

Detection of Entanglement with Polarized Photons: Experimental Realization of an Entanglement Witness

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We report on the first experimental realization of an entanglement witness, a method to detect entanglement with few local measurements. The present demonstration has been performed with polarized photons in Werner states, a well-known family of mixed states that can be either separable or nonseparable. The Werner states are generated by a novel high brilliance source of bipartite entangled states by which the state mixedness can be easily adjusted.

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One of the main issues of modern technology is the manipulation of *information*, its transmission, processing, storing, and computing with an increasingly high demand of speed, reliability, and security. Quantum physics has recently opened the way to the realization of radically new information-processing devices, with the possibility of guaranteed secure cryptographic communications and of huge speedups of some computational tasks. In this respect quantum *entanglement* represents the basis of the exponential parallelism of future quantum computers [1], of quantum teleportation [2–5], and of some kinds of cryptographic communications [6,7]. In practical realizations, however, entanglement is degraded by decoherence and dissipation processes that result from unavoidable couplings with the environment. Since entanglement is an expensive resource—it cannot be distributed between distant parties by classical communication means—it becomes crucial to be able to detect it efficiently, with the minimum number of measurements. Several methods have been proposed to assess the presence of entanglement for different types of quantum systems [8–13]. In particular, the method of “entanglement witness” is a simple and efficient protocol that uses only a few local measurements [10,14]. Different from quantum tomography, this method does not provide a full reconstruction of the quantum state, but it allows one to check, with a minimal number of measurements (three different settings instead of nine for the Werner states considered in our experiment), if the original kind of entanglement has been preserved. The need for such a procedure arises often in real applications, e.g., in connection with quantum communication or teleportation in a lossy channel. For such reason it is relevant to assess with a real experiment which are the practical sensitivity and precision limitations in the actual implementation of the method. In the present paper we report the first experimental implementation of entanglement witness for

polarization-entangled photons. The entangled two-photon state is generated by a new efficient method based on spontaneous parametric down-conversion [15], with entanglement being detected using only three independent quantum measurements.

In the present experiment the entanglement-witness method of Ref. [10] is implemented for a pair of polarized photons that can be in any of the Werner states [16]—a family of mixed quantum states that include both entangled and separable states. The states are generated at a high rate in any possible bipartite state in the Hilbert space $\mathcal{H}_1 \otimes \mathcal{H}_2$, where $\dim(\mathcal{H}_1) = \dim(\mathcal{H}_2) = 2$. These states can be generated as either pure or mixed, with complete control of the degree of mixing [17]. In particular, here we generate the Werner states

$$\rho_p = p|\Psi_{-}\rangle\langle\Psi_{-}| + \frac{1-p}{4}I, \quad (1)$$

which are mixtures with probability $p \in [0, 1]$ of the maximally chaotic state $\frac{1}{4}I$ (I is the identity operator on $\mathcal{H}_1 \otimes \mathcal{H}_2$) and of the maximally entangled singlet state:

$$|\Psi_{-}\rangle = \frac{1}{\sqrt{2}}(|HV\rangle - |VH\rangle), \quad (2)$$

where $|HV\rangle \equiv |H\rangle_1 \otimes |V\rangle_2$ denotes a two-photon state, with H and V representing horizontal and vertical polarizations, respectively. The method to establish whether a state is entangled or not is based on the concept of entanglement witness [18,19]. According to this framework a state ρ is entangled if and only if there exists a Hermitian operator W , a so-called *entanglement witness*, which has positive expectation value $\text{Tr}[W\rho_{\text{sep}}] \geq 0$ for all separable states ρ_{sep} , but nevertheless has negative expectation value $\text{Tr}[W\rho] < 0$ on the state ρ . For pairs of qubits (in our case polarized photons) also the non-positivity of the partial transpose of ρ gives a necessary

and sufficient criterion for entanglement [20,21], and in this case simple ways to construct entanglement witnesses are known [19]. The Werner states ([16]) that are tested in our experiment are particularly appropriate for entanglement detection, because they include both entangled ($p > \frac{1}{3}$) and separable ($p \leq \frac{1}{3}$) states.

The detection method proposed in [10] in the case of Werner states of Eq. (1) gives the following entanglement-witness operator:

$$W = \frac{1}{2}(|HH\rangle\langle HH| + |VV\rangle\langle VV| + |DD\rangle\langle DD| + |FF\rangle\langle FF| - |LR\rangle\langle LR| - |RL\rangle\langle RL|), \quad (3)$$

where $|D\rangle = (1/\sqrt{2})(|H\rangle + |V\rangle)$ and $|F\rangle = (1/\sqrt{2})(|H\rangle - |V\rangle)$ denote diagonally polarized single photon states, while $|L\rangle = (1/\sqrt{2})(|H\rangle + i|V\rangle)$ and $|R\rangle = (1/\sqrt{2})(|H\rangle - i|V\rangle)$ correspond to the left and right circular polarization states. The above operator can be locally measured by choosing correlated measurement settings that allow detection of the linear, diagonal, and circular polarization for both photons. It represents the most efficient witness, since it involves the minimum number of local measurements.

The experimental apparatus is shown in Fig. 1. A type I, 0.5 mm thick, β -barium-borate (BBO) crystal is excited by a V-polarized cw Ar^+ laser beam ($\lambda_p = 363.8$ nm) with wave vector $-\mathbf{k}_p$, i.e., directed towards the left in Fig. 1. The two degenerate ($\lambda = 727.6$ nm) spontaneous parametric down-conversion photons have common H polarization and are emitted with *equal probability* over a corresponding pair of wave vectors belonging to the surface of a cone with axis \mathbf{k}_p . The emitted radiation and the laser beam are then backreflected by a spherical mirror M with curvature radius $R = 15$ cm, highly reflecting both λ and λ_p , placed at a distance $d = R$ from the crystal. A zero-order $\lambda/4$ wave plate placed between M and the BBO intercepts twice both backreflected λ and

λ_p beams and then rotates by $\pi/2$ the polarization of the backreflected photons with wavelength λ while leaving in its original polarization state the backreflected pump beam $\lambda_p \approx 2\lambda$. The backreflected laser beam excites an identical albeit distinct down-conversion process with emission of a new radiation cone directed towards the right in Fig. 1 with axis \mathbf{k}_p . In this way, by optical back-reflection and a unitary polarization flipping each pair originally generated towards the left in Fig. 1 in the state $|VV\rangle$ is indistinguishable in principle from another pair originally generated towards the right and carrying the state $|HH\rangle$. The state of the overall radiation, resulting from the two overlapping indistinguishable cones, is then expressed by the pure entangled state:

$$|\Phi\rangle = \frac{1}{\sqrt{2}}(|HH\rangle + e^{i\phi}|VV\rangle) \quad (4)$$

with phase ($0 \leq \phi \leq \pi$) reliably controlled by metric displacements Δd of the spherical mirror M along \mathbf{k}_p . A positive lens transforms the overall *conical* emission distribution into a *cylindrical* one with axis \mathbf{k}_p , whose transverse circular section identifies the so-called *entanglement ring*. Then each couple of points symmetrically opposed through the center of the ring are entangled. An annular mask with diameter $D = 1.5$ cm and width $\delta = 0.07$ cm provides an accurate spatial selection of the ring. This is divided in two equal portions along a vertical axis by a prismatic two-mirror system and detected by two independent silicon-avalanche photodiodes, mod. SPCM-AQR14 at sites A and B. Typically, two equal interference filters, placed in front of the A and B detectors, with bandwidth $\Delta\lambda = 6$ nm, determine the coherence time of the emitted photons $\tau_{\text{coh}} \approx 140$ fsec. More than 4000 coincidences per second are detected for a pump power $P_p \approx 100$ mW.

Werner states of Eq. (1) are generated by selecting a convenient *patchwork* technique which implies the following steps [17]: (i) Starting from the original source state in Eq. (4), the *singlet* state $|\Psi_-\rangle$ is obtained by inserting a zero-order $\lambda/2$ wave plate in front of detector B; (ii) an antireflection coated glass plate G 200 μm thick, inserted between the mirror M and the BBO crystal with a variable transverse position Δx , introduces a decohering fixed time delay $\Delta t > \tau_{\text{coh}}$ that spoils the indistinguishability of the intercepted portions of the overlapping quantum-interfering radiation cones (see the inset in Fig. 1). In fact, after the BBO- M -BBO round-trip, the photon of the left-cone pair traveling through G is time delayed from the other, while no effect on the corresponding right-cone pair occurs. As a consequence, all nondiagonal elements of ρ_p given by the surface sectors $\mathbf{B} + \mathbf{C}$ of the entanglement ring, the ones optically intercepted by G , are set to zero while the non-intercepted sector \mathbf{A} expresses the singlet contribution to ρ_p . (iii) A $\lambda/2$ wave plate is inserted in the semicylindrical photon distribution reflected by the beam-splitting

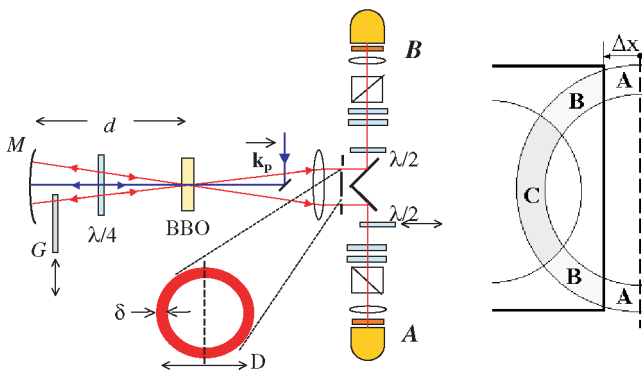


FIG. 1 (color online). Layout of the *universal, high-brilliance* source of polarization entangled photon states and of general mixed states. Inset: partition of the half entanglement ring into the spatial contributions of the emitted pair distribution to an output Werner state.

TABLE I. Table of the experimental values of the probabilities concurring to the evaluation of the entanglement witness.

p	0.965 ± 0.021	0.930 ± 0.017	0.619 ± 0.022	0.540 ± 0.026	0.319 ± 0.028	0.068 ± 0.031	0.004 ± 0.032
$\langle W \rangle$	-0.4932 ± 0.0082	-0.417 ± 0.0051	-0.281 ± 0.0035	-0.1431 ± 0.0035	0.0042 ± 0.0037	0.1510 ± 0.0037	0.2658 ± 0.0037
$ HH\rangle\langle HH $	0.0067 ± 0.0011	0.0165 ± 0.0011	0.0852 ± 0.0016	0.1069 ± 0.0018	0.1500 ± 0.0022	0.2084 ± 0.0031	0.2282 ± 0.0027
$ VV\rangle\langle VV $	0.0101 ± 0.0013	0.0179 ± 0.0011	0.1053 ± 0.0018	0.1232 ± 0.0020	0.1895 ± 0.0025	0.2555 ± 0.0032	0.2916 ± 0.0031
$ DD\rangle\langle DD $	0.0101 ± 0.0013	0.0441 ± 0.0018	0.1063 ± 0.0018	0.1512 ± 0.0022	0.2132 ± 0.0027	0.2419 ± 0.0028	0.2679 ± 0.0030
$ FF\rangle\langle FF $	0.0135 ± 0.0015	0.0289 ± 0.0014	0.0830 ± 0.0016	0.1389 ± 0.0021	0.2053 ± 0.0026	0.2346 ± 0.0028	0.2849 ± 0.0030
$ LR\rangle\langle LR $	0.476 ± 0.011	0.4683 ± 0.0069	0.4585 ± 0.0042	0.4037 ± 0.0042	0.3598 ± 0.0037	0.3141 ± 0.0029	0.2561 ± 0.0031
$ RL\rangle\langle RL $	0.551 ± 0.012	0.4738 ± 0.0069	0.4824 ± 0.0044	0.4026 ± 0.0040	0.3897 ± 0.0039	0.3241 ± 0.0032	0.2746 ± 0.0030

prism towards the detector A . Its position is carefully adjusted in order to intercept half of the $\mathbf{B} + \mathbf{C}$ sector, i.e., by making $\mathbf{B} = \mathbf{C}$. Note that only half of the ring which is represented in the inset in Fig. 1 needs to be intercepted by the optical plates. In summary, the sector \mathbf{A} contributes to ρ_p with the pure state $|\Psi_-\rangle\langle\Psi_-|$ with probability p , and the sector $\mathbf{B} + \mathbf{C} = 2\mathbf{B}$ with probability $1 - p$ contributes with the mixture

$$\frac{1}{4}[|HV\rangle\langle HV| + |VH\rangle\langle VH| + |HH\rangle\langle HH| + |VV\rangle\langle VV|]. \quad (5)$$

The experimental value of p is determined by the following procedure. First, the position Δx of the plate G is set according to a predetermined, zeroth-order value of $p(\Delta x)$. Then, after the insertion of the $\lambda/2$ wave plate, four values of the parameter $R = (1 - p/1 + p)$, i.e., $R_1 = (|HH\rangle\langle HH|/|VH\rangle\langle VH|)$, $R_2 = (|VV\rangle\langle VV|/|HV\rangle\langle HV|)$, $R_3 = (|HH\rangle\langle HH|/|HV\rangle\langle HV|)$, and $R_4 = (|VV\rangle\langle VV|/|VH\rangle\langle VH|)$, are obtained by measuring each term

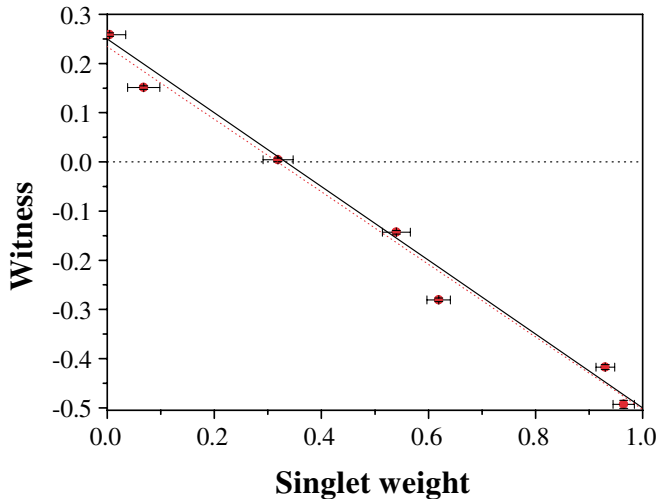


FIG. 2 (color online). Experimental results of the entanglement witness for Werner states, ranging from the pure singlet ($p = 1$) to the totally chaotic state at ($p = 0$). The straight line corresponds to the theoretical prediction $\text{Tr}[W\rho_p] = (1 - 3p/4)$, while the dotted line represents the experimental best fit with parameters 0.236 ± 0.026 and -0.739 ± 0.051 . The horizontal dashed line indicates the transition between separable and entangled Werner states, occurring at zero witness at $p = \frac{1}{3}$.

in (5) [17]. The probability p is then calculated by averaging over the four experimental values of R . It can be easily varied over its full range of values, going from $\rho_0 \equiv \frac{1}{4}I$ to $\rho_1 \equiv |\Psi_-\rangle\langle\Psi_-|$.

In order to detect whether the produced Werner state is entangled or not, the expectation value of the witness operator (3) has been measured, by performing local correlated measurements on each arm of the linear, diagonal, and circular polarizations. Each of the three settings (each corresponding to a measurement lasting an average time of 30 sec) contributes to the expectation of two projectors in Eq. (3). The polarization of the detected photon is selected by means of a sequence of $\lambda/4$ and $\lambda/2$ wave plates and a polarization beam splitter. Several different values of the singlet weight p have been tested in the experiment. The probabilities of each outcome for all polarization settings are reported in Table I. These have been obtained by normalizing the coincidence measurements to the sum of coincidence rates measured in the basis $|HH\rangle$, $|HV\rangle$, $|VH\rangle$, and $|VV\rangle$. Figure 2 shows the experimental expectation value of W as a function of the singlet weight p . The error bars on the horizontal axis correspond to the standard deviation of p , while those on the vertical axis have been evaluated by taking into account the statistical noise in the counting process. Figure 2 shows that agreement with the theoretical prediction $\text{Tr}[W\rho_p] = (1 - 3p/4)$ can be very good. Note that the experimental results precisely identify the transition between the separable and nonseparable Werner states, occurring at $p = \frac{1}{3}$, $\text{Tr}[W\rho_p] = 0$, in agreement with the theoretical expectations.

In conclusion, we have reported the results of the first experiment of entanglement detection based on the measurement of an entanglement witness, which is achieved with a minimal number of local measurement settings. We used polarized photons in Werner states generated by a high brilliance source of entanglement which is completely tunable to achieve any bipartite two-photon state. We found perfect agreement with the theoretical predictions. The present experiment represents a step towards the practical realization of all information-processing protocols, which due to unavoidable entanglement degradation need to test whether a state is entangled or not.

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