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NOISE IN NON-LINEAR AMPLIFIERS : QUANTUM LIMIT IN A SQUEEZED OPERATING STATE

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A quantum non-linear amplifier is analysed in which the operating state is coherent, squeezed and multi-boson squeezed. The quantum noise characterization of the amplifier is crucially connected to its phase sensitiveness: different types of amplifiers are available depending on their operating states.

The quantum mechanical description of amplifiers is customarily based on the following ingredients: a set of (bosonic) input modes, a set of (bosonic) output modes, the amplifier's degrees of freedom (which can be either bosonic or fermionic) and an interaction scheme of the latter which transfers input to output[1].

Restricting for simplicity our attention to the single mode case and denoting by a,a^+ and A,A^+ the input and output bosons respectively, the evolution equations of the amplifier - averaged over the internal degrees of freedom - are given by the mapping

$$A=\mathscr{D}(a,a^+) \qquad /1/$$

where the functional $\mathscr{D}$ is constrained to belong to a realization of the automorphism group of the Weyl algebra (namely $\left[a,a^+\right]=1$ should imply $\left[A, A^+\right]=1$).

The other relevant property of an amplifier is its operating state $|\Phi\rangle$ - which is assumed to be independent of the input state $|\Phi_{in}\rangle$ - describing how the amplifier's internal modes are prepared.

The relevant physical quantities describing how the amplifier actually operates are the mean-square fluctuation of the output amplitude

$$(\Delta A)^2= \langle\Phi_{in}|A^2|\Phi_{in}\rangle-\langle\Phi_{in}|A|\Phi_{in}\rangle^2 \qquad /2/$$

where the averages $\langle\bullet\rangle=\langle\Phi|\bullet|\Phi\rangle$ have been explicitly written in terms of input states only, because of the assumed factorization properties of the density matrix and in view of the averaging performed on the evolution equations; and the gain

$$G=|\langle A\rangle/\langle a\rangle|^2 \qquad /3/$$

In present communication we shall consider a variety of evolution functionals $\mathscr{D}$, both polynomial and analytic, as well as a set of different choices of input states selected to be coherent states and study the phase sensitivity of the amplifier, namely how the quantum input noise is processed.

More precisely Caves' definition of phase-insensitive linear amplifier will be extended to non-linear evolution equations, essentially by dropping the requirement of invariance of $\langle A\rangle$ under arbitrary phase transformations. The most general polynomial $\mathscr{D}$ of degree 2 reads[2].

$$\mathscr{D}\ :P(a+Qa^+)^2+Ma+Na^+ \qquad /4/$$

where the complex parameters P,Q,M,N satisfy the conditions

$$|M|^2-|N|^2=1=|Q|^2 \qquad /5/$$

$$\frac{P}{\bar{P}} = -\bar{Q}\frac{M-N\bar{Q}}{\bar{M}-\bar{N}\bar{Q}} \qquad /6/$$

Eqs./5/,/6/ are the constraints whereby /4/ indeed realizes an automorphism of Weyl's algebra.

We consider three different choices for $|\varphi_{in}\rangle$:

i) Coherent states[3]

$$|\alpha\rangle = D(\alpha)|0\rangle \qquad /7/$$

$$D(\alpha) = \exp(\alpha a^{+} - \bar{\alpha} a)$$

where $|0\rangle$ is the vacuum state and $\alpha \in \mathscr{C}$.

Such states are interesting in present application because they correspond to phase insensitive input noise, namely $(\Delta a)=0$.

ii) Squeezed states[4]

$$|z,\alpha\rangle = D(\alpha)S(z)|0\rangle \qquad /8/$$

$$S(z) = \exp\{z(a^{+})^2 - za^2\}$$

where $z \in \mathscr{C}$ is the squeezing parameter. They can never give a phase insensitive input noise, in that

$$(\Delta a)^2 = (z/2|z|)\sinh(4|z|) \qquad /9/$$

can be zero only for z=0, in which case /8/ transform into /7/.

iii) Multi-boson squeezed states[5]

$$|z,w;\alpha\rangle_k = D(\alpha)S_{(k)}(z,w)|0\rangle \qquad /10/$$

$$S_{(k)}(z,w) = \exp\{zb^{+}_{(k)} + iwa^{+}a - zb_{(k)}\}$$

$$w \in \mathscr{R}\ ;\quad z \in \mathscr{C}\ ;\quad k \in \mathscr{N}$$

where $b_{(k)}$, $b^{+}_{(k)}$ are multi-boson operators

$$\left[b_{(k)}, b^{+}_{(k)}\right] = 1\ ;\ \left[a^{+}a, b_{(k)}\right] = -kb_{(k)} \qquad /11/$$

which can be realized by the normal ordered series[6]

$$b_{(k)} = \sum_{j=0}^{\infty} \alpha_j^{(k)} (a^{+})^j a^{j+k} \qquad /12/$$

where

$$\alpha_j^{(k)} = \sum_{t=0}^{j} \frac{(-)^{j-t}}{(j-t)!}\left(\frac{1+[\![t/k]\!]}{t!(t+k)!}\right)^{1/2} e^{i\vartheta_t} \qquad /13/$$

$[\![x]\!]$ denoting the maximum integer $\leqslant x$ and ϑ_l, l=0,...,j a set of arbitrary phases. Note that $a^k \neq b_{(k)}$.

For the state /10/ the input noise is characterized by variance

$$(\Delta a)^2 = \sqrt{2}\ \ \delta_{k,2}\ \ \varphi_{(k)} zF(|\varphi_{(k)}|^2|z|^2) \qquad /14/$$

where

$$\varphi_{(k)} = \frac{\sin(kw/2)e^{ikw/2}}{kw/2} \qquad /15/$$

and the function F, whose properties where thoroughly discussed in reference 6, is defined by

$$F(x) = e^{-x}\sum_{n=0}^{\infty} \frac{x^n}{n!}\sqrt{2n+1} \qquad /16/$$

(Δa) as given by /16/ can vanish only for z=0 or $w = \frac{2m\pi}{k}$, $m \in \mathscr{Z}-\{0\}$ or $k > 2$.

In the former two cases the state /10/ turns into the coherent state[7]. The states $|z,w;\alpha\rangle_k$ are thus non-Gaussian wave packets[7], which, for k>2, are phase insensitive as well as coherent in a non-trivial way.

In case ii) the output noise is characterized by mean-square fluctuation

$$(\Delta A)^2 = 2P^2SC(2T^2+SC) + \sinh(2|z|)(Mz^2 + N\bar{z}^2)/2|z| + 2PT\{(\sinh|z|\ (MSz+NC\bar{z})/|z| + \cosh|z|(MC+NS)\} + MN\cosh(2|z|)$$

$$T = \alpha + Q\bar{\alpha} \qquad /17/$$

and gain

$$G = |P(T^2+SC)/\alpha \quad +M+N\,\bar{\alpha}/\alpha\,|^2 \qquad /18/$$

where

$$S=\cosh|z|+Q\,\bar{z}\,\sinh|z|/|z| \qquad /19/$$

$$C=Q\,\cosh|z|+z\,\sinh|z|/|z|$$

case i) is recovered from /17/,/18/ simply setting $z=0$, and the linear case for $P=0$.

One can check how the non-linearity forbids phase intensitivity. In the linear case, Caves' results $(\Delta A)^2 = MN$, $G = |M+N\,\bar{\alpha}/\alpha\,|^2$ are straightforwardly recovered adopting the vacuum as operating state. It is also interesting to notice how the gain given by /18/ has state dependence hinting (minimum of G vs. α) to different regimes of stability of the amplifier.

In case iii) the results are much more complicated and rich of features.

In view of the special role ascribed by eq./14/ to the choice $k = 2$, we shall confine here our attention to it.

The mean square fluctuation of the output amplitude is:

$$(\Delta A)^2 = 2P^2\Big[4\sqrt{2}\,R\,\Omega_1 F' + \Omega_2 H - \{\Omega_2 + 4\sqrt{2}\,R(\zeta^2+Q^2\bar{\zeta}^2)+2QR\}F^2 + 2\sqrt{2}\Big(\Omega_1(R+T^2+Q) + MT(\zeta+Q\bar{\zeta})/P + \frac{(M^2\zeta+N^2\bar{\zeta})}{2P^2}\Big)F + 4\Big(R^2+|\zeta|^2(5Q^2+2\mathbf{T})\Big) + \mathbf{T}\Big].$$

$$\Omega_1 = \zeta+Q^2\bar{\zeta} \;;\; \Omega_2 = \zeta^2+Q^4\,\bar{\zeta}^2 \qquad /20/$$

$$R=Q|\zeta|^2 \;;\quad \xi = |\varphi_k||z|$$

$$\mathbf{T} = Q(2T^2+Q) + T(N+MQ)/P + MN/2P^2$$

where $F = F(\xi^2)$ is given by /16/ and $H = H(\xi^2)$ is defined by

$$H(x) = e^{-x}\sum_{n=0}^{\infty}\frac{x^n}{n!}\sqrt{(2n+2)^2-1} \qquad /21/$$

The complex structure of the dependence of $(\Delta A)^2$ on P,Q,M,N and z,w , hints to the possibility of changing the characteristics of the amplifier (switching from phase-insensitive to phase-sensitive) by a suitable choice of the state parameters (z,w,k).

Another interesting case is that of the evolution equation given by:

$$A = Lb_{(k)} + Jb^+_{(k)} + Ma + Na^+ \qquad /22/$$

The more outstanding property of /22/ is that unlike /4/ it is possibly to put $M=N=0$ without infringing the restriction for $\mathscr{D}$ to be a realization of the automorphism group of the Weyl algebra. The calculations are a direct application of the method outlined above and the result are too long to be reported here.

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