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Experimental realization of 'universal homodyne tomography' with a single local oscillator

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Abstract. We report preliminary experimental measurement of the twin-beam quantum state via optical homodyne tomography using a single local oscillator. The experiment is a realization of the recently reported 'universal homodyne tomography' technique. The results agree well with theoretical predictions and reveal the non-classical photon-number correlation between the signal and idler photons of the twin-beam state.

1. Introduction

Optical homodyne tomography has become a very reliable and accurate method for the measurement of the quantum state of the radiation field [1]. If probability distributions $p(x, \phi)$ of the field quadrature $\hat{X}(\phi) = \hat{c} \exp(-i\phi) + \hat{c}^{\dagger} \exp(i\phi)$, where $\hat{c}$ is the annihilation operator of the field-mode of interest, are obtained through an appropriate homodyne technique, then any observable $\hat{O}$ of the unknown radiation-field mode can be measured by averaging a suitable unbiased estimator $\mathcal{E}[\hat{O}](x, \phi)$ over the experimental quadrature outcome x at the random phase value ϕ [2]. The technique of homodyne tomography has been extended to multimode states [3–6], which would be increasingly important considering the future applications of multimode quantum correlation such as the one exhibited by the Greenberger–Horne–Zeilinger state [7].

When the modes are separable, multimode tomography is a direct generalization of the single-mode case [3, 4]. The corresponding estimator is just the product of the estimators for each of the single-mode operators. However, when the modes are not spatio-temporally separated, such a simple generalization is not possible. Although special tomography methods have been developed for some

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limited situations [4–6], it is desirable to develop a more general multimode-tomography method which is widely applicable and highly accurate.

In a recent paper, D'Ariano *et al.* have described a new method of multimode homodyne tomography [8]. Their method is universal in the sense that an arbitrary observable of the multimode radiation field can be measured with a single local oscillator (LO). By scanning the LO over all possible linear combinations of the field modes and then averaging a suitable unbiased estimator over the collected quadrature outcomes, it is possible to measure any observable of the multimode-field state of interest. In this paper, we report the first experimental realization of multimode homodyne tomography of the type devised in [8].

2. Universal homodyne tomography with a single local oscillator

The primary focus of our experiment is to measure the joint photon-number distribution of the twin-beam state produced by a non-degenerate optical parametric amplifier (NOPA). In the NOPA, due to the nature of the parametric scattering process, a one pump photon splits into a pair of photons which belong to two different modes (the signal and the idler). The perfect quantum correlation between the two modes leads to the following joint probability distribution for n photons in one mode and m in the other:

$$P(n,m) = \frac{\delta_{nm}}{\bar{n}+1}\left(\frac{\bar{n}}{\bar{n}+1}\right)^n, \tag{1}$$

where $\bar{n} = g - 1$ is the average number of photons in each mode and g is the gain of the NOPA. The thermal character of the photon statistics of either of the two modes has been demonstrated previously by means of a self-generated matched-LO method [6, 9]. The joint distribution has also been measured by using a two-LO method [10], in which the two modes, which are orthogonally polarized due to type-II phase-matching in the NOPA, are spatially separated by a polarizing beamsplitter. However, in the present experiment, one does not need such a separation, and instead, we scan the polarization angle θ of a single-LO beam across the two orthogonally polarized modes of the twin-beam state.

In this case of two modes, the single-LO multimode tomography algorithm turns out to be rather simple. The estimator corresponding to the joint photon-number probability distribution is given by [8]

$$\begin{aligned}\mathcal{E}_\eta[|n,m\rangle\langle n,m|](x;\theta,\phi_0,\phi_1) \\ = \kappa^2\int_0^\infty \mathrm{d}t\,\exp\left[-t+\mathrm{i}2x(\kappa t)^{1/2}\right]tL_n(\kappa t\cos^2\theta)L_m(\kappa t\sin^2\theta),\end{aligned} \tag{2}$$

where $\kappa = 2\eta/(2\eta-1)$, η is the quantum efficiency, $L_n(z)$ denotes the customary Laguerre polynomial in z, and ϕ_0 and ϕ_1 are the relative phases between the LO and the two modes, respectively.

The estimator can then be averaged by taking the integral

$$\begin{aligned}\langle\hat{O}\rangle = \int_0^{2\pi}\frac{\mathrm{d}\phi_0}{2\pi}\int_0^{2\pi}\frac{\mathrm{d}\phi_1}{2\pi}\int_{-1}^{1}\frac{\mathrm{d}(\cos 2\theta)}{2} \\ \times\int_{-\infty}^{\infty}\mathrm{d}x\,p_\eta(x;\theta,\phi_0,\phi_1)\mathcal{E}_\eta[\hat{O}](x;\theta,\phi_0,\phi_1)\end{aligned} \tag{3}$$

over the polarization angle θ. Here $p_\eta(x;\theta,\phi_0,\phi_1)$ is the experimental homodyne probability for a $\cos^2(\theta)$ weighting of one polarization and a $\sin^2(\theta)$ weighting of the other mode with phases ϕ_0 and ϕ_1, respectively. The scaling of x is such that the variance of the vacuum input to the set-up is 1/4. Because the statistics of balanced homodyne measurements of any arbitrary state are Gaussian for a given θ, ϕ_0 and ϕ_1, the probability of events beyond a certain x become essentially zero, freeing the experimentalist from acquiring samples at a large x.

3. Experiment

A schematic of our experimental set-up is shown in figure 1. The NOPA, consisting of a 5 mm long KTP crystal, is pumped by the second harmonic of a Q-switched and mode-locked Nd:YAG laser. The laser output is a 100 MHz train of 120 ps duration pulses at 1064 nm wavelength (85 ps for the second harmonic at 532 nm) with a 205 ns wide Q-switch envelope (145 ns for the second harmonic) having a 1 kHz repetition rate. Owing to the type-II phase-matching employed in the KTP crystal, orthogonally polarized twin beams are generated in the same spatial mode at 1064 nm. The parametrically down-converted twin beams and the LO derived from the same laser are mixed at a 50/50 non-polarizing beamsplitter

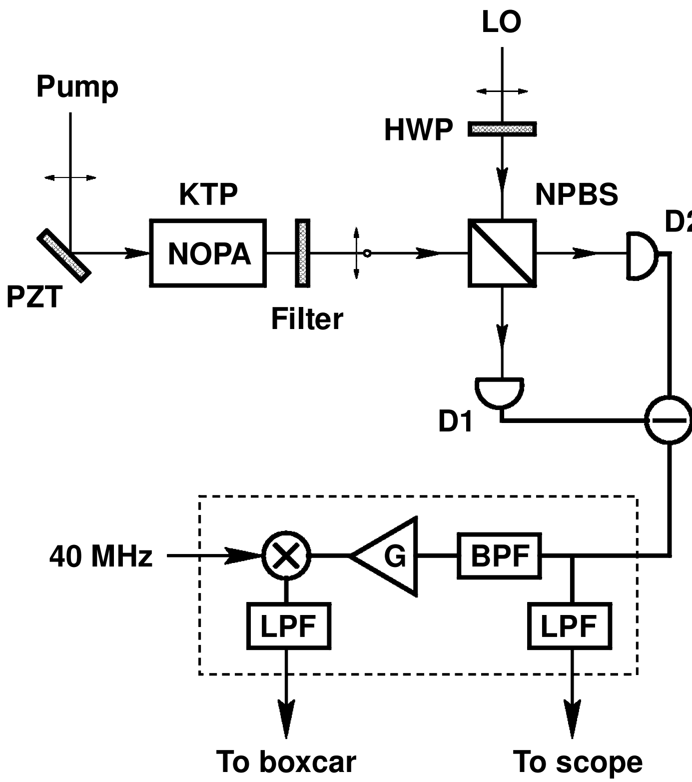

Figure 1. Schematic of the experimental set-up: NOPA, non-degenerate optical parametric amplifier; LO, local oscillator; HWP, half-wave plate; NPBS, non-polarizing 50/50 beamsplitter; PZT, piezoelectric transducer; BPF, bandpass filter; LPFs, low-pass filters; G, amplifier.

(NPBS) and then measured by a balanced set of photodetectors, D1 and D2. Transmittance and reflectance of the NPBS are measured to be $50 \pm 3\%$, independently of the input beam polarization. A 10 MHz wide band of radio frequencies near $\Omega/2\pi = 40$ MHz of the difference photocurrent from the photodiodes is selected by means of a bandpass filter and amplified with a low-noise electronic amplifier. This frequency is chosen to be higher than any remaining spectral component of the Q-switch pulse envelope so that only shot noise is analysed, and chosen to be lower than pickups from a strong 50 MHz rf (radio frequency) source which mode-locks the laser. Neglecting such technical reasons, extending the rf bandwith would produce a correspondingly similar increase to both the signal samples and the vacuum input samples. Because the signal samples are normalized to the standard deviation of the vacuum input, the measured quantum state does not change as long as this bandwidth remains within the optical bandwidth of the parametric fluorescence, which is the case in our experiment. One may narrow the RF bandwidth but only to the degree that it remains significantly larger than the linewidth of the LO, so the homodyne measurement remains a single mode measurement. The amplified noise photocurrent is then downconverted to the near-dc region by use of an rf mixer and sampled by a boxcar integrator. In this set-up, the detected mode is a superposition of two temporal modes having the same polarization as the LO, but shifted in frequency by $\pm\Omega$ from the LO frequency. If the optical frequency of the LO is $\omega/2\pi$, then the detected mode is $\hat{C}^{(\xi)} = [\exp(\mathrm{i}\xi)\hat{c}_{-\Omega} + \exp(-\mathrm{i}\xi)\hat{c}_{+\Omega}]/2^{1/2}$, where $\hat{c}_{\pm\Omega}$ are the annihilation operators at radian frequencies $\omega \pm \Omega$ and ξ is the phase of the rf LO entering the mixer. By scanning the phase of the twin beams with respect to that of the LO field, one can measure any quadrature of the $\hat{C}^{(\xi)}$ mode. In our set-up, the electronic noise level is about 11 dB below the shot-noise level.

For the twin-beam state, the quadrature probability distribution depends on the sum of the relative phases between the LO and the two orthogonally polarized modes: $\phi_0 + \phi_1$. In our experiment, the piezoelectric transducer (PZT) in the pump-beam path is used to scan this sum of the relative phases in a uniform fashion. The polarization scan between the two orthogonally polarized modes is achieved by using a half-wave plate (HWP) in the LO path, which is mounted in a computer-controlled rotating stage. This polarization scan is a key element in the implementation of the single-LO multimode-tomography procedure. To average the estimator of the joint photon-number distribution, one needs to know exactly the polarization angle at which each quadrature sample is measured. Because the angular resolution of the rotating stage is limited, we take 1000 quadrature-data samples for each setting of the HWP with a rotation-step size of $45^\circ/1206$. Therefore, the total number of samples acquired in a measurement run are 1206×1000.

The overall quantum efficiency in our experiment is estimated to be about $\eta = 48\%$, which includes the 80% propagation and the photo-detection efficiency, and the 70% (60%) homodyne-overlap efficiency of the LO with the signal (idler) mode. Estimation of the homodyne-overlap efficiency is a non-trivial issue in our experiment because of the travelling-wave nature of the interaction in the NOPA. To estimate the overlap efficiency, we used a coherent-state seed beam, derived from the 1064 nm fundamental output of the same laser, as input to the NOPA. The cited values of the overlap efficiency are then estimated from measurements of the interference contrast between the LO and the amplified signal (idler) beam.

4. Results

The reconstructed joint photon-number distribution is shown in figure 2 (*a*). The non-classical correlation of the two modes is evident when compared to two independent coherent-state modes with similar mean photon number as shown in figure 2 (*b*). In figure 2 (*a*), the distribution is spread around the main diagonal as is expected for less-than-unity quantum efficiency. However, this spreading is less than that obtained in [10] for similar average photon numbers, showing that the one-LO method is superior to the two-LO method for measuring multimode quantum states. One should, however, note the larger errors at higher sum photon numbers. As has already been noted in [8], the statistical errors grow larger at higher photon number in the one-LO technique than those obtained with the two-LO method.

In order to check the experimental results, we compare the mean sum-photon number obtained from the one-LO method with the sum of the mean photon numbers for each mode. To obtain the latter, i.e. $\langle \hat{a}^\dagger \hat{a} \rangle$ and $\langle \hat{b}^\dagger \hat{b} \rangle$, we perform the

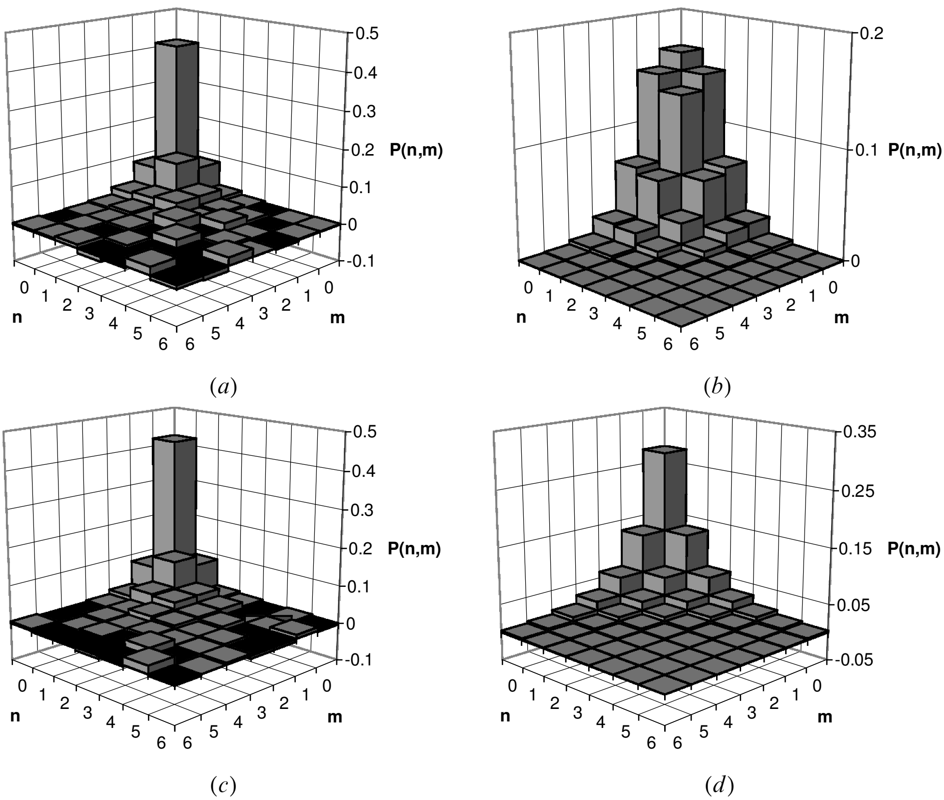

Figure 2. (*a*) Measured joint photon-number distribution for the twin-beam state of spontaneous parametric down-conversion. The mean photon-number is $\bar{n} = 0.94$ and $\bar{m} = 0.85$. 1206×1000 data samples are used with $90^\circ/1206$ polarization-angle resolution. (*b*) Theoretical joint photon-number distribution for two independent coherent states with mean photon numbers $\bar{n} = \bar{m} = 0.9$. (*c*) Monte Carlo simulation for the twin-beam state with $\bar{n} = \bar{m} = 0.9$ and $\eta = 48\%$ corresponding to the data shown in (*a*). (*d*) Theoretical joint photon-number distribution for two independent thermal states with mean photon numbers $\bar{n} = \bar{m} = 0.9$.

customary homodyne tomography on each mode by setting the LO polarization parallel to the proper mode of interest: $\theta = 0$ for the signal mode and $\theta = \pi/2$ for the idler mode. The consistency between the θ-scanned and θ-fixed quadrature measurements is verified since the mean value of the total photon number from the one-LO data $\langle \hat{a}^\dagger \hat{a} + \hat{b}^\dagger \hat{b} \rangle = 1.83$ (obtained by averaging the estimator in equation (3) over the θ-scanned quadrature data) is close to the mean photon number $\bar{n} + \bar{m} = 0.94 + 0.85 = 1.79$ obtained from the individual-mode tomography data with $\theta = 0$ and $\theta = \pi/2$, respectively.

To compare the experimental results with the theoretical predictions, we have performed Monte Carlo simulations of the measurement by using the theoretical homodyne-probability distribution for the twin-beam state. As in the experiment, we employed 1206×1000 data samples and chose $90^\circ/1206$ as the polarization-angle resolution. In the simulation, the experimental value $(\bar{n} + \bar{m})/2 = 0.9$ is used as the mean photon number in each mode and the quantum efficiency is chosen to be $\eta = 0.48$. Figure 2 (*c*) shows the joint photon-number probability distribution obtained from the theoretical quadrature distribution, with $\bar{n} = \bar{m} = 0.9$ and $\eta = 48\%$. One can see that the general features of the experimental and theoretical joint distributions are in good agreement. For comparison, in figure 2 (*d*) we also show the theoretical joint photon-number distibution for two independent thermal states with the same mean photon numbers.

The uniform scanning of the phase between the LO and the two modes was a critical ingredient in the success of the single-LO multimode-tomography experiment. For example, in our experiment with the twin-beam state, when the polarization angle is set to $\theta = \pi/4$ the detected mode is in a squeezed-vacuum state, whose quadrature value is highly phase sensitive. In order to demonstrate the uniformity of the phase scanning in the experiment, in figure 3 we show the phase scan which was obtained during the measurement of quadrature distribution

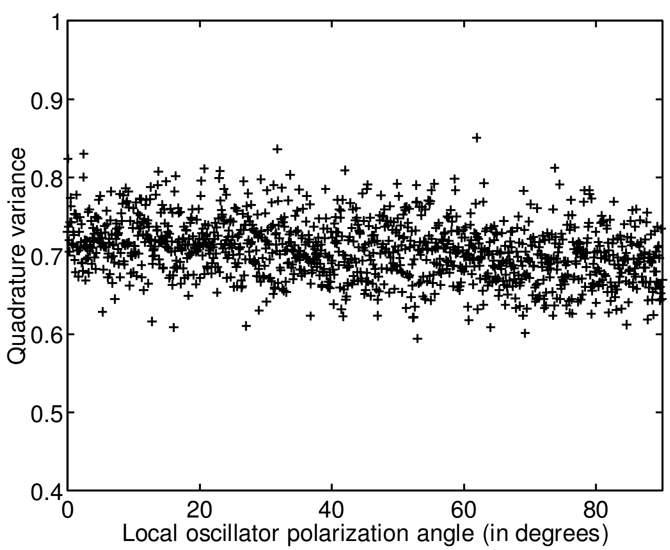

Figure 3. The distribution of quadrature variance during the polarization-angle scan. Each data point corresponds to the variance of 1000 data samples at each polarization angle.

for the data shown in figure 2 (*a*). Each data point in figure 3 corresponds to the variance of 1000 quadrature samples which were obtained at each polarization angle.

5. Discussion

In conclusion, we have demonstrated the method of universal homodyne tomography with a single LO for the case of the parametrically generated twin-beam state. The one-LO scheme has several advantages compared to the customary two-LO method. First, it provides a reliable alternative in practice when the modes of the field are not spatio-temporally separable. Second, because it uses fewer optical elements and a single electronic channel for noise measurement, the overall system loss is usually less than that obtainable with the two-LO method. Furthermore, one does not have to pay attention to electronic coherence in the detection channels, which was found to be critical in the two-LO experiment [10]. This benefit greatly simplifies the experiment. On the other hand, we find that the implementation of the new method needs further improvement. For example, in the case of the twin-beam state, it is more difficult to achieve the optimum homodyne overlap because the two modes are not spatio-temporally separated. The extraordinary beam in our experiment experiences the walk-off effect in the KTP crystal and, thus, there is no good way to simultaneously match both modes with the single LO. The method of matched LO [12] or walk-off compensation may be applied to improve the homodyne overlap.

We have presented initial measurements that show a successful application of the method of universal homodyne tomography to an NOPA with a gain of 2. Future work will include quantitative measurement of the degree of non-classical correlation between the two modes for different NOPA gains.

Acknowledgments

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