the quantities measured at the output also reflect the variations in the quantum correlations between the two output signals. In analogy with experiments involving correlated photon pairs, we may consider the detection of the coincidence counting rate at the output, namely of the fourth-order correlation function $K(\phi) = \langle \psi_{\text{out}} | a^\dagger a b^\dagger b | \psi_{\text{out}} \rangle$. However, as we will show in the following, this corresponds to low fringes visibility, and thus we sought for a more sensitive kind of measurement. The homodyne-like detection of the output difference photocurrent $\langle \psi_{\text{out}} | a^\dagger a - b^\dagger b | \psi_{\text{out}} \rangle$ is widely used in interferometry and generally results in a very sensitive measurement scheme [6]. Starting from this consideration, we suggest the squared difference photocurrent $H(\phi) = \langle \psi_{\text{out}} | (a^\dagger a - b^\dagger b)^2 | \psi_{\text{out}} \rangle$ as a suitable fourth-order quantity to be measured at the output of the interferometer. The fringes visibilities of both detection schemes are given by

$$V_K = \frac{K_{\max} - K_{\min}}{K_{\max} + K_{\min}}\,, \qquad V_H = \frac{H_{\max} - H_{\min}}{H_{\max} + H_{\min}}\,. \tag{10}$$

In Fig. 2 we report V_K and V_H as a function of the intensity N for different values of the input squeezing fraction γ. The H-measurement visibility V_H is larger than V_K in almost all situations, with the exception of the very low signals regime, where very few photons are present. The behavior of fringes visibility versus intensity N also confirms that V_H represents a good measure of the entanglement at the output. As it happens for the degree of entanglement, in fact, a couple of squeezed vacuum at the input corresponds to maximum visibility $V_H = 1$ independently on the intensity. On the other hand, the coincidence counting rate shows a visibility V_K that rapidly decreases versus N, and saturates to a value well below $1/2$. For non unit squeezing fraction, and moderate input intensities ($N < 10$), the behavior of V_H looks qualitatively similar to that of the degree of entanglement, whereas again V_K rapidly decreases. Remarkably, for highly excited states $N > 10$, the visibility V_H has the same asymptotic dependence of the degree of entanglement ϵ, in formula

$$\epsilon \overset{N \gg 1}{\simeq} 1 + \frac{A(\gamma)}{\log N}\,, \tag{11}$$

where the proportionality constant $A(\gamma) \simeq 1/5 \log \gamma$ is roughly proportional to that appearing in Eq. (9).

In conclusion, we have analytically evaluated the degree of entanglement at the output of an interferometer as a function of the input intensity and squeezing fraction, and of the internal phase-shift of the interferometer. By varying the input energy, we can produce

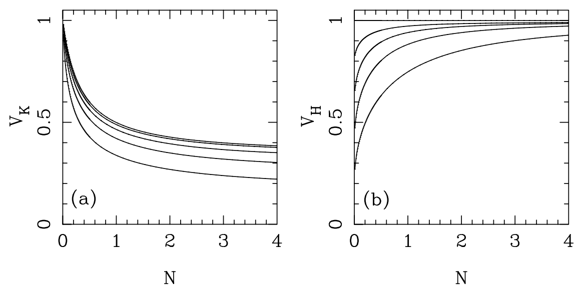

Fig. 2: Fringes visibility as a function of the intensity N for different values of the input squeezing fraction γ. In (a) the visibility of K-measurement V_K, and in (b) the visibility of H-measurement V_H. In both plots we report the visibility versus N for five values of the input squeezing fraction. From bottom to top we have the curves for $\gamma = 0.2, 0.4, 0.6, 0.8,$ and 1.0. As it is apparent, V_H is larger than V_K in almost all situations, with the exception of the very low signals regime.

entangled states of arbitrary large intensity, whereas the degree of entanglement can be tuned by varying the input squeezing fraction and the internal phase-shift. We have suggested an effective experimental characterization of the output entanglement through the measurement of the squared difference photocurrent between the output modes. The interference fringes that are observed by varying the internal phase-shift ϕ show, in fact, high visibility for the whole range of input squeezing parameter.

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