The Role of Quantum Efficiency in Impeding the Measurement of the Wave Function of a Single System

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Abstract. Homodyne detection provides a concrete method for "measuring the quantum state". The state measurement is achieved by repeating many measurements of inequivalent observables after re-preparing the system in the same state at each measurement. If the quantum efficiency η of the measurement is lower than 1/2 the density matrix of the state cannot be reconstructed. The presence of such lower bound for quantum efficiency prevents measuring the wave function of a single system using schemes of weak repeated indirect measurements on the same system. From one hand, this is another way to reassert the statistical meaning of the wave function in quantum theory. From the other hand it clarifies the fundamental role of the concept of quantum efficiency in measurement theory.

1 Introduction

The possibility of measuring the wave function of a quantum system has remained as a kind of *taboo* during the whole history of quantum mechanics. About 13 years ago E. P. Wigner wrote [1]: *There is no way to determine what the wave function of a system is—if arbitrarily given, there is no way to "measure" its wave function.* [...] *In order to verify the [quantum] theory in its generality, at least a succession of two measurements are needed. There is in general no way to determine the original state of the system, but having produced a definite state by a first measurement, the probabilities of the outcomes of a second measurement are then given by the theory.* As a matter of fact, the possibility of measuring quantum states has remained at the level of mere speculation for years [3], and entered the realm of experiments only less than three years ago [4] in the domain of quantum optics. These real experiments started desecrating the taboo, and reopened the discussion on this delicate issue. In the meanwhile, the experimental availability of quantum nondemolition measurements stimulated a new debate on the possibility of determining the wave function of a single quantum system [5, 6, 7, 8, 9, 10]. Astonishingly, with H. Yuen I discovered that, despite its fundamental relevance in the logical framework of quantum mechanics, the general impossibility of measuring a single-system wave function seem to have remained unproved, and thus we decided to give a set of proofs [11] of the impossibility of determining the state of a single quantum system for arbitrary measuring schemes, including any succession of measurements.

In this paper I will show in some detail the crucial role of quantum efficiency in impeding the measurement of the wave function of a single system. Homodyne detection provides a concrete method for "measuring" the quantum state. The state is reconstructed by repeating many measurements of inequivalent observables (the different quadratures of the field) after re-preparing the field in the same state at each measurement. In Ref. [12] an exact technique was derived, that produces the number-state matrix elements by averaging functions of homodyne data. This opened the route to a rigorous treatment of this new kind of measurement, and in Ref. [13] the possibility of homodyning the density matrix was recognized even for nonideal detector quantum efficiency $\eta < 1$. There, it was proved that there is a lower bound η_* for η that depends on the chosen representation for the matrix, and that at best $\eta_* = 1/2$. For quantum efficiency $\eta \leq \eta_*$ the density matrix of the state cannot be reconstructed. The presence of such lower bound for quantum efficiency prevents measuring the wave function of a single system using schemes of weak repeated indirect measurements on the same system. In fact, weak disturbance corresponds to low quantum efficiency, and the need of an efficiency above the bound necessarily leads to a state disturbance, either because the interaction between system and apparatus should be sufficiently strong, or because the preparation itself of the apparatus has to be "strong".

After analyzing the meaning of quantum efficiency in Sect. 2 for some indirect measurement schemes in quantum optics, the method of homodyning the state of the field is briefly reviewed in Sect. 3. A scheme for a chain of homodyne measurements on the same copy of the field is analyzed in detail in Sect. 4, before giving conclusions in Sect. 5.

2 Quantum efficiency of indirect measurements

The photon-count distribution for a photodetector (with a photo-tube small with respect to the coherence length of radiation) is given by the Mandel-Kelley-Kleiner formula

$$P_\eta(n) = \left\langle : \frac{(\eta a^\dagger a)^n}{n!} \exp(-\eta a^\dagger a) : \right\rangle , \qquad (2.1)$$

where $\langle \ldots \rangle = \mathrm{Tr}[\hat{\rho} \ldots]$ is the usual ensemble average for density operator ρ, :: denotes normal ordering, and η $(0 \leq \eta \leq 1)$ is the quantum efficiency of the detector. For simplicity in Eq. (2.1) the case of monochromatic field is considered, with a denoting the annihilator of the nonvacuum mode. For $\eta = 1$ Eq. (2.1)

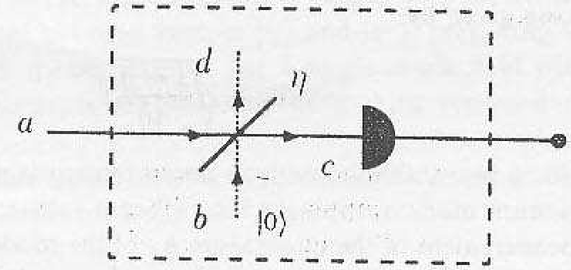

Figure 1: Equivalence of a nonideal ($\eta < 1$) detector with an ideal one preceded by a beam splitter of transmissivity η.

gives the output probability of ideal photon-number detection $P_1(n) = \langle |n\rangle\langle n|\rangle$. More generally, for $\eta < 1$, Eqs. (2.1) gives a Bernoulli convolution of the ideal probability. It is easy to show [14] that a detector with quantum efficiency $\eta < 1$ is equivalent to an ideal detector preceded by a beam splitter of transmissivity η, as schematically depicted in Fig. 1: the probability distribution of the photocurrent $c^\dagger c$ at the output can be rewritten in terms of the input field a as in Eq. (2.1). On the other hand, one can regard Fig. 1 also as a scheme for an indirect repeatable measurement of the number of photons $a^\dagger a$, by allowing mode a to interact with mode b, and then performing the measurement on mode c: mode d can thus be viewed as mode a after the evolution due to the measurement. Apart from trivial phase changes, the Heisenberg evolution of modes at the beam splitter is given by the unitary transformation of fields

$$\begin{pmatrix} c \\ d \end{pmatrix} = U^\dagger \begin{pmatrix} a \\ b \end{pmatrix} U = \begin{pmatrix} \eta^{1/2} & (1-\eta)^{1/2} \\ -(1-\eta)^{1/2} & \eta^{1/2} \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} . \tag{2.2}$$

It is clear that for $\eta = 1$ measuring c is equivalent to measuring a, and the measurement becomes exact. However, the field is totally absorbed, and no photon is available for a subsequent measurement on mode c. On the other hand, for $\eta = 0$ the measurement produces no disturbance, i. e. the output mode d is now equivalent to mode a. In this case no information on a is gained from the measurement on mode c. This simple example illustrates the *tradeoff between information gain and disturbance in quantum theory*: quantum efficiency parameterizes such tradeoff between information and disturbance. Nearly unit η means very informative measurement, however with strong disturbance on the field. Low η, on the contrary, means weakly disturbing measurements, however with scarce information on the field. This tradeoff can be taken as a fundamental principle of quantum theory, and in this sense the concept of quantum efficiency becomes a general feature of any repeatable measurement.

For homodyne detection the output photocurrent is just the quadrature x_ϕ of the field at phase ϕ with respect to the local oscillator (LO)

$$x_\phi = \frac{1}{2}\left(a^\dagger e^{i\phi} + a e^{-i\phi}\right) . \tag{2.3}$$

In this case, for detection with overall quantum efficiency $\eta < 1$ the probability distribution becomes a Gaussian convolution of the ideal probability, with additional r. m. s. noise given by

$$\Delta_\eta \equiv \sqrt{\frac{1-\eta}{4\eta}} . \tag{2.4}$$

A scheme for a repeatable homodyne measurement is depicted in Fig. 2. The field with nonvacuum mode a impinges into a beam splitter with reflectivity η. The repeatable measurement of the quadrature $\hat{a}_\phi$ of the mode a is achieved by measuring the quadrature $\hat{c}_\phi$ of mode c, namely by performing the measurement on another mode b after the interaction with a [one can regard mode c as the Heisenberg-picture evolution of b, with unitary transformation given by Eq. (2.2)]. As we will see shortly in Section 4 a repeatable homodyne measurement with reflectivity η has the same output probability distribution of a non-repeatable measurement with quantum efficiency η, namely the probability distribution is a Gaussian convolution of the ideal probability, with additional r. m. s. noise given by Eq. (2.4).

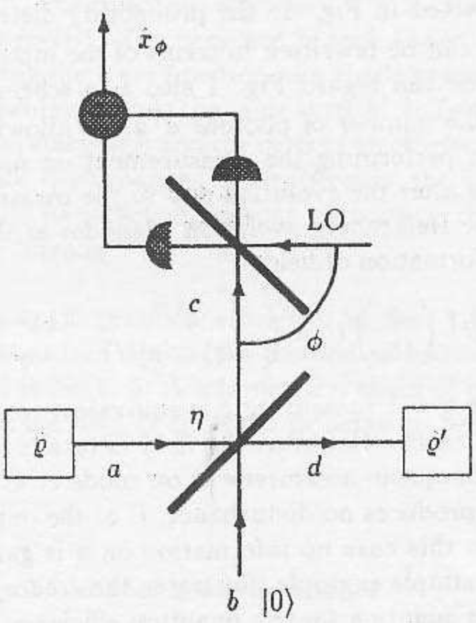

Figure 2: Scheme of a repeatable homodyne measurement.

3 Brief review of the method of homodyning the quantum state

Homodyning the quantum state of the field [12] is the only known experimental method for measuring the state of a quantum system. More precisely, the method allows the determination of the matrix elements $\langle\psi|\hat{\varrho}|\psi'\rangle$ of the density operator $\hat{\varrho}$ of the electromagnetic field between vectors $|\psi\rangle$ and $|\psi'\rangle$, preparing the field again in the same state at each measurement. For a single mode field with annihilator operator a the matrix elements are obtained by making repeated measurements of the quadrature operators $\hat{a}_\phi = \frac{1}{2}(a^\dagger e^{i\phi} + a e^{-i\phi})$ at different phases ϕ. Before analyzing a repeatable measurement scheme for a single system in Section 4 let us briefly recall the basics of the method.

The density operator is connected to the probabilities $p(x,\phi)$ of the outcomes of the quadratures $\hat{a}_\phi$ according to the identity [13]

$$\hat{\varrho} = \int_0^\pi \frac{d\phi}{\pi} \int_{-\infty}^{+\infty} dx\, p(x,\phi)\, K_\eta(x - \hat{a}_\phi)\,, \tag{3.5}$$

where the kernel $K_\eta(x)$ given by

$$K_\eta(x) = \frac{1}{2}\mathrm{Re}\int_0^{+\infty} dr\, r \exp\left(\frac{1-\eta}{8\eta}r^2 + irx\right) \tag{3.6}$$

depends parametrically on the detector quantum efficiency η. In a real experiment, according to Eq. (3.5) the density matrix elements $\langle\psi|\hat{\varrho}|\psi'\rangle$ are measured by averaging the kernels $\langle\psi|K_\eta(x-\hat{a}_\phi)|\psi'\rangle$ over the experimental data (x,ϕ), provided that $\langle\psi|K_\eta(x-\hat{a}_\phi)|\psi'\rangle$ are bounded as a function of x and ϕ. Despite the kernel $K_\eta(x)$ is not even a tempered distribution. the matrix elements $\langle\psi|K_\eta(x-\hat{a}_\phi)|\psi'\rangle$ are bounded if the following inequality is satisfied for all phases $0 \le \phi \le \pi$

$$\eta > \frac{1}{1+4\varepsilon^2(\phi)}\,, \tag{3.7}$$

where

$$\frac{2}{\varepsilon^2(\phi)} = \frac{1}{\varepsilon^2_\psi(\phi)} + \frac{1}{\varepsilon^2_{\psi'}(\phi)} \tag{3.8}$$

and $\varepsilon^2_\psi(\phi)$ is the "resolution" of the vector $|\psi\rangle$ in the x_ϕ-representation, namely:

$$|{}_\phi\langle x|\psi\rangle|^2 \simeq \exp\left[-\frac{x^2}{2\varepsilon^2_\psi(\phi)}\right]\,. \tag{3.9}$$

In Eq. (3.9) the symbol $\simeq$ stands for the leading term as a function of x, and $|x\rangle_\phi \equiv e^{ia^\dagger a\phi}|x\rangle$ denote eigenkets of the quadrature x_ϕ. Upon maximizing Eq. (3.7) with respect to ϕ one obtains the bound

$$\eta > \frac{1}{1+4\varepsilon^2}\,,\qquad \varepsilon^2 = \min_{0\le\phi\le\pi}\{\varepsilon^2(\phi)\}\,. \tag{3.10}$$

One can immediately see that the bound is $\eta > 1/2$ for both number-state and coherent-state representations, whereas for squeezed-state representations one has $\eta > (1+s^2)^{-1} \ge 1/2$, where $s < 1$, is the smallest squeezing semi-axe. From this last case one can see that that $\eta = 1/2$ is an absolute bound for η when homodyning the quantum state (see also Ref. [15]), and generally, for any representation there is a lower bound $\eta_* \geq 1/2$ above which the density matrix can be obtained. Hence, in order to measure the density matrix by homodyne tomography, a sizeable quantum efficiency $\eta > \eta_*$ is needed.

4 On the impossibility of measuring the density matrix of a single system

In the previous section the tomographic scheme that we have considered is based on "second kind" measurements, which completely destroys the quantum state of the system. Many measurements are performed, but the system is prepared in the same state $\hat{\varrho}$ before each measurement. We want now to consider a "first kind" version of the above scheme that in principle allows to measure the density matrix without destroying it. Similarly to any first kind measurement, this goal can be achieved using an indirect-measurement, namely by performing second-kind measurements on another "probe" mode b that interacts with a via a unitary operator U. The scheme for repeated measurements is depicted in Fig. 3, which is just a chain of indirect homodyne measuring devices as in Fig. 2. The present scheme physically corresponds to let the field mode a shine over a long chain of low-transmissivity mirrors, detecting the quadrature of the weak transmitted field at each mirror.

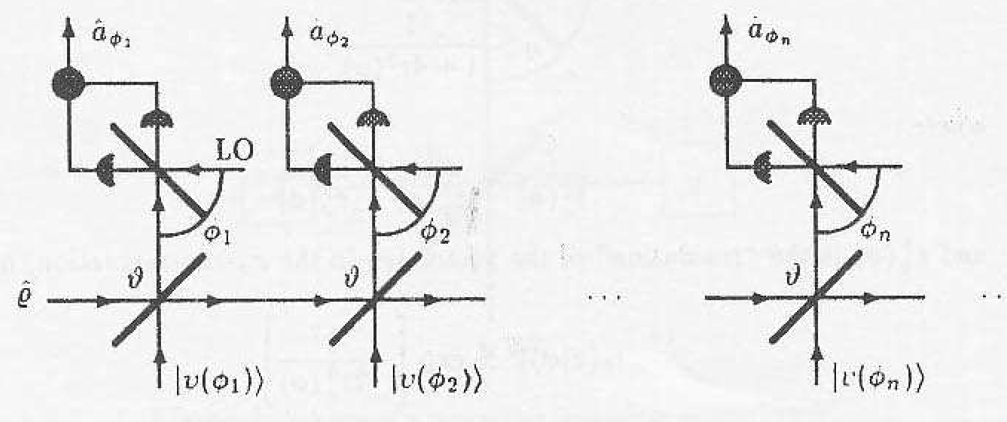

Figure 3: Scheme for repeated quadrature measurements on the same single system.

Without loss of generality, we consider that before every single measurement the probe is prepared in a pure state $|v(\phi)\rangle$, which is generally optimized as a function of the observable $\hat{b}_\phi$ that is measured. The generating function of the moments of $\hat{b}_\phi$ after the interaction with a is given by

$$X(\lambda, \phi) = \mathrm{Tr}\left[\exp\left(i\lambda U^\dagger \hat{b}_\phi U\right) \hat{\varrho} \otimes |v(\phi)\rangle\langle v(\phi)|\right], \qquad (4.11)$$

and is just the Fourier transform of the probability distribution of the experimental outcomes. We consider the interaction given in Eq. (2.2), but here with mirror transmissivity denoted by ϑ. The evolution of the b mode now reads

$$c = U^\dagger b U = \sin\kappa\, a + \cos\kappa\, b \equiv \vartheta^{1/2}\, a + (1-\vartheta)^{1/2}\, b \,. \tag{4.12}$$

Due to the linearity of Eq. (4.12), the moment generating function factorizes in the following way

$$X(\lambda,\phi) = \chi_a(\vartheta^{1/2}\lambda,\phi)\chi_b((1-\vartheta)^{1/2}\lambda,\phi)\,, \tag{4.13}$$

where $\chi_a(\lambda,\phi) = \mathrm{Tr}[\exp(i\lambda a_\phi)\varrho]$ is the generating function for the noninteracting mode a only, and, analogously, $\chi_b(\lambda,\phi)$ for the mode b. Using the operator identity

$$\varrho = \int \frac{d^2\alpha}{\pi}\,\mathrm{Tr}(\varrho e^{-\bar{\alpha}a+\alpha a^\dagger})\, e^{-\alpha a^\dagger + \bar{\alpha} a} \tag{4.14}$$

the density operator $\hat{\varrho}$ is written in terms of the generating function $\chi_a(\lambda,\phi)$

$$\varrho = \int_0^\pi \frac{d\phi}{\pi}\int_{-\infty}^{+\infty}\frac{d\lambda\,|\lambda|}{4}e^{-i\lambda\hat{a}_\phi}\chi_a(\lambda,\phi)\,, \tag{4.15}$$

and using Eq. (4.13) one obtains

$$\varrho = \int_0^\pi \frac{d\phi}{\pi}\int_{-\infty}^{+\infty} dx\, p_\vartheta(x,\phi)\Xi_\vartheta(x - \hat{a}_\phi)\,, \tag{4.16}$$

where $p_\vartheta(x,\phi)$ is the probability of the measured quadrature b_ϕ rescaled by $\vartheta^{1/2}$

$$p_\vartheta(x,\phi) = \vartheta^{1/2}\int_{-\infty}^{+\infty}\frac{d\lambda}{2\pi}e^{-i\lambda\vartheta^{1/2}x}X(\lambda,\phi)\,. \tag{4.17}$$

The kernel $\Xi_\vartheta(x)$ in Eq. (4.16) is given by

$$\Xi_\vartheta(x) = \frac{1}{2}\mathrm{Re}\int_0^{+\infty} d\lambda\,\lambda e^{i\lambda x}[\chi_b(\lambda\sqrt{(1-\vartheta)/\vartheta},\phi)]^{-1}\,, \tag{4.18}$$

and generally depends on the coupling parameter ϑ and on the probe state $|v(\phi)\rangle$. One can easily see that when the probe mode b is in the vacuum state the kernel (4.18) is identical to $K_\eta(x)$ in Eq. (3.6) with $\vartheta \equiv \eta$, namely, the transmissivity ϑ plays the role of the overall quantum efficiency of the indirect measurement. However, the effective quantum efficiency can be decreased at will by squeezing the probe mode b in the direction of the quadrature b_ϕ. More precisely, one prepares the probe in the squeezed vacuum

$$|v(\phi)\rangle \equiv S_\phi|0\rangle\,, \tag{4.19}$$

where

$$S_\phi = e^{ib^\dagger b\phi}S_0 e^{-ib^\dagger b\phi}\,, \quad S_0 = \exp\left[-\frac{r}{2}\left(b^{\dagger 2} - b^2\right)\right]. \tag{4.20}$$

$r > 0$ denoting the squeezing parameter. One has

$$S_0 b_0 S_0^\dagger = e^r b_0 \, , \qquad S_0^\dagger |x\rangle = e^{r/2} |e^r x\rangle \, . \tag{4.21}$$

with $|x\rangle$ denoting the eigenvector of b_0 for eigenvalue x: the rescaling (4.21) more generally holds for the quadrature b_ϕ and its eigenvectors $e^{ib^\dagger b\phi}|x\rangle$ when using the rotated squeezing operator S_ϕ in place of S_0. With the help of transformations (4.20) and (4.21) it is easy to check that the kernel $\Xi_\vartheta(x)$ in Eq. (4.18) coincides with $K_\eta(x)$ in Eq. (3.6) with effective efficiency

$$\eta \equiv \frac{e^{2r}\vartheta}{e^{2r}\vartheta + 1 - \vartheta} \, . \tag{4.22}$$

Therefore, by increasing the squeezing parameter r it is possible to enhance the effective quantum efficiency η beyond the allowed bound for measuring the density matrix ($\eta > 1/2$ for number and coherent states). At this point one may think that squeezing the vacuum of b allows one to consider weaker and weaker interactions with $\vartheta \to 0$, with the possibility of performing repeated measurements on the same system with vanishing perturbation at each measurement. However, as we will show immediately, this cannot be attained, because the squeezing needed to keep η as constant also amplifies the perturbation back to a finite extent. This can be seen upon analyzing the limiting behavior of the transition operator $\Omega(x,\phi)$, that gives the state reduction after each measurement

$$\varrho' = \frac{\Omega(x,\phi)\varrho\Omega^\dagger(x,\phi)}{\mathrm{Tr}[\varrho\Omega^\dagger(x,\phi)\Omega(x,\phi)]} \, . \tag{4.23}$$

Here, the transition operator $\hat\Omega(x,\phi)$ is given by

$$\hat\Omega(x,\phi) = \vartheta^{1/4}\langle\vartheta^{1/2}x|e^{-ib^\dagger b\phi}e^{\kappa(ab^\dagger - a^\dagger b)}S_\phi|0\rangle \, , \tag{4.24}$$

where the powers of ϑ account for quadrature rescaling, and one should keep in mind that the matrix element is evaluated between vectors $|\vartheta^{1/2}x\rangle$ and $|0\rangle$ in the Hilbert space of mode b only, so that $\hat\Omega(x,\phi)$ is an operator on the Hilbert space of mode a. One has

$$\hat\Omega(x,\phi) = \vartheta^{1/4}\langle\vartheta^{1/2}x|\hat S_0 e^{-ib^\dagger b\phi}e^{\kappa(a_r b^\dagger - a_r^\dagger b)}|0\rangle \, , \tag{4.25}$$

where

$$a_r = \cosh r\, a + e^{2i\phi}\sinh r\, a^\dagger \, . \tag{4.26}$$

Using Eq. (4.21) and normal ordering the interaction operator with respect to b, one obtains

$$\hat\Omega(x,\phi) = (e^{2r}\vartheta)^{1/4}\langle(e^{2r}\vartheta)^{1/2}x|e^{-ib^\dagger b\phi}e^{\tan\kappa\, a_r b^\dagger}|0\rangle|\cos\kappa|^{a_r^\dagger a_r} \, . \tag{4.27}$$

Eq. (4.27) can be rewritten as follows

$$\hat{\Omega}(x,\phi) = \left(\frac{2e^{2r}\vartheta}{\pi}\right)^{1/4} \exp\left[-\left(e^{2r}\vartheta x - \tan\kappa e^{-i\phi}a_r\right)^2\right] \times \exp\left[\frac{1}{2}\tan^2\kappa e^{-2i\phi}a_r^2\right] |\cos\kappa|^{a_r^\dagger a_r} . \quad (4.28)$$

We can now finally evaluate the asymptotic form of the transition operator in Eq. (4.28) in the simultaneous limits of vanishing transmission coefficient $\vartheta \to 0$ and infinite squeezing parameter $r \to \infty$ keeping the effective quantum efficiency η as constant according to Eq. (4.22). In Eq. (4.28), for $\vartheta \to 0$ using Eq. (4.12) one obtains $e^{2r}\vartheta \to \Delta_\eta^{-2}$, where

$$\Delta_\eta^2 \equiv \frac{1-\eta}{\eta} . \quad (4.29)$$

Moreover, one has $e^{-r}e^{-i\phi}a_r \to a_\phi$, and $|\cos\kappa|^{a_r^\dagger a_r} \to \exp[-a_\phi^2/(2\Delta_\eta^2)]$. Therefore, the asymptotic form of the transition operator is simply given by

$$\hat{\Omega}(x,\phi) = \left(\frac{2}{\pi\Delta_\eta^2}\right)^{1/4} \exp\left[-\frac{(x-\hat{a}_\phi)^2}{\Delta_\eta^2}\right] . \quad (4.30)$$

Equation (4.30) is the typical form of a von Neumann "reduction" of the state. Therefore, one concludes that despite the interaction has been tuned as vanishingly small, the state is in fact "reduced" at each measuring step.

5 Conclusions

In conclusion we have seen in which way any scheme of repeated weak measurements on the same single quantum system is doomed to fail in determining the wave function. The lower bound for quantum efficiency for measuring the state of an ensemble of systems is of fundamental relevance in impeding the measurement of the individual wave function. There is always a tradeoff between information and disturbance: weak disturbance means low quantum efficiency, hence low information; high quantum efficiency means maximum information, however, it means also strong disturbance. High quantum efficiency can be achieved even with vanishingly weak interaction between system and apparatus, but the overall perturbation remains sizeable, due to the need for "strong" preparation of the apparatus in order to enhance the efficiency. If the density matrix could be detected for vanishingly small quantum efficiency, then there would be no need for strong preparation of the apparatus, and the density matrix of a single system could be detected by successions of repeated vanishingly weak measurements.

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