Multiple Path Interferometers

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Abstract. Linear optical multi-port couplers are used in implementing interferometers with M paths. We show that they provide a phase sensitivity that re-scales as $\Delta\varphi \propto M^{-1}$ versus the number of paths.

Linear optical couplers are the simplest devices that one can use to couple different modes of the radiation field. The beam splitter is the lowest dimensional example, yet it allows to show a number of interesting features of nonclassical states of light [1]. Recently, novel nonclassical effects have been investigated and higher dimensional Bell-type experiments have been suggested that involve multi-port linear couplers [2]. Here we will analyze higher dimensional interference from such multi-port couplers, for application in high sensitive interferometry [3]. We will show that an interferometer with M paths shows a phase sensitivity that re-scales as $\Delta\varphi \propto M^{-1}$, even when the interferometer is supplied with customary coherent laser light.

In conventional Mach-Zehnder interferometers, impinged with coherent light, sensitivity in monitoring phase-shifts is limited by the shot-noise fluctuations $\Delta\varphi \propto N^{-1/2}$, N being the mean photon number of the incoming beam. This allows to measure minute variations of the optical path of a light beam, corresponding to tiny shifts. However, in recent applications—as for detecting gravitational waves—extremely accurate measurements are needed. Many efforts have been made in order to optimize the quantum state at the input of the interferometer [4, 5, 6, 7], improving sensitivity up to the ultimate limit $\Delta\varphi \propto N^{-1}$ of quantum estimation theory [9, 10]. Indeed, it has been recently shown that such sensitivity can be reached in heterodyne interferometry [8], upon suitable preparation of the two-mode (signal + idler) input state. The approaches of Refs. [4, 5, 6, 7, 8] provide nice and powerful examples on how manipulate and redirect quantum fluctuations: however, they still suffer stability problems, as it is still difficult to retain the required quantum correlation within the decoherence time, which has to be compared with the relevant time scale of the phenomena under study (the period of a gravitational wave $\tau \simeq 10^{-3} sec.$, as an example).

Here we deal with a different approach to the interferometry problem. We consider a multipath arrangement where a coherent input signal is split into many beams by a $2M$-port multi-splitter. As it will be shown in the following, interference among an increased number M of available paths leads to an improvement of the phase sensitivity of a factor M^{-1}.

Multipath interferometers can be built by commercial optical components, and our result on sensitivity has been already checked for the lowest case $M = 3$ [14]. In this paper we briefly describe the operation of the general M case: readers interested in a detailed analysis are addressed to Ref. [3]. First we deal with optical M-

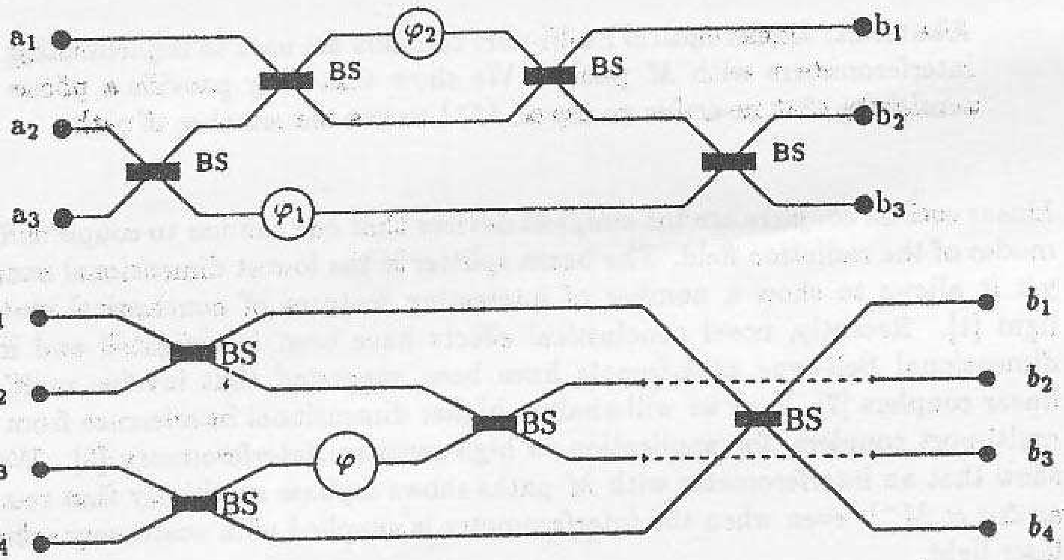

Figure 1: How to implement the lowest dimensional multi-splitter $M = 3, 4$ by suitable configuration of beam slitter and phase shifter. For the tritters we need $\varphi_1 = \arccos(1/3)$ and $\varphi_2 = \varphi_2/2$ whereas the quarter requires $\varphi = \pi/2$.

port couplers. These are generalizations of the customary lossless beam splitter, which connect a set of M input modes $\mathbf{a} = (a_1, ..., a_M)$ to a new set of output modes $\mathbf{b} = (b_1, ..., b_M)$ by a linear transformation. Quantum mechanics imposes that such transformations belong to the matrix group $SU(M)$ [the beam splitter is described by the $SU(2)$ group]. The further requirement of full symmetry among input modes leads to the canonical unitary matrix for the multi-splitter

$$A_{kj} = \frac{1}{\sqrt{M}} \exp\left[\frac{2\pi i}{M}(k-1)(j-1)\right] . \qquad (1)$$

This mathematical description has a very profitable physical consequence. In fact, a $SU(M)$ transformation can always be decomposed into the product of $SU(2)$ transformations and $U(1)$ phase shifts [12] (the decomposition being not unique).

This means that a multi-splitter can be built as a suitable setup of beam splitters and phase shifters. In Fig.1 we report the schematic diagrams of the lowest dimensional cases for $M = 3$ and 4, realized by symmetric beam splitters and suitable phase shifters. Tritter couplers have been recently implemented also by optical fiber technology [14], whereas quarters are well known in quantum optics in form of eight-port homodyne detectors [11], and have been experimentally used for an operational definition of the quantum phase [10, 13]. A multipath interferometer can

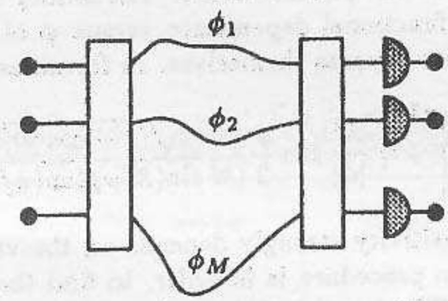

Figure 2: Schematic diagram of a multipath interferometer experimental setup.

be implemented by two multi-splitters, generalizing the customary Mach-Zehnder interferometer. A schematic diagram of the experimental setup is reported in Fig. 2. In the first part of the device a coherent beam provided by a stable laser source is symmetrically split into M modes. Each of these modes undergoes a different phase shift $\exp\{i\hat{n}_k\phi_k\}$ due to a different optical path, $\hat{n}_k$ being the photon number operator of the considered mode. Modes are then recombined by a second multi-splitter, and subsequently detected by M identical photodetectors. The whole device is described by an unitary matrix $\mathbf{T}$ whose elements are given by

$$T_{kj}(\phi_1, ..., \phi_M) = \frac{1}{M}\sum_{l=1}^{M} \exp\left\{ i \left[\phi_l + \frac{2\pi}{M}(l-1)(k+j-2)\right]\right\} . \qquad (2)$$

As input state of the interferometer we consider a coherent state entering the first port $|\Psi\rangle = |\alpha\rangle_1 \otimes_{k=2}^{M} |0\rangle_k$, α being the amplitude of the laser beam. This leads to a straightforward evaluation of the output probability distribution, which can be written as product of independent Poisson distributions for each photocurrent as follows

$$P(I_1, ..., I_M) = \prod_{k=1}^{M} e^{-|\beta_k|^2} \frac{|\beta_k|^{2I_k}}{I_k!} , \qquad (3)$$

with $|\beta_k|^2 = \langle \hat{I}_k \rangle = \langle \widehat{\Delta I_k^2} \rangle = |\alpha|^2 |T_{1k}(\phi_1, ..., \phi_M)|^2$. We now focus on a special configuration, that corresponds to monitor the same fixed phase shift φ among contiguous pairs of modes, i. e. $(\phi_{k+1} - \phi_k) = \varphi$, $k = 1, ..., M-1$. The relevant matrix elements are now given by

$$|T_{1k}(\varphi, 2\varphi, ..., M\varphi)|^2 \equiv F_M^k(\varphi) = \frac{1}{M^2} \frac{1 - \cos[M(\varphi + \theta_k)]}{1 - \cos(\varphi + \theta_k)}, \tag{4}$$

where $\theta_k = \frac{2\pi}{M}(k-1)$, $k = 1, ..., M$. Any perturbation of φ will produce changes in the output distribution of photocurrents. The ability in resolving these changes depends on both the functional dependence versus φ of the mean value, and the fluctuations of the photocurrents themselves. In formulas, one has

$$\Delta\varphi \equiv \sqrt{\langle \Delta \hat{I}_1^2 \rangle} \left| \frac{\delta \langle \hat{I}_1 \rangle}{\delta \varphi} \right|^{-1} = \frac{M}{|\alpha|} \tan\frac{\varphi}{2} \frac{\sqrt{(1 - \cos\varphi)[1 - \cos(M\varphi)]}}{|M \sin(M\varphi)\tan(\varphi/2) + \cos(M\varphi) - 1|} .$$

Clearly, the actual sensitivity strongly depends on the value φ of the phase shift. Hence, an optimization procedure is in order, to find the best working point φ_{wp} of the interferometer. The functional dependence versus φ is identical for all photocurrents, with just additional shifts θ_k. We now optimize only one particular photocurrent: optimization could be obviously repeated for each current (however, the final result would be just a set of M equivalent optimal configurations). For two and three-path interferometers the minimization procedure can be carried out analytically, and the result agrees with the known results for the Mach-Zehnder interferometer and the tritter in Ref. [14]. In the general case $M \geq 4$ the optimization procedure is easily performed numerically. The absolute minimum of sensitivity $\Delta\varphi_M$ in Eq. (5) is given by

$$\varphi_{wp}^{(M)} \simeq \frac{A}{M}, \qquad \Delta\varphi_M \simeq \frac{1}{|\alpha|} \frac{A-2}{M}, \tag{5}$$

where A is a numerical constant (from a best fit in the range $4 \leq M \leq 1000$ we obtained $A \simeq 4.27$).

We now illustrate this result on the basis of numerically simulated experiments. The corresponding plots of the I_1 photocurrent outcomes are reported in Fig. 3. We consider three different multipath interferometers, with $M = 3, 30, 60$, all of them fed by a laser beam of mean photon number $|\alpha|^2 = 100$ (a quite low intensity) each operating at the best working point. In a time interval $t_2 - t_1$ (very large relative to the duration of each measurement shot) some perturbation is "switched on", changing the phase shift between neighboring paths from φ_{wp} to $\varphi_{wp} + 10^{-2} rad$. Correspondingly, the statistics at the output is changed, and the benefit from the increasing number of paths becomes apparent. For a three-path interferometer the distribution of I_1 outcomes is slightly changed (the same perturbation would be utterly undetectable in a conventional Mach-Zehnder interferometer supplied by the same laser intensity): in this case, a careful analysis would be needed, in order to distinguish an actual perturbation from a false alarm. For $M = 30$ and $M = 60$ the phase shift becomes detectable very easily, and this is well evident in Fig. 3. In conclusion, a high sensitive interferometric scheme based on multi-splitter linear

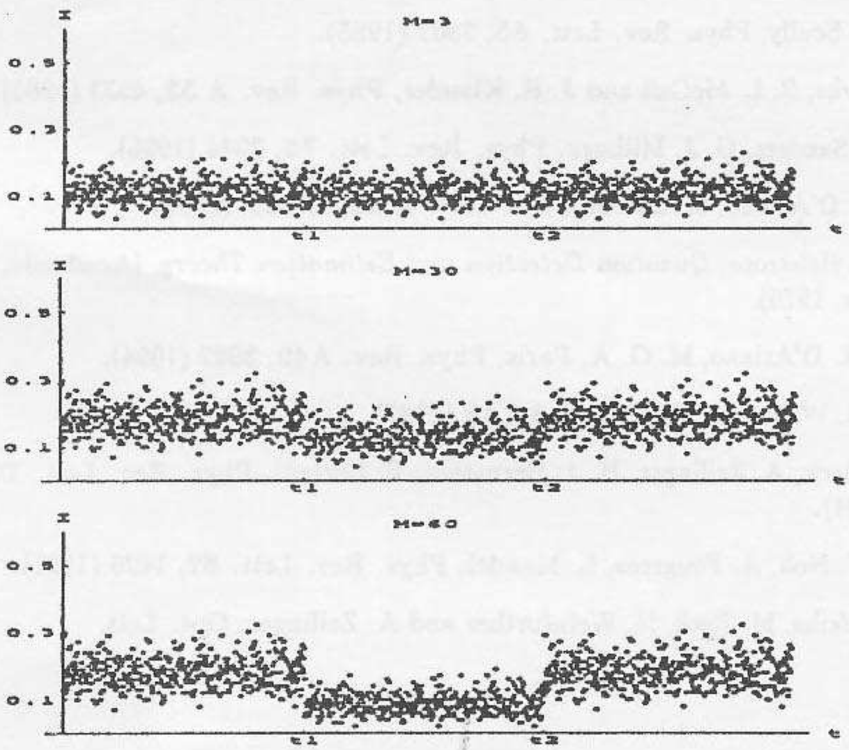

Figure 3: Numerical simulations of real trial experiments. The outcome of photocurrent I_1 are reported for three different multipath interferometers $M = 3, 30, 60$ subjected, in the time interval $(t_2 - t_1)$ to the same shift of $10^{-2} rad$ from their respective working point. The intensity of the incoming coherent beam is $|\alpha|^2 = 100$ and the photocurrent outcomes are reported in unit of $|\alpha|^2$.

couplers has been presented. It can be implemented either as an *all-fiber* device, or by discrete optical components (beam splitters and phase shifters). The increasing number M of paths makes the interferometer more sensitive, with sensitivity linearly improving versus M. Thus, the present proposed scheme provides a way to achieve arbitrary precision at fixed amount of energy impinged into the apparatus.

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