SQUEEZING-SYMMETRY OF THE BALANCED HOMODYNE DETECTOR

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1. INTRODUCTION

Detection of squeezed light requires high quantum efficiency, phase sensitivity, and freedom from amplifier noise or any other kind of disturbance. At present, the homodyne detector is the optimum device for detection of a quadrature component of the field [1]. Its phase sensitivity comes from combining the signal beam through a beam splitter (BS) with an intense 'local oscillator' (LO) field operating at the same frequency. The combined field is then directed to a photodetector and the amplitude component of the field is revealed as the beating between the two input fields. Noise from intensity fluctuations of the LO is canceled by means of the balanced configuration of Fig.1, involving two photodetectors with equal responsivity and a 50-50 BS: the difference photocurrent $\hat{I}_D$ between the two photodetectors D1 and D2 measures the interference between the signal beam and the LO, the interference being constructive at one photodetector and destructive at the second one.

In this paper, a novel symmetry of the balanced homodyne detector is presented, which relates different input pairs of beams to the same output photocurrent $\hat{I}_D$. I call the symmetry 'squeezing symmetry' because the input pairs of beams—which are equivalent in producing the same current $\hat{I}_D$—are related through a squeezing transformation of the fields. The symmetry is presented in Sec. 2, where the physical meaning of the squeezing transformation of the fields is also discussed. In Sec. 3 the extension of the symmetry to four-port linear devices is briefly sketched, and an application to a simple interferometer, build up as a cascade of linear devices, is illustrated.

2. SYMMETRY OF THE DIFFERENCE PHOTOCURRENT IN THE BALANCED HOMODYNE DETECTOR

In the following, a single-mode analysis is given, in the assumption of lossless BS and ideal photodetectors (having unit quantum efficiency). The input fields a and b combine at the 50-50 BS giving the sum and the difference fields c and d in the output arms. After tuning the overall phases (by adjusting the path lengths), one has

$$c = \frac{a+b}{\sqrt{2}}, \qquad d = \frac{a-b}{\sqrt{2}}. \tag{1}$$

The output photocurrents $\hat{I}_1$ and $\hat{I}_2$ are proportional to the number operators $c^\dagger c$ and

Lecture Notes in Physics, Vol. 378
C. Bendjaballah, O. Hirota, S. Reynaud (Eds.),
Quantum Aspects of Optical Communications

$d^\dagger d$ and the difference photocurrent $\hat{I}_D$ has the form

$$\hat{I}_D = \hat{I}_1 - \hat{I}_2 \propto a^\dagger b + b^\dagger a \ . \tag{2}$$

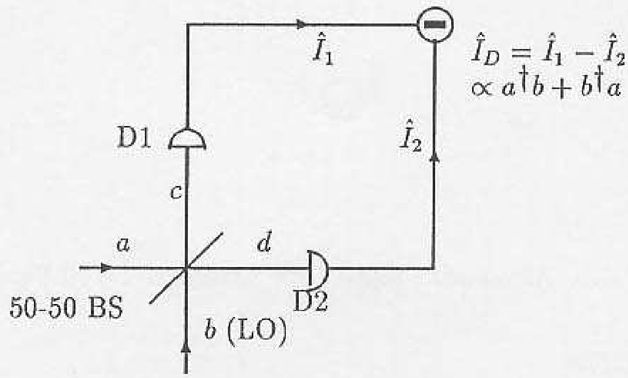


Figure 1: Scheme of the balanced homodyne detector.

The particular form of $\hat{I}_D$ in Eq.(2) is highly symmetrical, as a consequence of the balanced scheme. Besides the trivial symmetry under permutation of the input fields, $\hat{I}_D$ commutes with the unitary operator $\hat{U}(\mu,\nu)$

$$[\hat{I}_D, \hat{U}(\mu,\nu)] = 0 \ , \tag{3}$$

where

$$\hat{U}(\mu,\nu) = \hat{S}_a^\dagger(\overline{\mu},\nu)\hat{S}_b(\mu,\nu) \ . \tag{4}$$

Here $\hat{S}_{a,b}(\mu,\nu)$ are the squeezing operators of Yuen [2] acting on the input fields as follows

$$\hat{S}_a(\mu,\nu) a \hat{S}_a^\dagger(\mu,\nu) = \mu a + \nu a^\dagger \ , \qquad \hat{S}_b(\mu,\nu) b \hat{S}_b^\dagger(\mu,\nu) = \mu b + \nu b^\dagger \ . \tag{5}$$

and the complex numbers μ and ν satisfy the relation $|\mu|^2 - |\nu|^2 = 1$. Invariance (3) can be verifed by using the identity $\hat{S}_{a,b}^\dagger(\mu,\nu) = \hat{S}_{a,b}(\overline{\mu},-\nu)$. For a real parameter μ Eqs.(3) and (4) state that *the difference photocurrent $\hat{I}_D$ is invariant under inverse squeezing of the two input beams.* Due to the form of Eq.(4) the symmetry transformation $\hat{U}(\mu,\nu)$ does not affect the correlation between the input beams, as opposed to the $SU(2)$ symmetries of the beam splitter derived in Ref.[3].
In the Schrödinger picture the invariance (3) means that the quantum statistics of the difference photocurrent $\hat{I}_D$ does not change if the input state is symmetry-transformed as

$$\hat{R}_{in} \longrightarrow \hat{U}^\dagger(\mu,\nu)\hat{R}_{in}\hat{U}(\mu,\nu) \ . \tag{6}$$

where $\hat{R}_{in}$ denotes the density matrix of the input (with the two beams which are in general quantum-correlated). For the particular case of uncorrelated beams, namely

$\hat{R}_{in} = \hat{\rho}_a\hat{\rho}_b$, the symmetry transformation (6) is equivalent to the following pair of single-mode transformations

$$\hat{\rho}_a \longrightarrow \hat{S}_a(\overline{\mu},\nu)\hat{\rho}_a\hat{S}_a^\dagger(\overline{\mu},\nu)\,, \qquad \hat{\rho}_b \longrightarrow \hat{S}_b^\dagger(\mu,\nu)\hat{\rho}_b\hat{S}_b(\mu,\nu)\,. \tag{7}$$

Here, some remarks regarding the physical meaning of the squeezing transformations (7) are in order. The present squeezing transformation corresponds to a simultaneous squeezing of both the noise and the signal, whereas in the usual squeezed states, the unsqueezed signal is superimposed to the squeezed fluctuations (see also Ref.[4]). In practice, the present squeezing is equivalent to an ideal noisless phase-sensitive amplification [5], which enhances a phase component of the field, while reduces the conjugated one. As a consequence of the squeezing symmetry, one infers that, for example, the use of a squeezed LO is equivalent to a phase-sensitive amplification of the remote signal. However, no improvement of the signal-to-noise ratio is achieved, and the homodyne with squeezed LO is essentially equivalent to the homodyne with coherent LO, where the homodyne gain is shared by both the signal and the squeezing power components of the LO.

3. GENERALIZATION TO OTHER FOUR-PORT LINEAR DEVICES

In this section I briefly present the derivation of the squeezing symmetries of a linear four-port device with output photocurrent quadratic in the output fields. In particular, I suggest an application to a four-port device, built up as a cascade of elementary four-port devices.

By a linear device I mean that the Heisenberg equations relating the output to the input fields are linear. The beam splitter provides an example of linear four-port device, with Heisenberg equations

$$\begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} ma+nb \\ -\overline{n}a+\overline{m}b \end{pmatrix}\,, \qquad |m|^2+|n|^2=1\,. \tag{8}$$

The degenerate four wave mixer (FWM) with classical (non depleted) pump waves is an other example of four-port linear device, with evolution equations for the signal and idler waves given by [6]

$$\begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} ma+nb^\dagger \\ na^\dagger+mb \end{pmatrix}\,, \qquad |m|^2-|n|^2=1\,. \tag{9}$$

$|m|^2$ being the signal gain. If one of the waves is assumed to have a constant field amplitude, the parametric amplifier (PA) can be regarded as a four-port device and Eqs.(9) apply to this case as well. (The symmetrical form of Eqs.(8) and (9) pertains to a suited choice of the field phases related to the path lengths).

For a general four-port linear device the Heisenberg equations can be written in the form

$$\mathbf{M}\begin{pmatrix} a \\ b \end{pmatrix} = \hat{M}\begin{pmatrix} a \\ b \end{pmatrix}\hat{M}^\dagger\,, \tag{10}$$

where $\mathbf{M}$ denotes the linear transformation corresponding to the unitary operator $\hat{M}$.

Table 1: Some examples of linear transformations and conserved currents.

Symbol	*Linear transformations of the fields a, b*	*Constraints*	*Some conserved currents*
$\mathbf{V}(\mu,\nu,e^{i\phi})$	$\mu a+\nu b^\dagger$ $\mu b+e^{i\phi}\nu a^\dagger$	$\|\mu\|^2-\|\nu\|^2=1$	$a^\dagger a-b^\dagger b\,,\quad(\phi=0)$
$\mathbf{W}(\mu,\nu,e^{i\phi})$	$\mu a+\nu b$ $\overline{\mu}b+e^{i\phi}\overline{\nu}a$	$\|\mu\|^2+\|\nu\|^2=1$	$a^\dagger a+b^\dagger b\,,\quad(\phi=\pi)$
$\mathbf{U}(\mu,\nu,e^{i\phi})$	$\mu a+\nu a^\dagger$ $\mu b+e^{i\phi}\nu b^\dagger$	$\|\mu\|^2-\|\nu\|^2=1$	$a^\dagger b+b^\dagger a\,,\quad(\phi=\pi)$
$\mathbf{Z}(\mu,\nu,e^{i\phi})$	$\mu a+\nu a^\dagger$ $\overline{\mu}b+e^{i\phi}\overline{\nu}b^\dagger$	$\|\mu\|^2-\|\nu\|^2=1$	$a^\dagger b^\dagger+ab\,,\quad(\phi=\pi)$

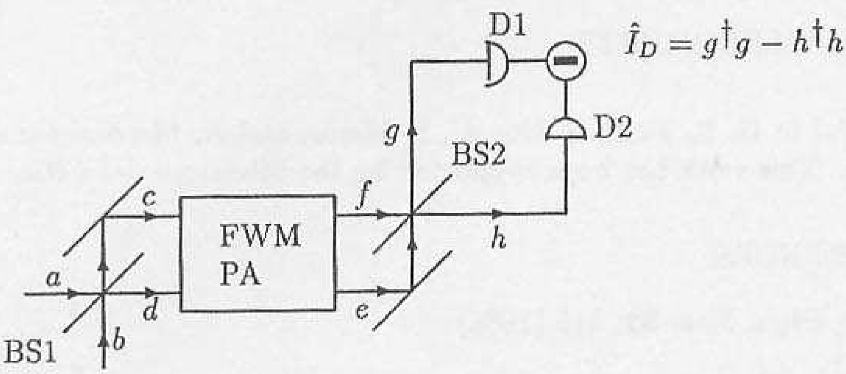
Table 2: Symmetry transformations of the four beams leaving the Heisenberg equations invariant. (The phases ϕ and ψ are given by: $\phi=\arg(m)$, $\psi=\arg(n)$).

Device	*Heisenberg evolutions of the input fields*	*Symmetry transformations*	
		Input fields	*Output fields*
BS	$\mathbf{W}(m,n,-1)$	$\mathbf{V}(\mu,\nu,-1)$	$\mathbf{V}(\mu,\nu,-1)$
		$\mathbf{U}(\mu,\nu e^{-2i\phi},e^{2i(\phi-\psi)})$	$\mathbf{U}(\mu,\nu,e^{-2i(\phi+\psi)})$
50-50 BS		$\mathbf{U}(\mu,-\nu e^{-i(\phi-\psi)},-e^{2i(\phi-\psi)})$	$\mathbf{V}(\mu,\nu,1)$
FWM/PA	$\mathbf{V}(m,n,1)$	$\mathbf{V}(\mu,\nu e^{-2i\phi},e^{2i(\psi-\phi)})$	$\mathbf{V}(\mu,\nu,e^{2i(\phi+\psi)})$
		$\mathbf{W}(\mu,\nu,1)$	$\mathbf{W}(\mu,\nu,1)$
		$\mathbf{Z}(\mu,\nu e^{-2i\phi},e^{2i(\psi-\phi)})$	$\mathbf{Z}(\mu,\nu,e^{2i(\phi+\psi)})$

Some examples of linear transformations, which are relevant in the present context, are defined in Table 1, where the transformations (8) and (9) are represented by the linear operators $\mathbf{W}(m,n,-1)$ and $\mathbf{V}(m,n,1)$.
The derivation of the squeezing symmetries of a four-port linear device with measured photocurrent $\hat{I}_o$ can be divided into three steps:
i) evaluation of the symmetry transformations (ST) of the output beams which preserve the current $\hat{I}_o$ (some examples can be found in Table 1);
ii) evaluation of the ST of the four beams which preserve the Heisenberg equations of the device; in other words: evaluation of the ST of the output beams corresponding to a given ST of the input beams;
iii) matching between the current-preserving ST of step i) and the Heisenberg-preserving ST of step ii).
Step i) through step iii) allow one to obtain the squeezing transformation of the input beams leaving the output photocurrent invariant. In Table 2, some ST leaving the Heisenberg equations invariant are given, for both the BS and FWM/PA devices. They can be simply derived by a linear analysis, or by means of the group theoretical approaches of Refs.[3],[7],[8].
Using the above method with the help of Tables 1 and 2, the squeezing symmetry of the 50-50 BS ($\phi=\psi=0$) can be obtained as follows: a) the output photocurrent $\hat{I}_D=c^\dagger c-d^\dagger d$ is preserved by $\mathbf{V}(\mu,\nu,1)$; b) the ST $\mathbf{V}(\mu,\nu,1)$ of the output is related

to the ST $\mathbf{U}(\mu, -\nu, -1)$ of the input: the second transformation corresponds to the unitary operator of Eq.(4).

If a cascade of linear four-port devices is considered (where, for example, the output beams of an element coincide with the input beams of the following), it is not necessary to derive its Heisenberg equations in order to apply the above procedure. In fact, the evaluation of the Heisenberg-preserving ST in step ii) for every element of the cascade allows one to connect the ST of the inputs of two consecutive devices, and the process can be iterated, until the input beams of the whole cascade are reached. Eventually, the compatibility of the ST of two consecutive elements may restrict the class of the symmetry or impose phase-matching between the devices. This mechanism actually corresponds to a smaller symmetry class of the whole cascade, and it occurs when the dynamical groups of the contiguous devices are different, as, for example, when a BS is followed by a FWM/PA [3],[8].



As an example illustrating the above concepts I consider the interferometer in Fig. 2, which is built up as a cascade of a BS followed by a FWM/PA and finally by a balanced homodyne detector. I denote by $\phi_{B1}, \psi_{B1}, \phi_{B2}, \psi_{B2}$ the phases of the beam splitters and by ϕ_F, ψ_F those of the four-wave mixer (for the conventions, see Table 2). Choosing, for simplicity, $\phi_{B2} = \psi_{B2} = 0$, as in the previous example, the ST of the BS2 input is $\mathbf{U}(\mu, -\nu, -1)$. The transformation $\mathbf{U}$ is not in the Table 2 at the entry of the FWM: as a consequence, the symmetry matching between the BS2 and the FWM requires a restriction of the symmetry class. This is attained through the identity

$$\mathbf{U}(\mu, \nu, e^{i\theta}) = \mathbf{Z}(\mu, \nu, e^{i(\theta + 2\arg(\nu))}), \qquad \mu \text{ real}, \tag{11}$$

which implies that the ST for f and e coincides with $\mathbf{Z}(\mu, -\nu, -e^{2i\arg(\nu)})$. From Table 2 one can see that the phase of ν is constrained to the values $\arg(\nu) = \phi_F + \psi_F + \pi/2 + k\pi$ (k integer), namely the restricted symmetry for e and f is $\mathbf{Z}(\mu, i\nu e^{i(\phi_F + \psi_F)}, e^{2i(\phi_F + \psi_F)})$, where now both μ and ν are real. The corresponding ST for the input beams c and d is $\mathbf{Z}(\mu, i\nu e^{i(\psi_F - \phi_F)}, e^{2i(\psi_F - \phi_F)})$, which is the global squeezing symmetry of the partial cascade FWM/PA-BS2.

In order to obtain the squeezing symmetry of the whole cascade one has to match the ST of BS1 and FWM. Eq.(11) leads to $\mathbf{Z}(\mu, i\nu e^{i(\psi_F - \phi_F)}, e^{2i(\psi_F - \phi_F)}) = \mathbf{U}(\mu, i\nu e^{i(\psi_F - \phi_F)}, -1)$, and the complete cascade has a squeezing symmetry only if the phases of BS1 are constrained by $\phi_{B1} + \psi_{B1} = \pi/2 + k\pi$. In this case the ST of a and b is $\mathbf{U}(\mu, i\nu e^{i(\psi_F - \phi_F - 2\phi_{B1})}, -e^{4i\phi_{B1}})$.

In conclusion, the interferometer in Fig.2 has a squeezing symmetry only if the phases of the beam splitter BS1 are related by the equation

$$\phi_{B1} + \psi_{B1} = \pi/2 + k\pi \, . \tag{12}$$

The output photocurrent $\hat{I}_D$ is invariant under the squeezing transformation of the input beams

$$a \to \mu a + i\nu e^{(\psi_F - \phi_F - 2\phi_{B1})} a^\dagger \, , \qquad b \to \mu b - i\nu e^{(\psi_F - \phi_F + 2\phi_{B1})} a^\dagger \, , \tag{13}$$

where μ and ν are real. The same result can also be obtained by evaluating the Heisenberg equations of the cascade and deriving the related ST: this direct way, however, is lengthy, and the symmetry breaking mechanism due to the matching of different devices is not evident.

4. ACKNOWLEDGMENTS

I am grateful to H. P. Yuen, S. Donati, S. Merlo, and A. Mecozzi for discussions and suggestions. This work has been supported by the Ministero della Ricerca Scientifica.

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