MEASUREMENT OF JOINT PHOTON-NUMBER DISTRIBUTION OF A TWIN-BEAM STATE BY MEANS OF OPTICAL HOMODYNE TOMOGRAPHY

Michael Vasilyev, Sang-Kyung Choi, Prem Kumar, and G. M. D'Ariano

Department of Electrical and Computer Engineering,
Northwestern University, Evanston, Illinois 60208-3118

We report the first measurement of the joint photon-number probability distribution for a two-mode quantum state created by a nondegenerate optical parametric amplifier. The measured distribution exhibits inherently quantum correlations between the signal and idler photon numbers, whereas the marginal distributions are thermal as expected for parametric fluorescence.

INTRODUCTION

The method of optical homodyne tomography (OHT), developed in the last several years, has become an important tool in studying the quantum properties of light.[1-3] Its reconstruction algorithm has evolved from a computationally unstable inverse-Radon transform to a powerful direct sampling approach.[4] The latter allows one to directly determine the density-matrix elements ρ_{nm} of the quantum state in the Fock representation by averaging the so-called pattern functions

$$F_{nm}(X_\phi, \phi) = f_{nm}(X_\phi) \exp[i(n - m)\phi] \tag{1}$$

over the experimental quadrature outcomes X_ϕ of the balanced-homodyne detector and over the local-oscillator (LO) phases ϕ. This procedure is greatly simplified if one is interested only in the diagonal elements of the density matrix, which give the photon-number distribution of the quantum state.[2] Since the pattern functions for the diagonal elements ρ_{nn} are independent of ϕ, they can be averaged over quadrature outcomes that are taken at random LO phases, thus simplifying the experiment by eliminating the need for phase locking.

For the measurement of photon-number distributions, the direct sampling approach represents a very powerful alternative to the direct detection method. It allows one to use fast, high-quantum-efficiency p-i-n photodiodes instead of the slower and less efficient avalanche photodetectors. The issue of high quantum efficiency becomes extremely important in studies of the states of light possessing inherently quantum features. These features, for example, the recently-observed oscillations in the photon-number distribution of squeezed vacuum,[3] wash out very rapidly with degradation of the quantum efficiency from unity.

In this paper, we report the first measurement of the joint photon-number distribution of a two-mode state—the twin-beam state emerging from a nondegenerate optical parametric amplifier (NOPA)—with inherently quantum features. This state is of great interest to researchers owing to the quantum correlation that is imposed onto the two modes by the parametric interaction. In a NOPA, one pump photon simultaneously produces a pair of parametrically down-converted photons that belong to two different (signal and idler) modes. This corresponds to the creation of the following two-mode state:[5]

$$|\Psi\rangle = \frac{1}{\sqrt{\bar{n}+1}} \sum_{n=0}^{\infty} \left(\frac{\bar{n}}{\bar{n}+1}\right)^{n/2} |n,n\rangle , \tag{2}$$

where $\bar{n} = g - 1$ is the average number of photons in each mode with g being the gain of the parametric amplifier. The total photon number in the two modes is always even because of the pairwise nature of the photon creation process. Hence, its probability distribution exhibits even-odd oscillations similar to those for a squeezed-vacuum state.[3] Although the photons in the signal and idler modes are perfectly correlated, their statistics in each mode alone is thermal, yielding the following joint probability distribution:

$$P(n,m) \equiv |\langle n,m|\Psi\rangle|^2 = \frac{\delta_{nm}}{\bar{n}+1} \left(\frac{\bar{n}}{\bar{n}+1}\right)^n . \tag{3}$$

The distribution (3) has zero probabilities everywhere except along the main diagonal, as shown in Fig. 1(left).
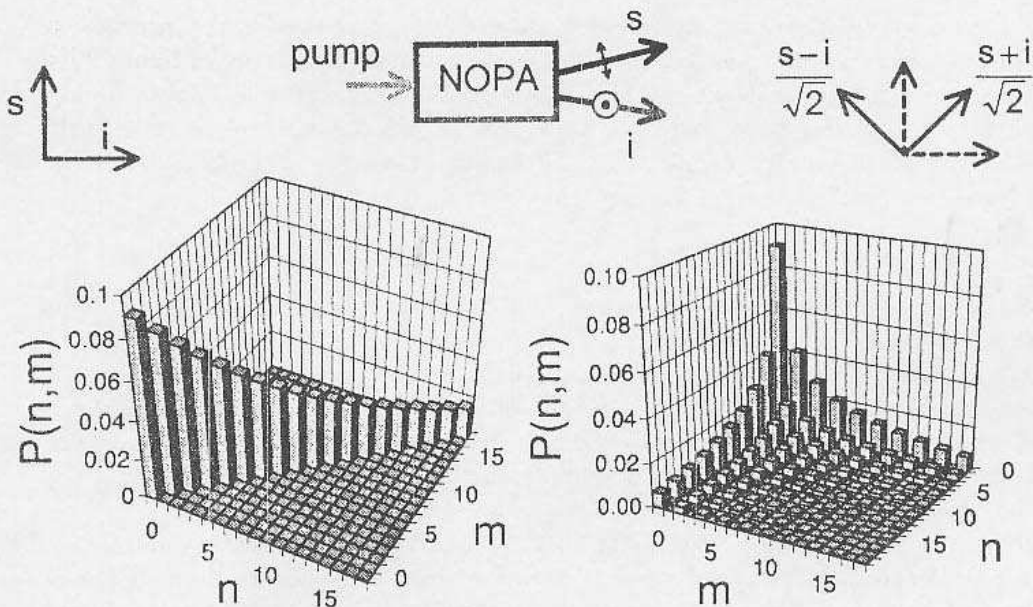
Figure 1. Theoretical plots of the joint photon-number distributions for the signal and idler modes (left), and the +45° and −45°-polarized modes (right). The average number of photons in each mode is 10.

In a polarization non-degenerate parametric amplifier, the signal and idler modes are orthogonally polarized. Such an amplifier can be shown to be equivalent to two independent degenerate parametric amplifiers for modes that are polarized at ±45° with respect to the signal polarization. Each of these modes is in a squeezed-vacuum

state, and Figure 1(right) shows their joint distribution

$$P(n,m) = \begin{cases} 0 & \text{for } n = 2k+1 \text{ or } m = 2l+1 \,, \\ \frac{(2k-1)!! \ (2l-1)!!}{2^{k+l} \ k! \ l!} \ \frac{1}{\bar{n}+1} \left(\frac{\bar{n}}{\bar{n}+1}\right)^{k+l} & \text{for } n = 2k, \ m = 2l \,, \end{cases} \qquad (4)$$

where $\bar{n}$ is the same as that in Eq. (2). It is easy to see that the distribution (4) factorizes into a product of marginal distributions.

EXPERIMENTAL SETUP

The schematic of our experimental setup is shown in Fig. 2. The NOPA, consisting of a 5-mm-long KTP crystal, is pumped by the second harmonic of a Q-switched and mode-locked Nd:YAG laser. The laser produces a 100-MHz train of 120-ps pulses at 1064 nm (85 ps for the second harmonic at 532 nm) with a 205-ns Q-switch envelope (145 ns for 532 nm) having a 1 kHz repetition rate. The 1064-nm orthogonally-polarized
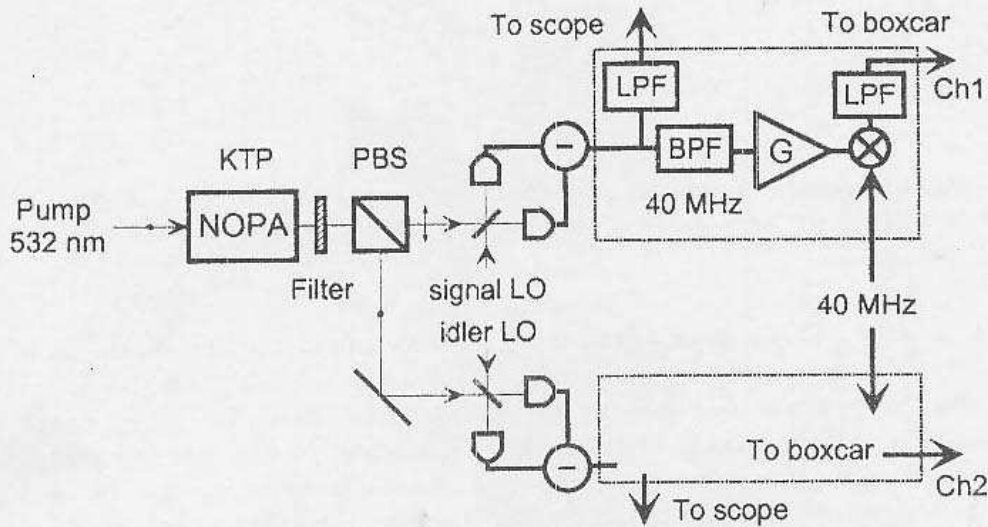

Figure 2. A schematic of the experimental setup.

twin beams emitted by the NOPA are detected separately by two balanced-homodyne detection setups using two independent LO's.[6] Low- and high-frequency parts of the photocurrents are separated. The peak amplitudes of the 5-MHz low-pass-filtered photocurrents in the signal and the idler arms are monitored by the oscilloscope. A 10-MHz-wide band of radio frequencies near $\Omega/2\pi = 40$ MHz is selected in each arm by means of a bandpass filter and amplified with a low-noise amplifier. The amplified noise photocurrent is then down-converted to the near-DC region by use of an RF mixer and sampled by the boxcar integrator (signal arm—by channel 1, idler arm—by channel 2). The outputs of the boxcar channels are a measure of the quadrature amplitudes X_ϕ^S and X_ψ^I, where ϕ and ψ are the phases of the signal and idler LO's, respectively.[7] The joint photon-number probability distribution $P(n,m)$ of the twin beams is then obtained by averaging the two-mode pattern function $f_{nn}(X_\phi^S)f_{mm}(X_\psi^I)$ over the quadrature samples X_ϕ^S and X_ψ^I and over the independently randomly-varying LO phases ϕ and ψ.[5]

The overall quantum efficiencies for the twin beams, including propagation losses, homodyne efficiencies, and detector efficiencies, are estimated to be $\eta_S = 0.38$ and $\eta_I = 0.35$, respectively. The most significant factor in deviation of our quantum efficiency from unity is the efficiency of homodyne overlap, which is approximately 0.45
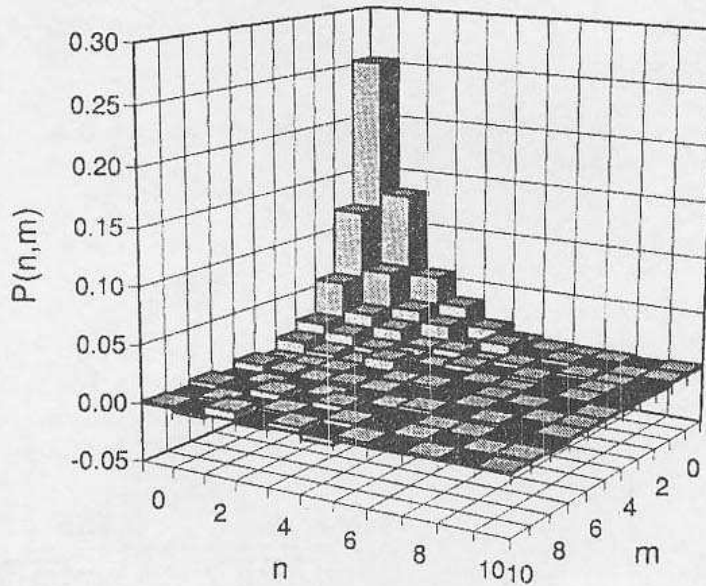
Figure 3. Joint photon-number distribution of the twin-beam state reconstructed from 320,000 homodyne data samples. The mean photon numbers are: $\bar{n}$=1.1 for the signal beam and $\bar{m}$=1.0 for the idler beam.

in our present setup. This is because the traveling-wave pulsed amplification process causes a change in the spatio-temporal profile of the amplified field, resulting in its mismatch with the LO. We have shown previously that the photon statistics of each of the parametric beams alone can be measured very efficiently using a self-generated matched LO (self-homodyne tomography).[8] This approach, however, can not be used for a joint measurement of the twin beams because it renders the NOPA phase-sensitive, which would distort the self-generated LO, making its matching with the mode of the quantum state of interest inefficient. The observation of the even-odd oscillations of the total photon number is extremely sensitive to the quantum efficiency and requires the overall efficiency to be at least 0.8.[5] The correlation between the signal and the idler beams, on the other hand, is less sensitive to η_S and η_I, and could be observed with our current setup.

MEASUREMENT RESULTS

The measured joint photon-number distribution is shown in Fig. 3. The detected mean photon numbers are $\bar{n} = 1.1$ for the signal beam and $\bar{m} = 1.0$ for the idler beam. The non-unity quantum efficiency results in spreading of the distribution around the main diagonal $n = m$, where a delta-like correlation is expected for $\eta_S = \eta_I = 1$. The marginal distributions for the signal or the idler beam alone are shown in Fig. 4. They indicate good agreement with the theoretically-predicted thermal distributions for the same mean photon numbers.

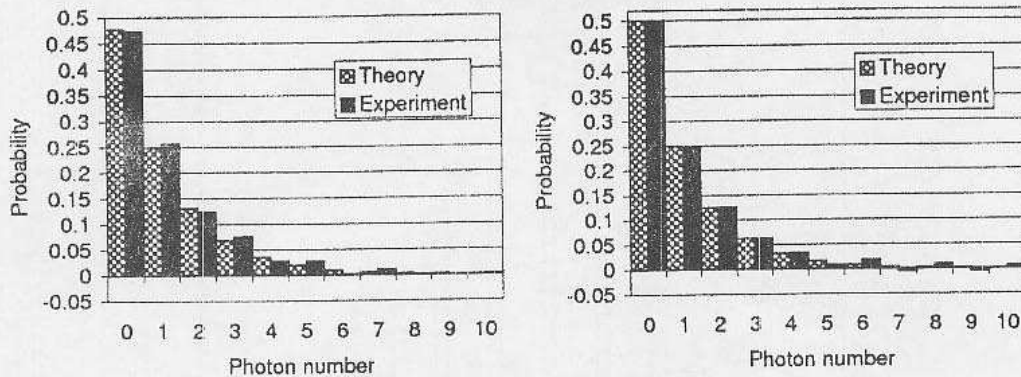To show the quantum character of the measured distribution, we used it to find

Figure 4. Marginal distributions for the signal (left) and the idler (right) beams reconstructed from the same data as the distribution in Fig. 3. Theoretical distributions for the same mean photon numbers $\bar{n}=1.1$ (left) and $\bar{m}=1.0$ (right) are also shown.

the photon-number correlation $d(n)$ between the two modes:

$$d(n) = \sum_{k=\max(-n,0)}^{N} P(k, k+n) \,, \qquad (5)$$

which is shown in Fig. 5. The number $N = 18$ is determined by the size of our reconstructed distribution. In the limit of $N \to \infty$, $d(n)$ is the probability of finding the difference between the signal and the idler photon numbers to be n. In the case of ideal homodyne detection, $d(n)$ is expected to be the Kronecker δ_{n0}. For a non-unity quantum efficiency, however, the correlation $d(n)$ is no longer a delta-function; it spreads around $n = 0$. Nevertheless, it remains narrower than the correlation function for two independent coherent-state beams having the same mean photon numbers as the twin beams, which represents the standard quantum limit of correlation between classical states. We compare the photon-number correlation observed in our measurement with the standard quantum limit. The twin-beam correlation function shown in Fig. 5 is narrower than the coherent-state correlation function, which indicates the inherently quantum character of the twin-beam state. For substantial deviations $|n|$ from the main diagonal, $d(n)$ becomes randomly oscillating due to the increasingly large contribution of the statistical errors in the measurement of the joint distribution.

CONCLUSIONS

We have measured, for the first time, the joint photon-number distribution of a twin-beam state. While the marginal distributions of the signal or the idler beam alone are thermal, the photon-number correlation between them shows an inherently quantum character. An improvement of the homodyne efficiency will allow us to also observe the oscillations of the total photon number. A detailed study is underway to overcome the mode-matching limitations of the current setup.
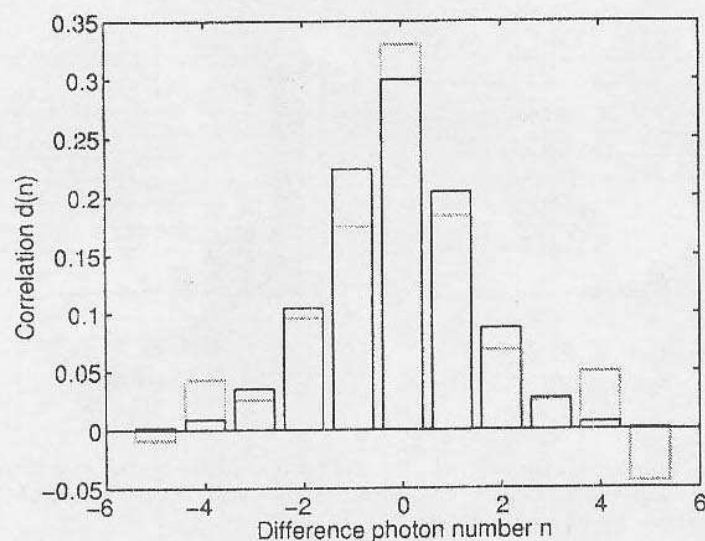
Figure 5. Correlation function $d(n)$ of the signal and idler photon numbers (solid gray line), obtained from the joint distribution of Fig. 3. Here $N = 18$. Solid black line — correlation of two independent coherent states with mean photon numbers $\bar{n}=1.1$ and $\bar{m}=1.0$.

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