Chapter 11
Quantum-Informational Principles for Physics

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Abstract It is time to to take a pause of reflection on the general foundations of physics, for re-examining the logical solidity of the most basic principles, as the relativity and the gravity-acceleration equivalence. The validity at the Planck scale of such principles is under dispute. A constructive criticism engages us in seeking new general principles, which reduce to the old ones only in the already explored domain of energies. At the very basis of physics there are epistemological and operational rules for the same formulability of the physical law and for the computability of its theoretical predictions. Such rules give rise to new solid principles, leading us to a quantum-information theoretic formulation, that hinges on the logical identification of the experimental protocol with the quantum algorithm.

The information-theoretic program for physics foundations has already been advocated in the past by several authors [1]. Recently the program succeeded in deriving the full structure of quantum theory from informational principles [2–5], and we will very briefly examine them here, as exemplars of good principles. The problem is now to extend the informational program to relativistic quantum field theory, the most fundamental theoretical structure of physics. The plan here proposed is to ground quantum field theory on two new principles pertaining only the formulability and computability of the physical law: (1) the Deutsch-Church-Turing principle, and (2) the topological homogeneity of interactions. As we will see, in conjunction with the principles of quantum theory, these two new principles entail a quantum cellular automata extension of quantum field theory.

The quantum automaton extends field theory in the sense that it includes localized states and measurements, for whose description quantum field theory is largely inadequate. The quantum automaton doesn't suffer any formal violation of causality, e.g.

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superluminal tails of the probability distributions. It is not afflicted by any kind of divergence, being exactly computable by principle. Relativistic covariance and other symmetries are violated, but are recovered at the usual scale of energy.

The generality of the new principles does not deplete them of physical content. On the contrary, the Dirac automaton—the most elementary theory of this kind—despite its simplicity leads us to unexpected interesting predictions, e.g. it anticipates a maximum mass for the Dirac particle just as a consequence of the unitarity of quantum evolution, without invoking black-hole general-relativity arguments. It also opens totally unexpected routes for redefining mechanical notions. As regards gravity, the theory seems to suggest the route of the emergent thermodynamic force of Jacobson-Verlinde [6, 7], here, specifically, as a purely quantum-digital effect of the Dirac automaton.

Good and Bad Principles

Which kinds of principles are good and which are bad? We can limit ourself to four main different types of principles: (1) dogmatic, (2) empirical, (3) simplifying (or conventional), and (4) epistemological.

The dogma. This is definitely the worst case. Do we have dogmas in physics? We have few subtle ones. It is not a blasphemy to regard the non existence of an absolute reference frame as a dogma. What about the reference frame of the background radiation? We indeed always invoke the frame of "fixed stars" for establishing if a frame is inertial. The denial of the existence of an absolute frame is a relic of the anthropocentrism repudiation that followed the Keplerian revolution. We will come back to this dogma later.

The empirical principle. A principle is empirical if it has no logical motivation other than its empirical evidence. A typical example is the Einstein's equivalence principle: the identity between inertial and gravitational mass is an observed fact. But do we have a good reason for it? The principle implies that the trajectory of a mass in a gravitational field is independent on the mass, and this leads us to reinterpret gravity as a property of space—the starting point of general relativity theory, which is then a re-interpretation of the principle, not a logical motivation. Another relevant example of empirical principle is the invariance of the speed of light with the reference frame—quite an odd one, isn't it? This lead Einstein to his first formulation of special relativity. The principle was later recognized by Einstein himself to be only an instance of the more general Galilei principle (the invariance of the physical law with the reference frame) upon including the laws of electromagnetism: this was definitely a great logical improvement. The empirical ones are good temporary practical principles when we relinquish further explanation.

The simplifying principle. A simplifying principle is an unfalsifiable conventional assumption that abridges the formulation of the physical law. An example of such kind of principle is the assumption of homogeneity of time (it is impossible to compare two different time-intervals in temporal sequence). But a purported non-homogeneity of time would introduce only an unnecessary functional time-parametrization in the physical law. Another example is the assumption that the speed of light be isotropic in space. Reichenbach [8] correctly argued that in order to determine simultaneity of distant events we need to know the light speed, but in order to measure the light speed we need to establish simultaneity of two different events for synchronizing clocks, and this leaded to a logical loop. What we can do? Using a single clock we can only determine the two-way average speed of light on a closed path. Reichenbach wrote indeed unconventional Lorentz transformations for non-isotropic light speed, with the only result of introducing an additional anisotropy parameter that is utterly irrelevant in practice. In conclusion: the simplifying principles are good ones, but we must keep in mind their conventional nature.

The epistemological principle. This is the most solid kind of principle: a principle that cannot be violated, even "in-principle", because its violation would involve contradicting the scientific method itself. Somebody would argue that claiming principles only of this kind would be equivalent to claiming an "ultimate theory". This is true. But should this be a good reason for not seeking principles of this kind, and for evaluating their ultimate logical consequences? Clearly, to be a principle for physics it cannot involve only pure logic: it must also incorporate the basic axiomatic of the physical experiment. G. Ludwig has been a great advocate of such kind of principles [9]. Einstein himself formulated special relativity in terms of precise protocols for synchronizing clocks in order to establish coordinate systems. In the recent literature operational axiomatic frameworks of this kind have emerged for quantum theory, later converging to a unified framework [2, 10–12]. The basic notions—tests, events and systems—make the framework equivalent to a category theory for physics [13]. At the same time, it is also the skeleton axiomatization of a general information theory, and, as mentioned, it ultimately leads to the informational axiomatization of quantum theory [2–5]. A remarkable fact about the operational approach is that it logically identifies the experimental protocol with the computer algorithm, providing a stronger logical connection between theory and experiment.

The Relativity Principle

The relativity principle of Galilei and Einstein seems to possess a definite epistemological character, since it establishes the independence of the physical law from the reference system, apparently a necessary requirement for the law formulation and experimentation. The principle instead is based on the "no-absolute" dogma, and nothing forbids defining the law within an absolute frame as long as we are able to translate it to any other frame (which is what we actually do when we invoke the "fixed stars" frame). This viewpoint may look as a sacrilege, it is the only logical possibility for violations of Lorentz covariance.

The Causality Principle

Causality has been always a taboo in physics. It is a principle underlying all modern physics, and has been central to debates on the foundations of relativity and quantum mechanics for over a century. Despite this, there is still a philosophical train of thought arguing that the causality notion should be removed from physics. B. Russell was one of the major advocates of this opinion [14].

On the other hand, causality is such a natural assumption that is often overlooked as an axiom (see e.g. the first quantum axiomatization work of Hardy [10]). Instead, it is the first of the informational axioms of quantum theory [2], also referred to as "no signaling from the future". In simple words it says: in a cascade of measurements on the same system, the outcome probability of a measurement does not depend on the choice of the measurement performed at the output. The principle also implies no-signaling without interaction—shortly "no-signaling", and also commonly known as "Einstein causality". I should make now clear that, being causality an axiom of quantum theory, any information purportedly originated in the future, as a time travel, would logically constitute a falsification of the theory. For example, it would mean to require nonlinearities in state evolution, or other variations of the theory.

As we will see later, in the present informational context special relativity emerges as an approximate principle due to the joint implication of three principles: (1) the causality principle, (2) the Deutsch-Church-Turing principle, and (3) the principle of topological homogeneity of interactions.

The problem of physical causation is a huge topic in philosophy, and a thorough discussion would take a thick volume. For the philosopher disbeliever I just want to add that the reconciliation with the Humean position (that causality is just a human way of looking at phenomena) passes through the probabilistic nature of the causal link stated in the axiom, which involve the comparison between two probabilities: the Humean viewpoint corresponds to the Bayesian interpretation of probability [15].

If causality cannot be proved, it can be falsified, as for any other scientific theory. How? By considering any binary test that is granted to be deterministic, namely to have zero probability for one outcome: if operating at the output of the test we can make this same outcome to happen, then we can logically claim a signaling from the future, given for granted the apparatus and its preparation.

Causal reasoning has always been a basic methodology in physics and in science generally, but the romantic dream of a time travel keeps a sentiment against it alive.

Informational Principles for Quantum Theory

In addition to causality, there are five other informational principles that are needed for deriving quantum theory [2]: (ii) local tomography, (iii) perfect distinguishability, (iv) atomicity of composition, (v) ideal compressibility, and (vi) purification. All six principles apart from (vi) hold for both classical and quantum information: only the purification one singles out quantum theory.

The information-theoretical framework hinges around the notion of event, which can occur probabilistically and has inputs and outputs systems. A complete collection of such events with overall unit probability is what is called *test*—physically a measurement instrument. The systems are just the usual physical systems. Informationally, tests and events represent subroutines, whereas the systems are registers on which information is read and written. Axiom (ii) (stating that joint states of multiple systems can be discriminated by measurements on single systems) has become popular [16], since it reconciles the holism of quantum theory with the reductionism of the experimental approach [17]. Axiom (iii) is crucial for hypothesis falsification, and reconciles probabilism with logic. Axiom (iv) establishes that maximal knowledge of two transformations implies maximal knowledge of their composition, a requirement that seems obvious indeed. The compression axiom (v) is the one that leads to the notion of sub-systems (e.g. the *qubit* is a subsystem of the *qutrit*). It entails the possibility of addressing separately the unknown from the perfectly known. Finally, the purification postulate (vi) informally speaking is the principle of "conservation of information". In simple words it says that irreversibility and mixing can be always regarded as the result of discarding an environment, otherwise everything is describable in terms of pure states and reversible transformations. Another informal way of stating the principle is that ignorance about a part is always compatible with the maximal knowledge about the whole.

The six principles for quantum theory have nothing of "mechanical" nature: what I call "quantum theory" is just the "theory of systems", i.e. the mathematical framework of Hilbert spaces, algebra of observables, unitary transformations. It has no bearing on the "mechanics", namely particles, dynamics, quantization rules: for these the name "quantum mechanics" would be more appropriate. Quantum mechanics, however, is just a small portion of the more general quantum field theory, which itself is a theory of systems: the quantum fields. The only mechanical elements remaining in quantum field theory are the so-called "quantization rules" (or the path-integral) that one may want to avoid in order to make the theory completely autonomous from the classical theory, whereas, reversely, it should be classical mechanics to be derived as an approximation of quantum field theory via a "classicalization" rule. But, how can we formulate a field theory that is quantum *ab initio*? We need more informational principles, in addition to the six ones of quantum theory. Those principles, which will substitute the relativity principles, are: the Deutsch-Church-Turing principle, and the principle of topological homogeneity.

Substitutes for the Relativity Principle

The Deutsch-Church-Turing principle. Rephrasing Deutsch [18]: "Every physical process describable in finite terms must be perfectly simulated by a quantum computer made with a finite number of qubits and a finite number of gates". In the logic of specularity between experimental protocols and algorithms (both include also outcomes), I would say: Every finite experimental protocol is perfectly simulated by a finite quantum algorithm. It is immediate to see that the principle implies two sub-principles: (a) the density of information is finite, and (b) interactions are local. The kind of information that we are considering here is quantum, whence the assertion that the density of information is finite means that the dimension of the Hilbert space for any bounded portion of reality is finite. This means that e.g. there are no Bosons, and the bosonic particle is only an asymptotic approximate notion. Richard Feynman himself is reported to like the idea of finite information density, because he felt that "There might be something wrong with the old concept of continuous functions. How could there possibly be an infinite amount of information in any finite volume?" [1]. The finite dimension of the Hilbert space also implies locality of interactions, namely that the number of quantum systems connected to each gate is finite.

Topological homogeneity of interactions. The principle states that the quantum algorithm describing a physical law is a periodic quantum network. In the informational paradigm the physical law is represented by a finite set of connected quantum gates, corresponding to a finite protocol, theoretically specular of a finite quantum algorithm. Thus locality of interactions is required in order to define a physical law in terms of a finite protocol under the local control of the experimenter, whereas homogeneity represents the universality of the law, which is assumed to hold everywhere and ever. It follows that algorithmically the physical law is represented by a quantum unitary cellular automaton [19]. The "space"-period and the "time"-period of the automaton correspond to the minimum space and time units $\mathrm{l_P}$ and $\mathrm{t_P}$—the Planck distance and the Planck time, respectively. At some very small scale—the Planck scale—the world is discrete.

The Quantum Cellular Automaton

Causality together with the Deutsch-Church-Turing principle imply that information propagates at finite speed, the maximum speed being the "speed of light" $c = l_P/t_P$—the causal speed of the automaton. The two principles together thus imply that the state of any finite set of systems can be evaluated exactly as the evolution for of finite number of time-steps of a larger but still finite number of systems in the past causal cone, regardless the quantum network being unbounded. We take as vacuum state any state that is locally invariant under the automaton evolution. The localized states are then those that differ from the vacuum, only for a finite number of systems. The future causal cone of these state-supporting systems is then the place where only we need to evaluate the evolution, again with no need of boundary conditions. We do not have any divergence, nor ultraviolet (no continuum), nor infrared (no calculation for infinite extension): the Deutsch-Church-Turing principle excludes tout court the continuum and the infinite dimension.

Recovering the old quantum field theory. The old field theory is re-covered as an approximation via an analytical asymptotic evaluation of the automaton evolution in the relativistic limit of small wave vectors and for delocalized states, which correspond to the customary quantum particles. In this way one can both derive the Dirac equation in the relativistic regime, but also describe the physics of very large Planckian masses and in the ultra-relativistic regime of huge momenta [20].

Emerging physics. It must be stressed that the homogeneity of interactions is a purely topological property, not a metrical one: "to be near" for systems means just "to be interacting", and the length of the graph links has no physical meaning. Space-time metric emerges from the pure topology by event counting, and the Planck length $\mathrm{l_P}$ and time $\mathrm{t_P}$ conceptually are only digital-analog conversion factors. Also the particle mass m of the Dirac automaton is a pure number $0 \leq \mathrm{m} \leq 1$, and the Planck mass $\mathrm{m_P}$ is the conversion factor to customary kilograms.

Universal automata constants. The three quantities $\mathrm{l_P}$, $\mathrm{t_P}$, $\mathrm{m_P}$, are the irreducible universal constants of the automata theory, and the adimensional mass is the only free parameter of the Dirac automaton. The Planck constant can be rewritten in terms of the automata universal constants as $\hbar = mp\, lp^2 tp^{-1}$.

Inertial mass. As I already explained in my previous FQXi essay [21, 22], the inertial mass is reinterpreted as the slowing down of the information flow via the coupling between the modes flowing along the directions in the network at maximal speed c (for $d > 1$ space-dimensions is a coupling between different chiralities [23]).

Particle speed and Planck mass as bound on mass. The speed of a zero-mass particle depends on the wave-length, and approaches zero at Planckian wavelengths anisotropically in space (see Fig. 11.1). For massive particles the speed of light in the Dirac equation decreases also versus the mass for very large Planckian masses, the automaton evolution becoming stationary at the Planck mass [24], since for larger masses the evolution would be non unitary. It follows that the particle mass is mounded by the Planck mass, at which it behaves essentially as a mini black hole. It is remarkable how these conclusions are reached without using general relativity, just a result of quantum theory.

Energy and momenta are finite in the digital world. The maximum momentum is the De Broglie relation $\hbar\pi/\mathrm{l_P}$. We can have only one particle and one antiparticle per Planck cell, and the bound on how much energy can be crammed into a unit of

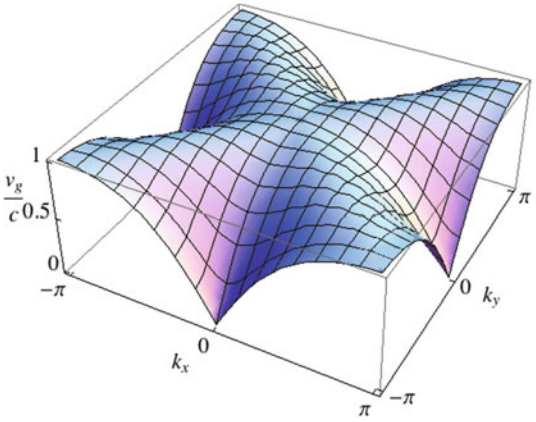

Fig. 11.1 Group velocity v_g (normalized to c) for a zero-mass particle automaton versus the adimensional momentum (k_x, k_y) (from Ref. [23]). The speed is approximately isotropic for low momentum (relativistic regime), and becomes anisotropic for very large momenta (ultra-relativistic regime)

space is determined by the maximum energy per particle, which cannot be more that $\hbar\pi t p^{-1} = 6.14663{*}10^9$ J, a huge energy! This is the energy for achieving 2 ops [25] of the automaton during the Planck time, as given by the Margulus-Levitin theorem [26] (each step of the automaton is obtained with two rows of quantum gates).

A Quantum-Digital Space-Time

The quantum nature of the automaton is crucial for the emergence of space-time. There are two main points against using a classical automaton.

First point against a classical automaton. With a classical automaton one cannot have isotropic space emerging from an homogeneous classical causal network, due to the Weyl Tile argument [27]: we count the same number of tiles in a square lattice both along the diagonal and in the direction of the square sides: where the $\sqrt{2}$ comes from? Indeed, the maximal speed of information in bcc-lattice automaton, as in the Dirac case, would be faster by a factor $\sqrt{2}$ or $\sqrt{3}$ along diagonals than along lattice axes, ending up with an anisotropic space for any homogeneous lattice [28], (the problem is not cured by the continuum limit). Instead, in a quantum network isotropy is recovered through quantum superpositions of different paths (see e.g. Fig. 11.2c), and we have again isotropy of max-speed in the relativistic regime of small momenta (Fig. 11.1), whereas anisotropy would be in principle visible only in the ultra-relativistic regime of huge momenta (Figs. 11.1,11.2b) or for ultra-localized states (Fig. 11.2d). In a similar manner the quantum nature of the network provides the mechanism for restoration of all continuum symmetries in the relativistic regime. The digital version of Lorentz transformations for a classical homogeneous causal network can be found in Ref. [29]: the usual Lorentz covariance cannot be restored from them. Recovering Lorentz covariance from a classical causal network (i.e.

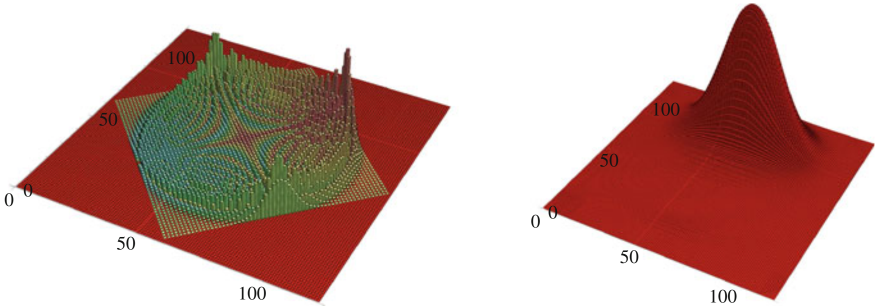

Fig. 11.2 How particles would look in a digital world made by a quantum automaton: the Dirac automaton for $d = 2$ space dimensions. The height of the plot is proportional to the absolute amplitude of finding a particle with up-spin. Colors represents the spin state. The two figures depict the evolved state after 60 steps of an initial state centered in the center of the plane. *Left* spin-up localized state. *Right* Gaussian spin-up particle state, with $\Delta_{x^2} = 2\Delta_{y^2} = 8lp$. (Theory in Ref. [23])

describing a causal ordering partial relation) conflicts with the homogeneity principle, and needs a random topology, as in the causal set program of Sorkin.

Second point against a classical automaton. The second reason against classical automata is that quantum superposition of localized states provides a mechanism for directing information in space, in a continuum of directions, by superimposing localized states at neighboring locations with constant relative phase between them, thus giving momentum to the information flow. Such mechanism is not possible in a classical causal network with finite information density. It is the interplay between quantum coherence and nonlocality that plays the crucial role of keeping information going along a desired direction with minimal spreading, a task that cannot be accomplished by a classical automaton.

Emergence of classical mechanics. The Hamiltonian for the classical field theory corresponding to the quantum automaton can be reversely derived from the unitary operator of the automaton [21, 22]. Customary quantum particles are Gaussian coherent superposition of single-system states with constant relative phase between neighboring systems, corresponding to the particle momentum: the classical trajectory is the "typical path" along the quantum network, namely the path with maximum probability of the Gaussian packet.

Where Is Gravity?

The big question is now where gravity comes from. I still don't have a definite answer, but I believe that the equivalence principle must be rooted in the automaton mechanism: the gravitational force must emerge at the level of the Dirac free theory, which itself defines the inertial mass. This does not occur in customary quantum field theory, but may happen in the quantum automaton theory, in terms of a tiny "thermodynamic" effect that can occur even for few particles: a purely quantum-digital effect. Indeed, the digital nature of the quantum automaton seems to make it the natural scenario for the generalized holographic principle at the basis of the Jacobson-Verlinde idea of gravity as entropic force [6, 7]. The hypothesis of gravity as a quantum-digital effect is very fascinating: it would mean we are indeed experiencing the quantum-digital nature of the world, in everyday experience: through gravity!

Postscriptum

All predictions contained in this Essay has been later derived, and are now available in technical papers. The reader should look at Ref. [23]. Other results can be found in Ref. [20, 30, 31].

The main result is contained in manuscript [23], entitled "Derivation of the Dirac equation from informational principles". There it is proved the remarkable result that from the only general assumptions of locality, homogeneity, isotropy, linearity and unitarity of the interaction network, only two quantum cellular automata follow that have minimum dimension two, corresponding to a Fermi field. The two automata are connected by CPT, manifesting the breaking of Lorentz covariance. Both automata converge to the Weyl equation in the relativistic limit of small wave-vectors, where Lorentz covariance is restored. Instead, in the ultra-relativistic limit of large wave-vectors (i.e. at the Planck scale), in addition to the speed of light one has extra invariants in terms of energy, momentum, and length scales. The resulting distorted Lorentz covariance belongs to the class of the *Doubly Special Relativity* of Amelino-Camelia/Smolin/Magueijo. Such theory predicts the phenomenon of *relative locality*, namely that also coincidence in space, not only in time, depends on the reference frame. In terms of energy and momentum covariance is given by the group of transformations that leave the automaton dispersion relations unchanged. Via Fourier transform one recovers a space-time of quantum nature, with points in superposition. All the above results about distorted Lorentz covariance are derived in the new Ref. [32].

The Weyl QCA is the elementary building block for both the Dirac and the Maxwell field. The latter is recovered in the form of the de Broglie neutrino theory of the photon. The Fermionic fundamental nature of light follows from the minimality of the field dimension, which leads to the Boson as an emergent notion [33].

The discrete framework of the theory allows to avoid all problems that plague quantum field theory arising from the continuum, including the outstanding problem of localization. Most relevant, the theory is quantum *ab initio*, with no need of quantization rules.

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