JOHN TEMPLETON FOUNDATION

SUPPORTING SCIENCE~INVESTING IN THE BIG QUESTIONS

Project: *A Quantum-Digital Universe*, Grant ID: 43796

Informationalism
as a solution of the Sixth Hilbert problem

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Workshop "Hilbert's Sixth Problem"
University of Leicester, May 02-04, 2016

The sixth Hilbert problem

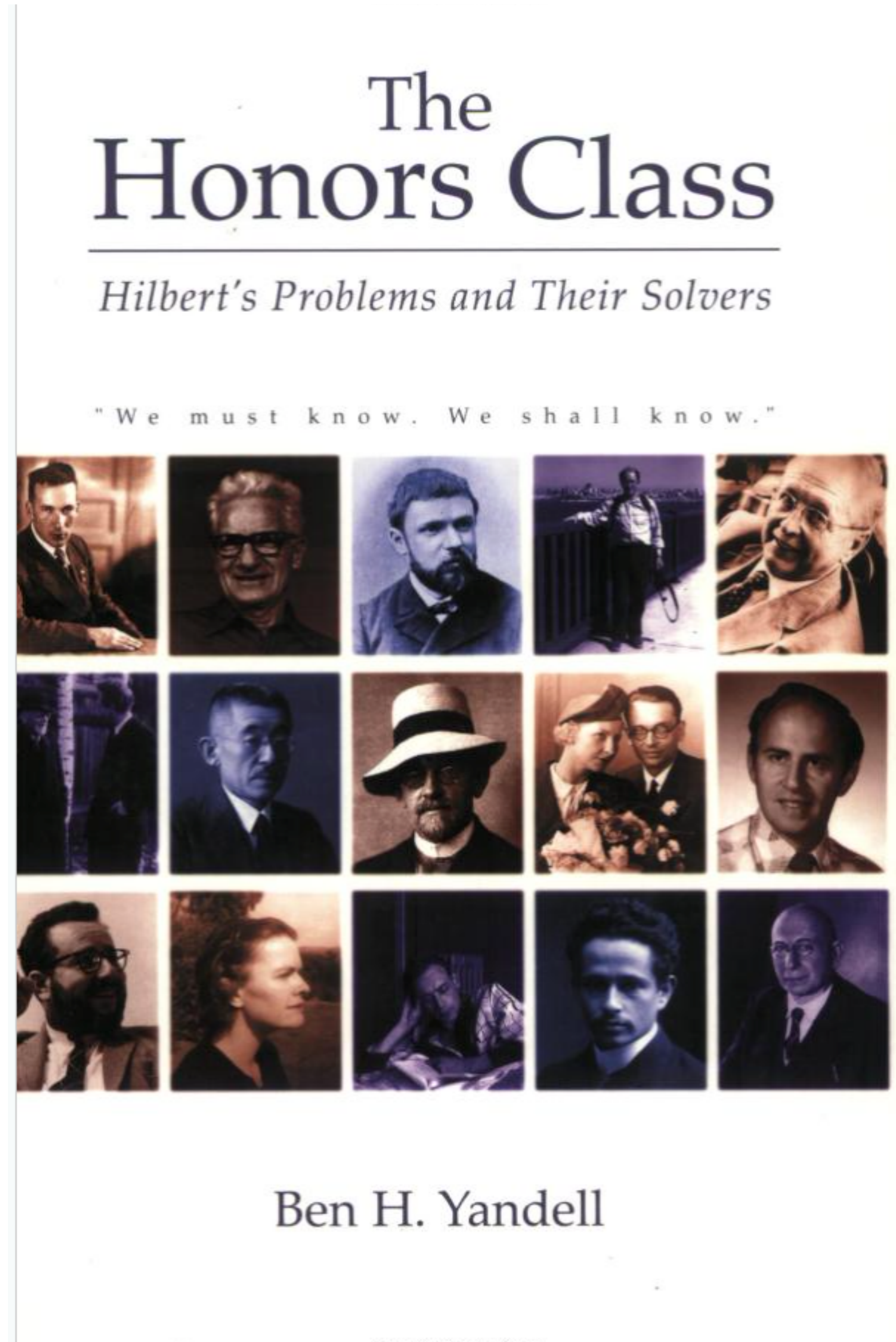
The investigations on the foundations of geometry suggest the problem: To treat in the same manner by means of axioms, those physical sciences in which mathematics plays an important part; in the first rank are the theory of probabilities and mechanics.

David Hilbert

Mechanics: the Trojan horse

Axiomatizing the theory of probabilities was a realistic goal: Kolmogorov accomplished this in 1933. The word ‘mechanics’ without a qualifier, however, is a Trojan horse.”

Benjamin Yandell



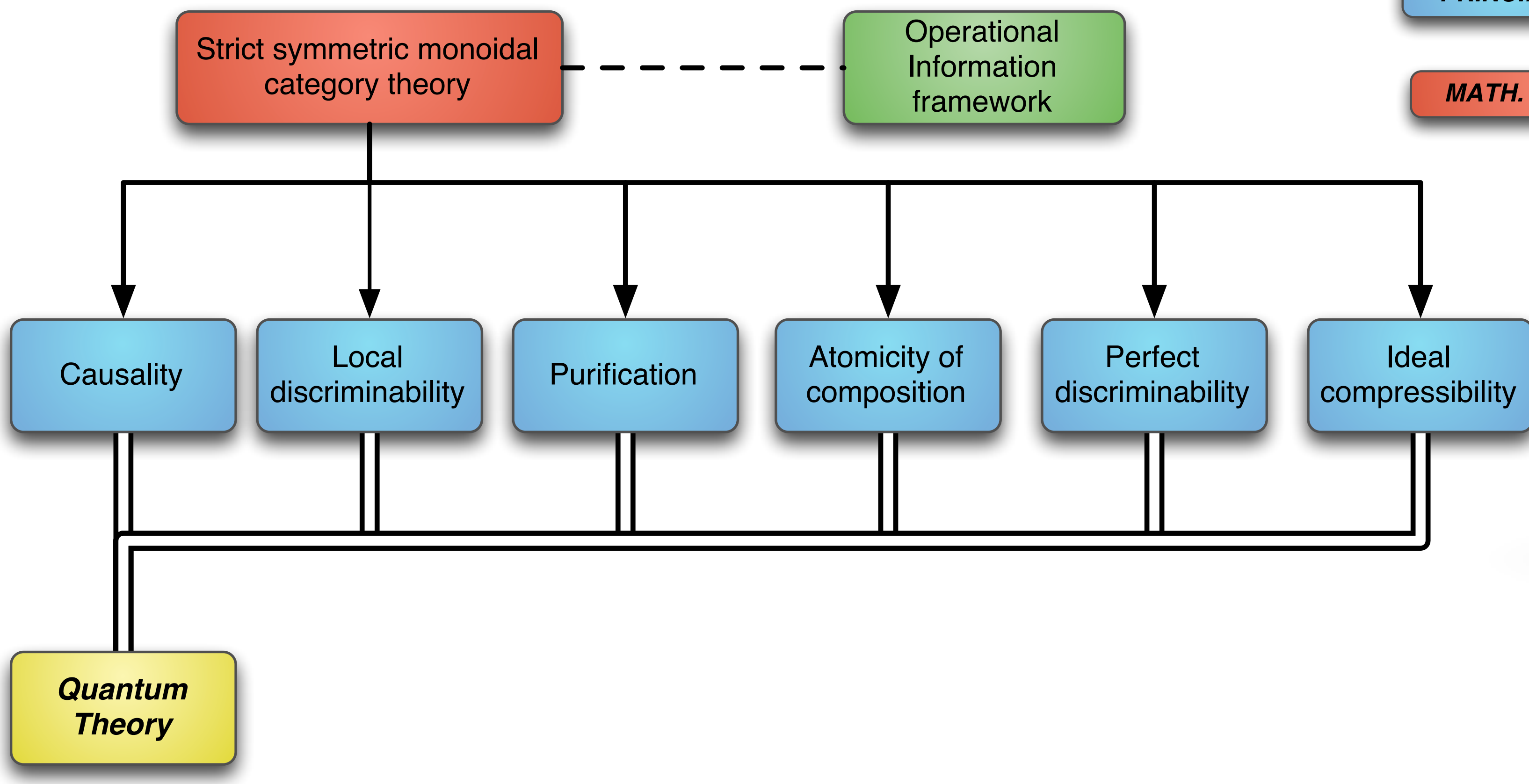
Program

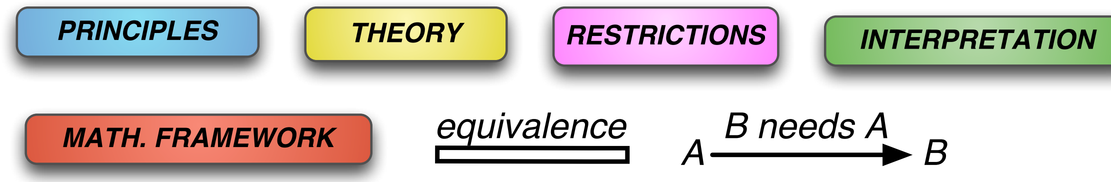
To derive the whole Physics axiomatically

from “principles” stated in form of purely mathematical axioms
without physical primitives,
but having a thorough physical interpretation.

Solution: informationalism

physical primitives: mass, force, rods, clocks,…





Principles for Quantum Theory

Selected for a Viewpoint in *Physics*

PHYSICAL REVIEW A **84**, 012311 (2011)

Informational derivation of quantum theory

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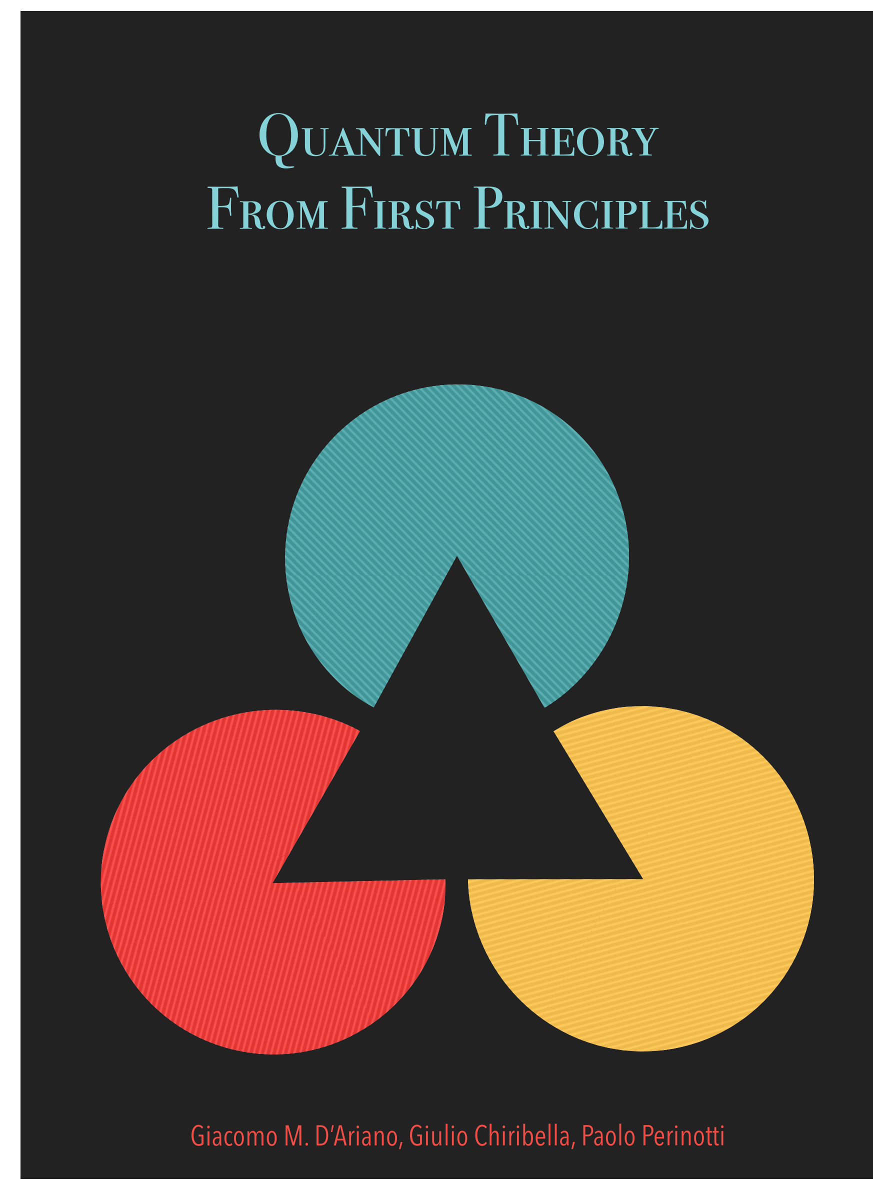
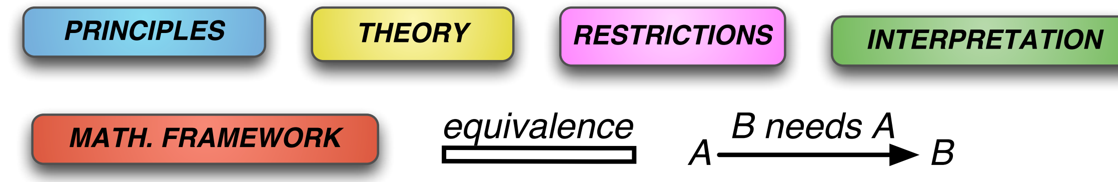
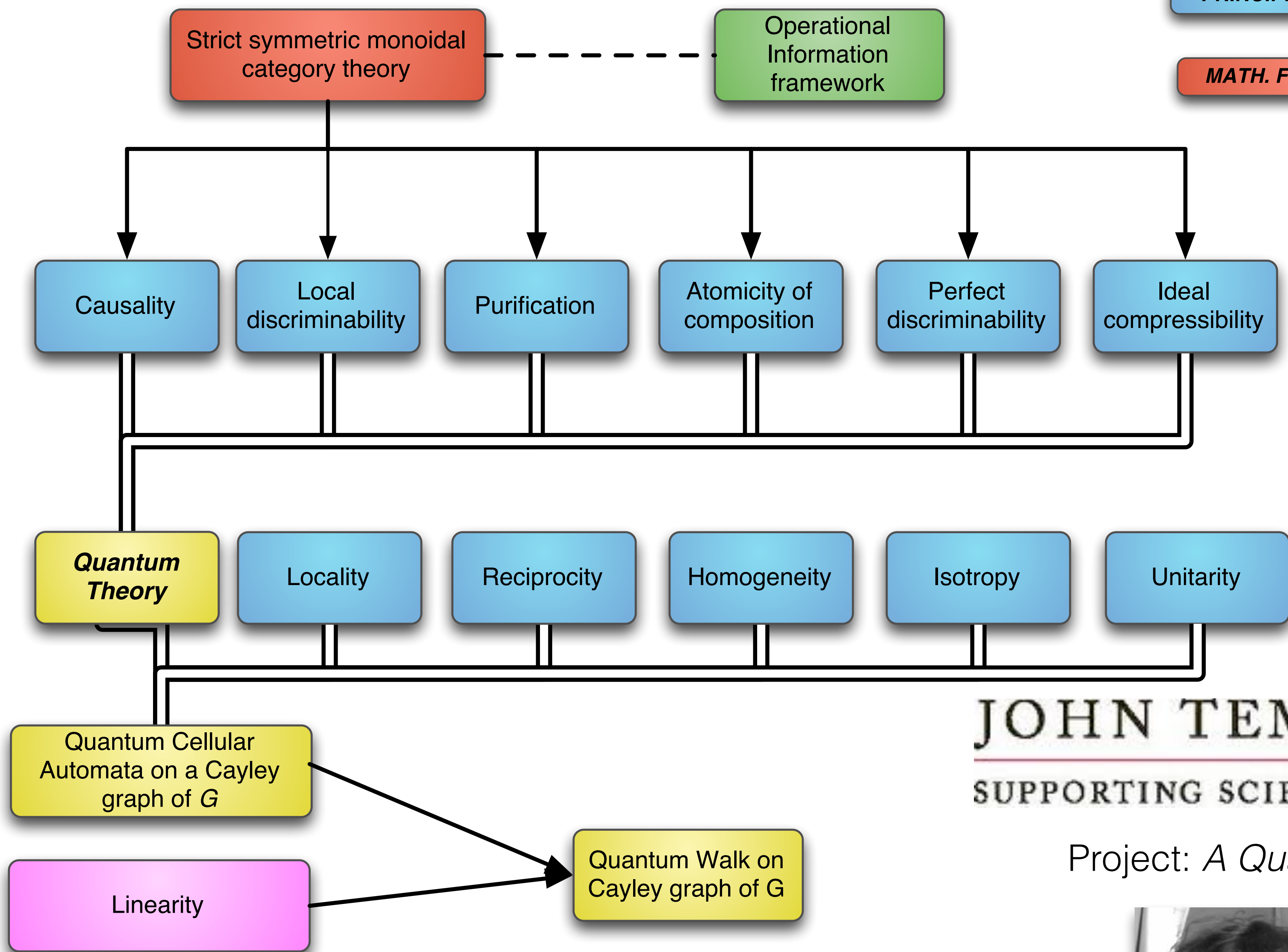
(Received 29 November 2010; published 11 July 2011)

We derive quantum theory from purely informational principles. Five elementary axioms—causality, perfect distinguishability, ideal compression, local distinguishability, and pure conditioning—define a broad class of theories of information processing that can be regarded as standard. One postulate—purification—singles out quantum theory within this class.

DOI: 10.1103/PhysRevA.84.012311 PACS number(s): 03.67.Ac, 03.65.Ta







Principles for Mechanics

- *Mechanics (QFT) derived in terms of countably many quantum systems in interaction*

add principles

Min algorithmic complexity principle

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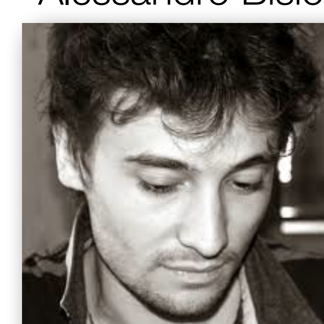
Paolo Perinotti

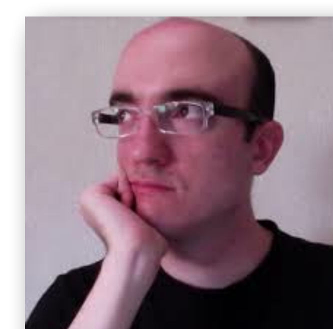
Alessandro Bisio

Alessandro Tosini

Marco Erba



Franco Manessi



Nicola Mosco

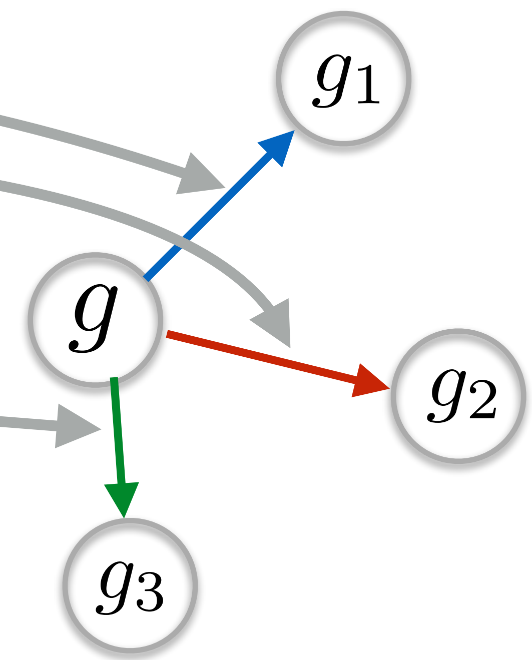
Quantum walk on Cayley graph

Hilbert space: $\mathcal{H} = \bigoplus_{g\in G} \mathbb{C}^{s_g} \quad |G| \leqslant \aleph, \; s_g \in \mathbb{N}$

Evolution

$$\psi_g(t+1) = \sum_{g'\in S_g} A_{gg'}\psi_{g'}(t)$$

$$\sum_{g'} A_{gg'}A^{\dagger}_{g''g'} = \sum_{g'} A^{\dagger}_{gg'}A_{g''g'} = \delta_{gg''} I_{s_g}$$



Build a <u>directed graph</u> with an arrow from *g* to *g' wherever they* are connected by $A_{gg'} \neq 0$

D'Ariano, Perinotti,
PRA **90** 062106 (2014)

Quantum walk on Cayley graph

w.l.g. Hilbert space $\mathcal{H} = \oplus_{g\in G}\mathbb{C}^{s_g}$ $\quad |G| \leqslant \aleph,\ s_g \in \mathbb{N}$

Evolution

$$\psi_g(t+1) = \sum_{g'\in S_g} A_{gg'}\psi_{g'}(t)$$

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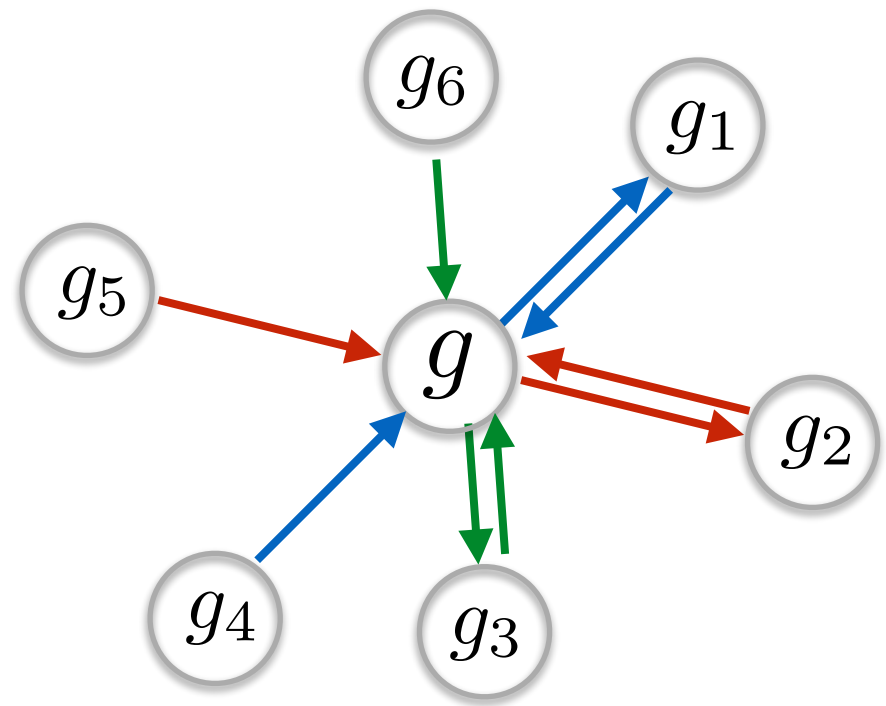


1) Locality: S_g uniformly bounded

2) Reciprocity: $A_{gg'} \neq 0 \implies A_{g'g} \neq 0$

3) Homogeneity: all $g \in G$ are "equivalent"

$S_g = S,\ s_g = s$ ... label $A_{gg'} =: A_h,\ h \in: S$

define the "action" on the set of vertices *G*: $gh := g'$ whenever $A_{gg'} = A_h$

skip proof

D'Ariano, Perinotti,
PRA **90** 062106 (2014)

D'Ariano, Perinotti,
PRA **90** 062106 (2014)

Quantum walk on Cayley graph

w.l.g. Hilbert space $\mathcal{H} = \oplus_{g\in G}\mathbb{C}^{s_g}$ $\quad |G| \leqslant \aleph, \; s_g \in \mathbb{N}$

Evolution

$$\psi_g(t+1) = \sum_{g'\in S_g} A_{gg'}\psi_{g'}(t)$$

$$\sum_{g'} A_{gg'}A^\dagger_{g''g'} = \sum_{g'} A^\dagger_{gg'}A_{g''g'} = \delta_{gg''}I_{s_g}$$

The following operator over the Hilbert space $\ell^2(G)\otimes\mathbb{C}^s$ is unitary

$$A = \sum_{h\in S} T_h \otimes A_h$$

where T is the right regular representation of G on $\ell^2(G)$ acting as

$$T_g|g'\rangle = |g'g^{-1}\rangle$$

$$\left.\begin{array}{l}\text{1) } \underline{\textit{Locality}}\text{: } S_g \text{ uniformly bounded} \\ \text{2) } \underline{\textit{Reciprocity}}\text{: } A_{gg'} \neq 0 \implies A_{g'g} \neq 0 \\ \text{3) } \underline{\textit{Homogeneity}}\text{: all } g \in G \text{ are equivalent}\end{array}\right\} = \text{Quantum Walk on Cayley graph}$$

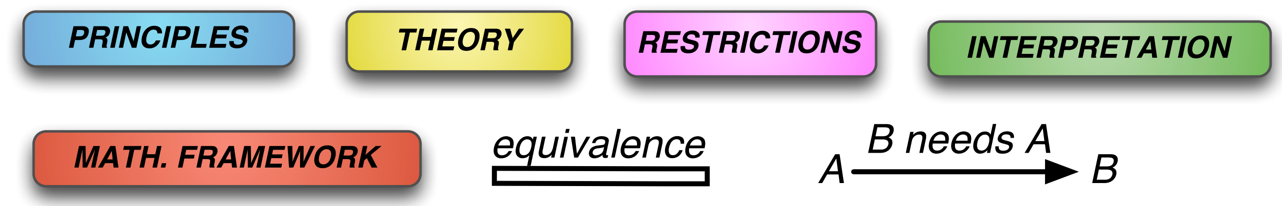
4) *Isotropy:*

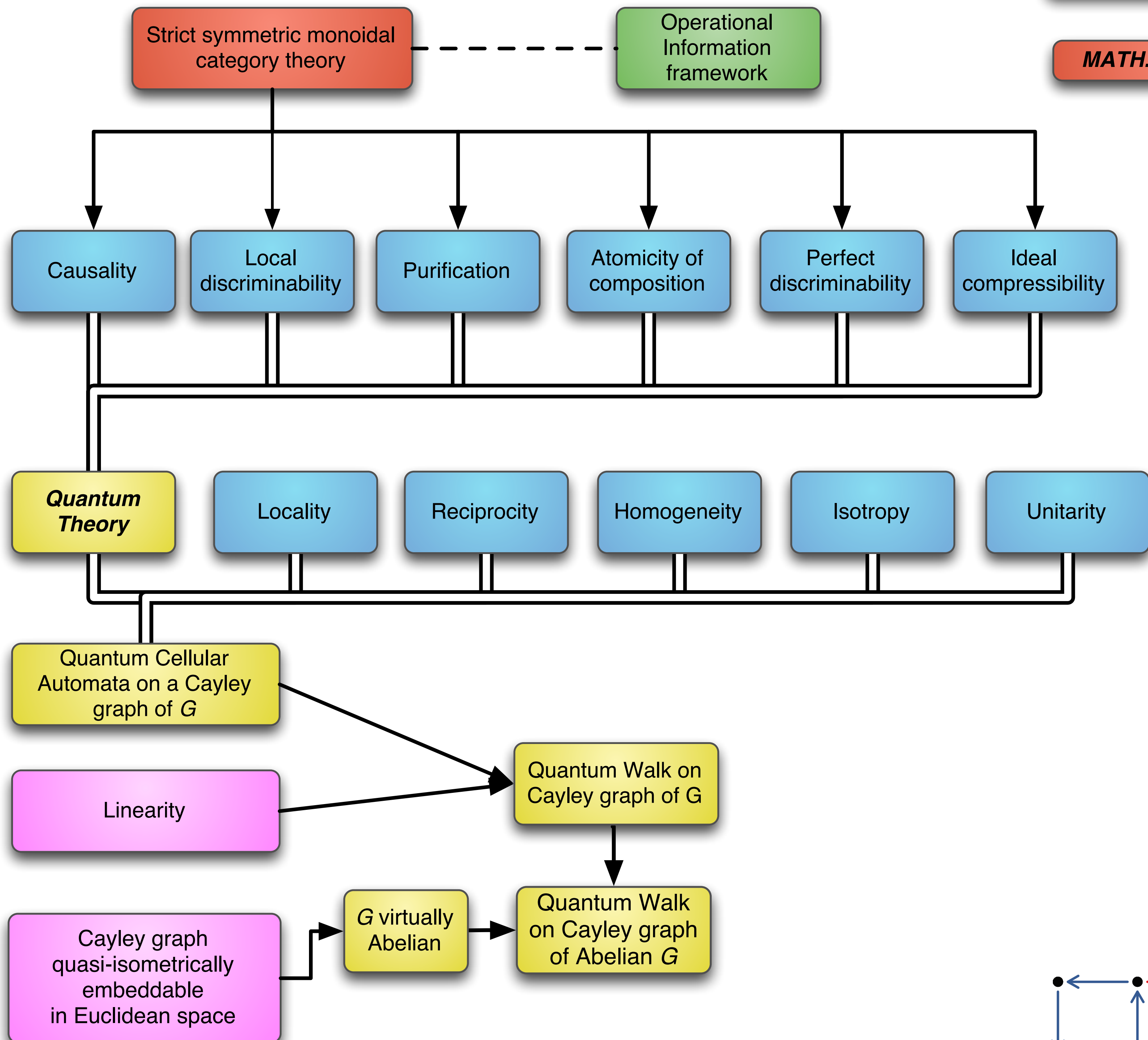
There exist:

- a group L of permutations of S_+, transitive over S_+ that leaves the Cayley graph invariant
- a unitary s-dimensional (projective) representation $\{L_l\}$ of L such that:

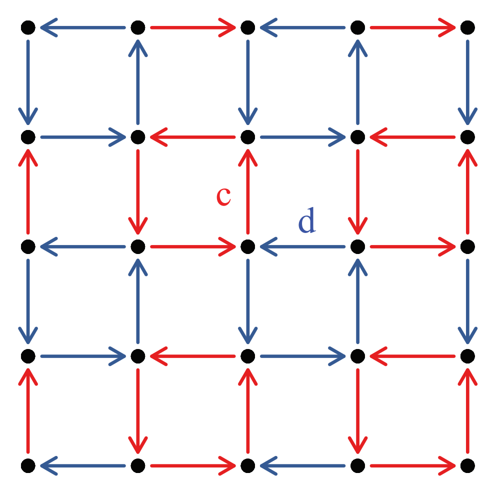
$$\boxed{A = \sum_{h\in S} T_h \otimes A_h = \sum_{h\in S} T_{lh} \otimes L_l A_h L_l^\dagger}$$

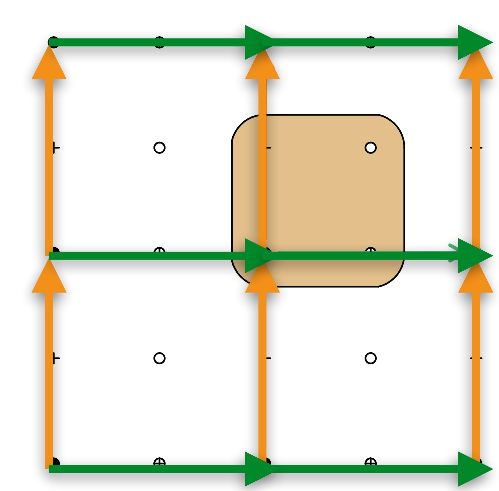
Principles for Quantum Theory





$$\langle\, c, d \mid c^4, d^4, (cd)^2 \,\rangle$$

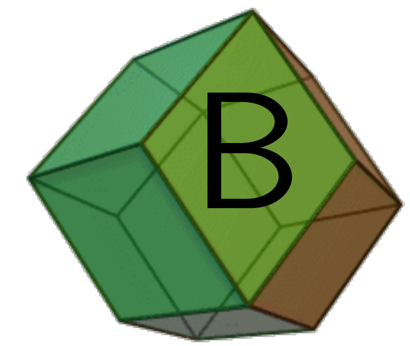




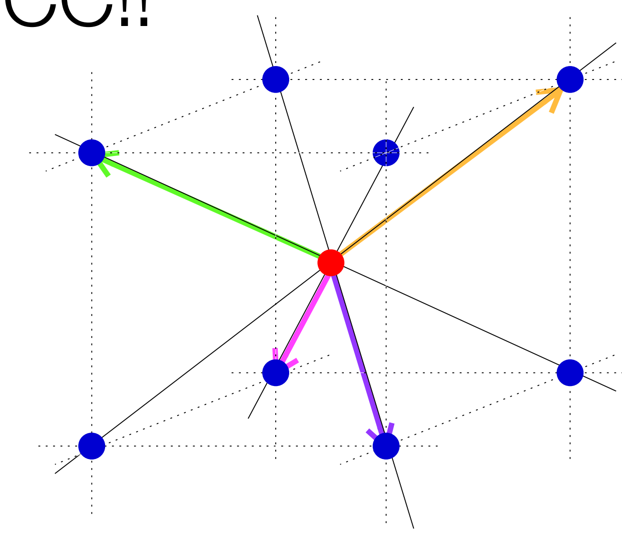
D'Ariano, Perinotti, PRA **90** 062106 (2014)

The Weyl QW

☞ Minimal dimension for nontrivial unitary *A:* s=2



Unitarity + isotropy ⇒ for d=3 the only Cayley is the BCC!!



Unitary operator: $$A = \int_B^{\oplus} d\mathbf{k}\, A_{\mathbf{k}}$$

Two QWs connected by P

$$\boxed{\begin{aligned} A_{\mathbf{k}}^{\pm} = & -i\sigma_x(s_x c_y c_z \pm c_x s_y s_z) \\ & \mp i\sigma_y(c_x s_y c_z \mp s_x c_y s_z) \\ & - i\sigma_z(c_x c_y s_z \pm s_x s_y c_z) \\ & + I(c_x c_y c_z \mp s_x s_y s_z) \end{aligned}}$$

$$s_\alpha = \sin\frac{k_\alpha}{\sqrt{3}}$$

$$c_\alpha = \cos\frac{k_\alpha}{\sqrt{3}}$$

The Weyl QW

D'Ariano, Perinotti,
PRA **90** 062106 (2014)

$$i\partial_t \psi(t) \simeq \tfrac{i}{2}[\psi(t+1) - \psi(t-1)] = \tfrac{i}{2}(A - A^\dagger)\psi(t)$$

$$\begin{aligned} \tfrac{i}{2}(A^{\pm}_{\mathbf{k}} - A^{\pm\dagger}_{\mathbf{k}}) = &+\sigma_x(s_x c_y c_z \pm c_x s_y s_z) \\ &\pm \sigma_y(c_x s_y c_z \mp s_x c_y s_z) \\ &+ \sigma_z(c_x c_y s_z \pm s_x s_y c_z) \end{aligned}$$

"Hamiltonian"

$k \ll 1$ $\Rightarrow$ $\boxed{i\partial_t \psi = \tfrac{1}{\sqrt{3}} \boldsymbol{\sigma}^{\pm} \cdot \mathbf{k}\, \psi}$ ☜ Weyl equation! $\boldsymbol{\sigma}^{\pm} := (\sigma_x, \pm\sigma_y, \sigma_z)$

Two QCAs connected by P

$$\boxed{\begin{aligned} A^{\pm}_{\mathbf{k}} = &-i\sigma_x(s_x c_y c_z \pm c_x s_y s_z) \\ &\mp i\sigma_y(c_x s_y c_z \mp s_x c_y s_z) \\ &- i\sigma_z(c_x c_y s_z \pm s_x s_y c_z) \\ &+ I(c_x c_y c_z \mp s_x s_y s_z) \end{aligned}}$$

$$s_\alpha = \sin \frac{k_\alpha}{\sqrt{3}}$$

$$c_\alpha = \cos \frac{k_\alpha}{\sqrt{3}}$$

D'Ariano, Perinotti, PRA **90** 062106 (2014)

Bisio, D'Ariano, Perinotti, Ann. Phys. **368** 177 (2016)

Dirac QW $\oplus$

Local coupling: $A_{\mathbf{k}}$ coupled with its inverse with off-diagonal identity block matrix

$$\boxed{E^{\pm}_{\mathbf{k}} = \begin{pmatrix} nA^{\pm}_{\mathbf{k}} & imI \\ imI & nA^{\pm\dagger}_{\mathbf{k}} \end{pmatrix} \qquad n^2 + m^2 = 1 \qquad n, m \in \mathbb{R}}$$

$E^{\pm}_{\mathbf{k}}$ CPT-connected!

$$\omega^{E}_{\pm}(\mathbf{k}) = \cos^{-1}[n(c_x c_y c_z \mp s_x s_y s_z)]$$

Dirac in relativistic limit $k \ll m \ll 1$

m: mass, $m^2 \leq 1$

n^{-1}: refraction index

± 1

m !

0

Maxwell QW $\otimes$

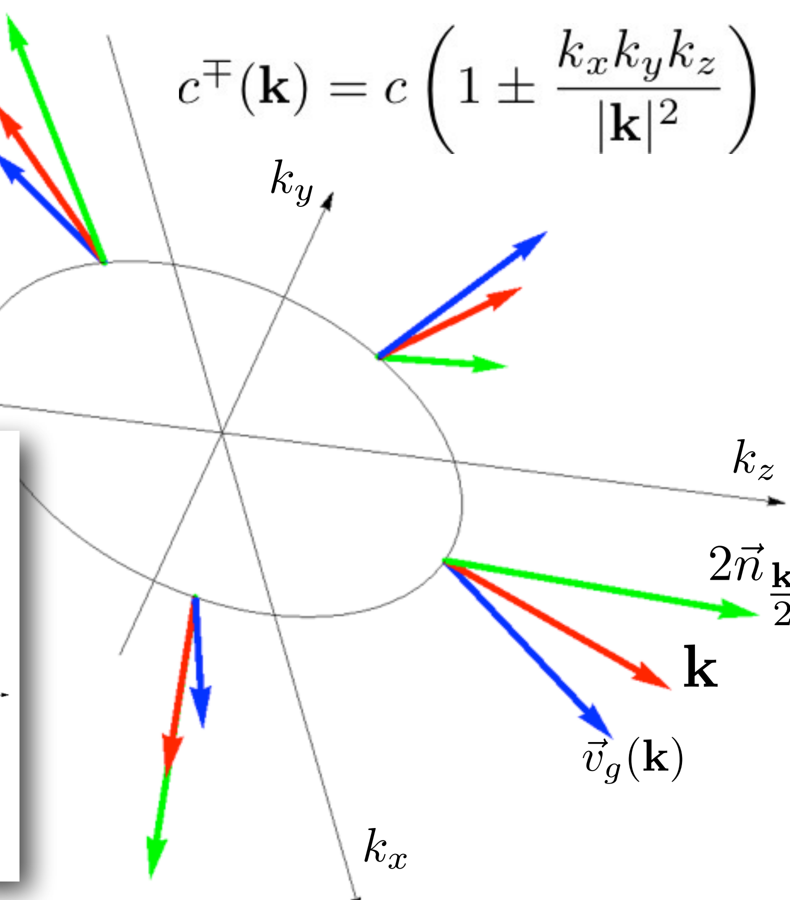
$$\boxed{M^{\pm}_{\mathbf{k}} = A^{\pm}_{\mathbf{k}} \otimes A^{\pm *}_{\mathbf{k}} \qquad F^{\mu}(\mathbf{k}) = \int \frac{\mathrm{d}\,\mathbf{q}}{2\pi} f(\mathbf{q}) \tilde{\psi}(\tfrac{\mathbf{k}}{2} - \mathbf{q}) \sigma^{\mu} \varphi(\tfrac{\mathbf{k}}{2} + \mathbf{q})}$$

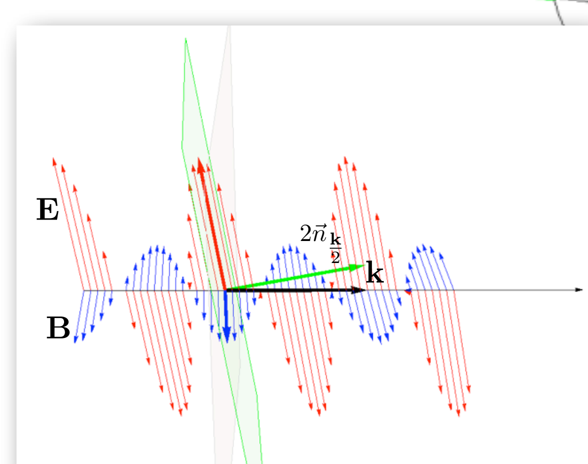
Maxwell in relativistic limit $k \ll 1$

Boson: emergent from convolution of fermions

(De Broglie neutrino-theory of photon)

$$c^{\mp}(\mathbf{k}) = c\left(1 \pm \frac{k_x k_y k_z}{|\mathbf{k}|^2}\right)$$





Dirac emerging from the QCA

D'Ariano, Perinotti,
PRA **90** 062106 (2014)

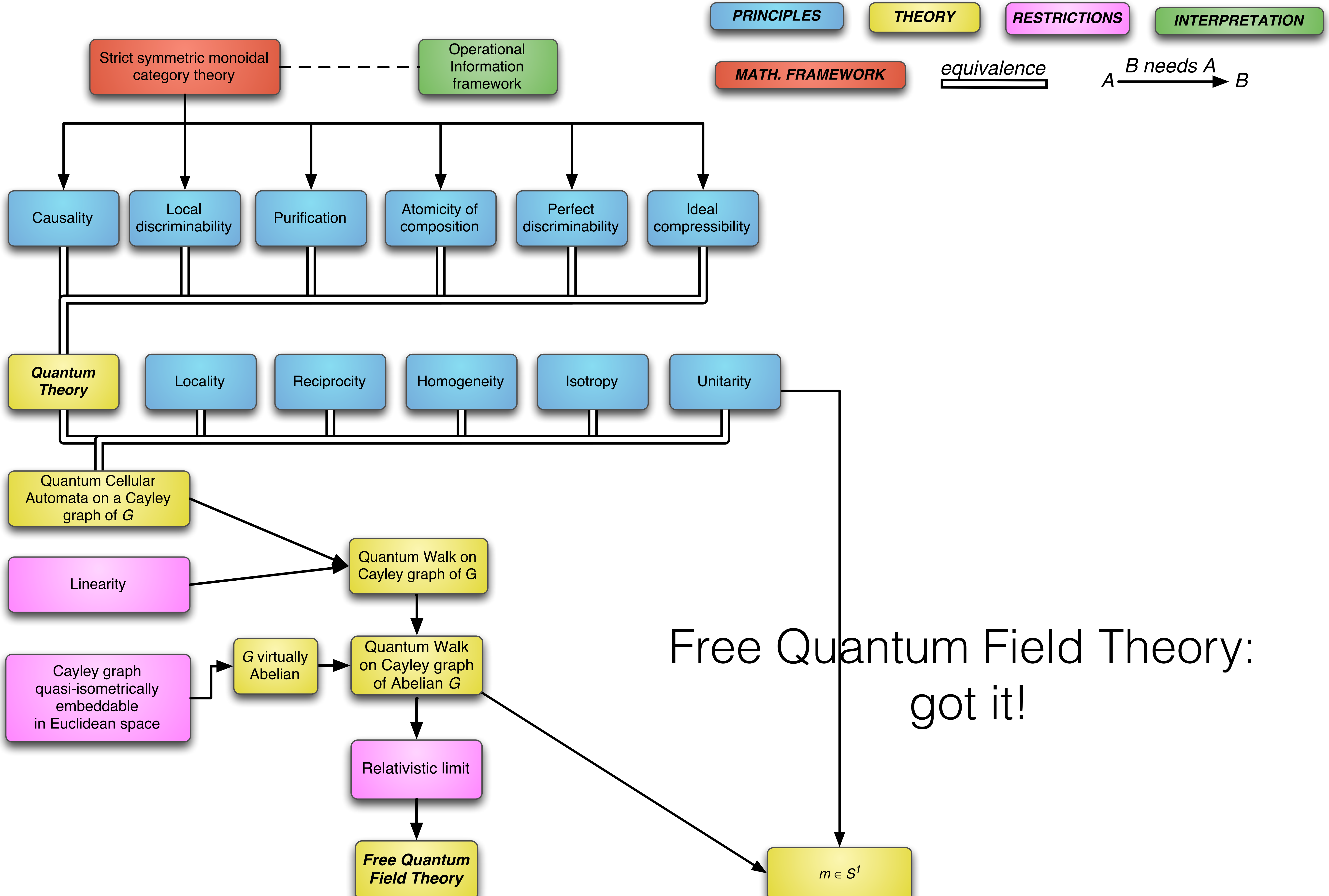
Fidelity with Dirac for a narrowband packets in the relativistic limit $k \simeq m \ll 1$

$$F = |\langle \exp\left[-iN\boldsymbol{\Delta}(\mathbf{k})\right]\rangle|$$

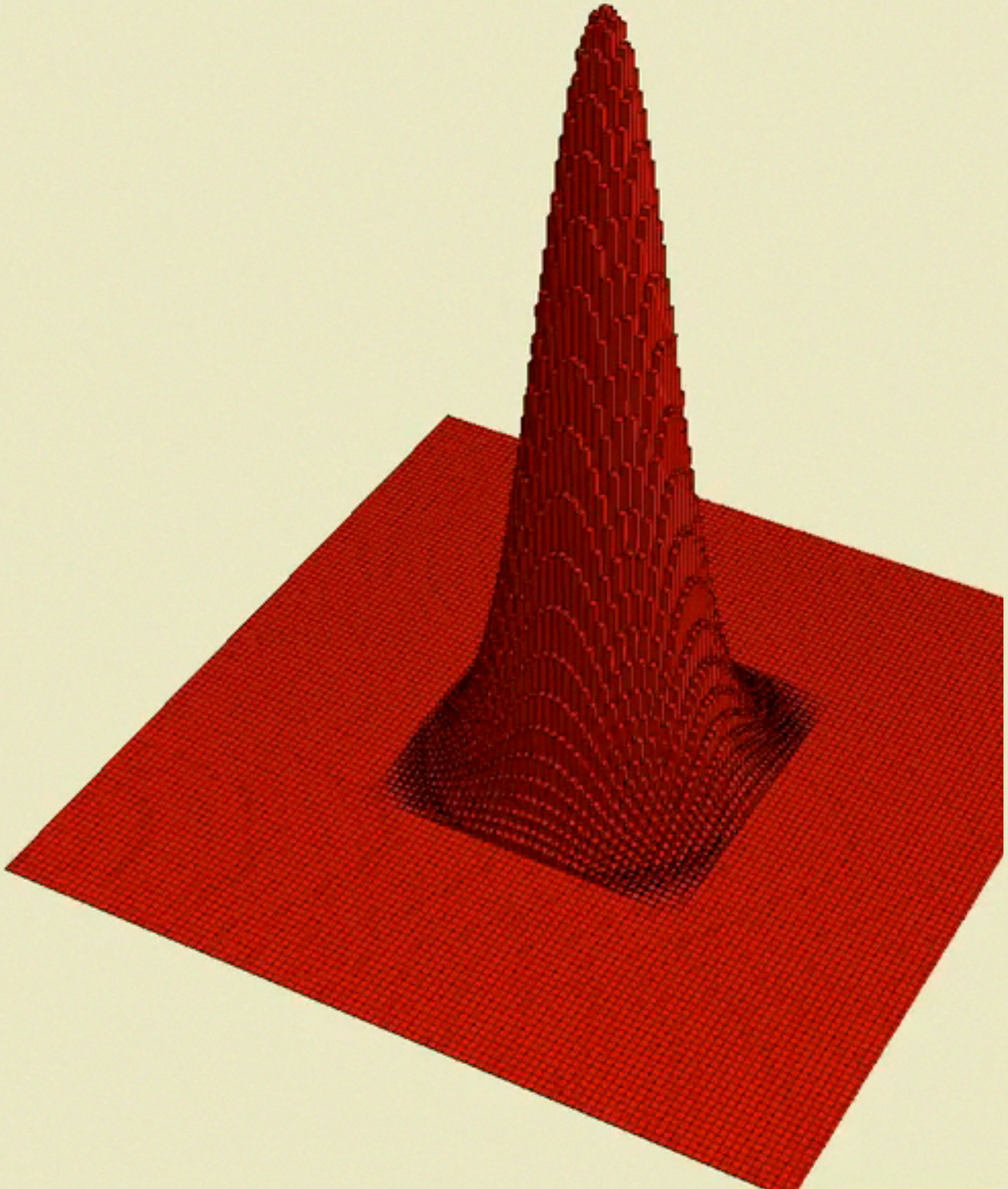
$$\begin{aligned}\Delta(\mathbf{k}) &:= (m^2 + \tfrac{k^2}{3})^{\frac{1}{2}} - \omega^E(\mathbf{k}) \\ &= \frac{\sqrt{3}k_x k_y k_z}{(m^2 + \frac{k^2}{3})^{\frac{1}{2}}} - \frac{3(k_x k_y k_z)^2}{(m^2 + \frac{k^2}{3})^{\frac{3}{2}}} + \tfrac{1}{24}(m^2 + \tfrac{k^2}{3})^{\frac{3}{2}} + \mathcal{O}(k^4 + N^{-1}k^2)\end{aligned}$$

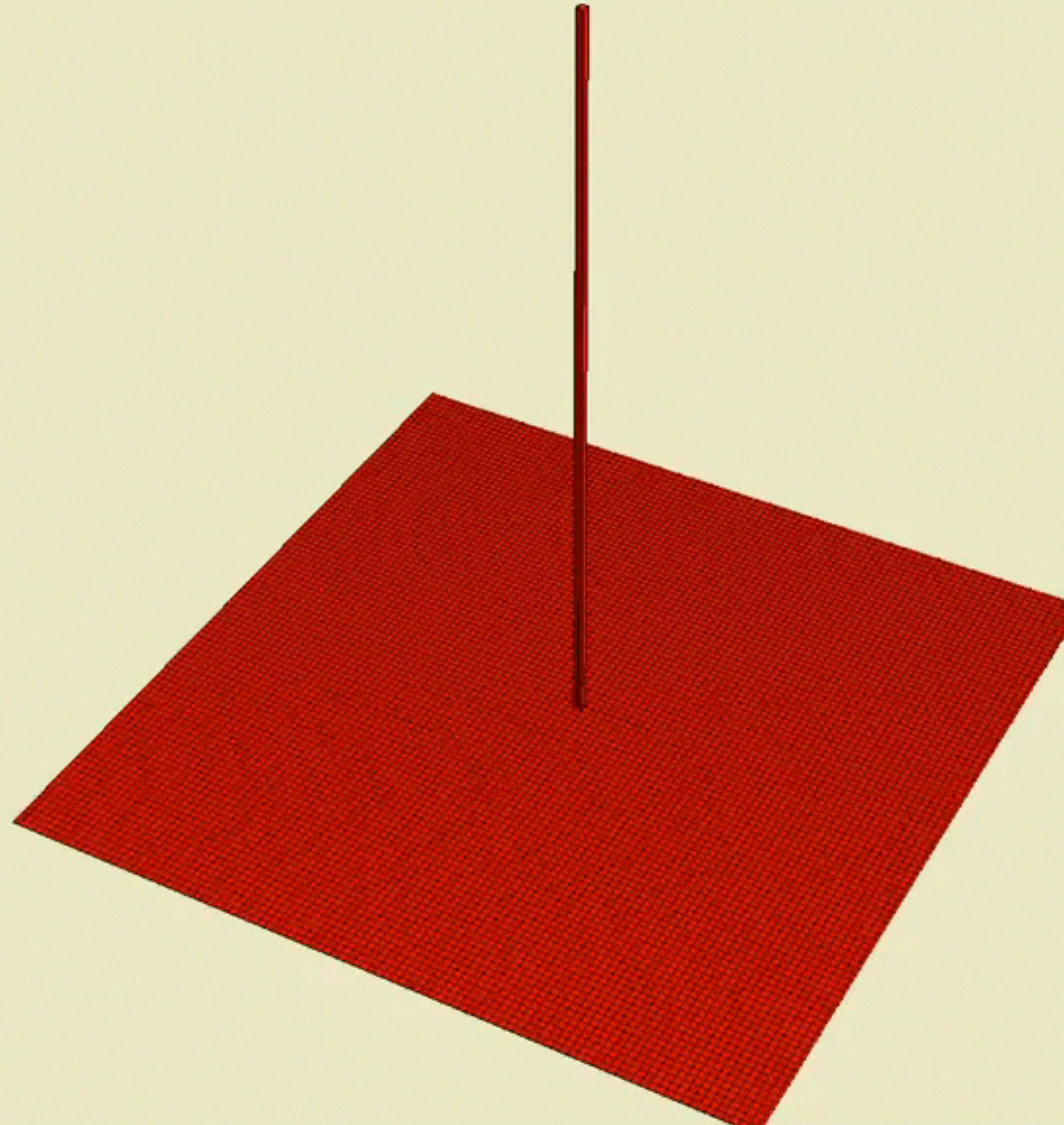
relativistic proton: $N \simeq m^{-3} = 2.2 * 10^{57} \;\Rightarrow\; t = 1.2 * 10^{14}\mathrm{s} = 3.7 * 10^6$ y

UHECRs: $k = 10^{-8} \gg m \;\Rightarrow N \simeq k^{-2} = 10^{16} \;\Rightarrow\; 5 * 10^{-28}$ s



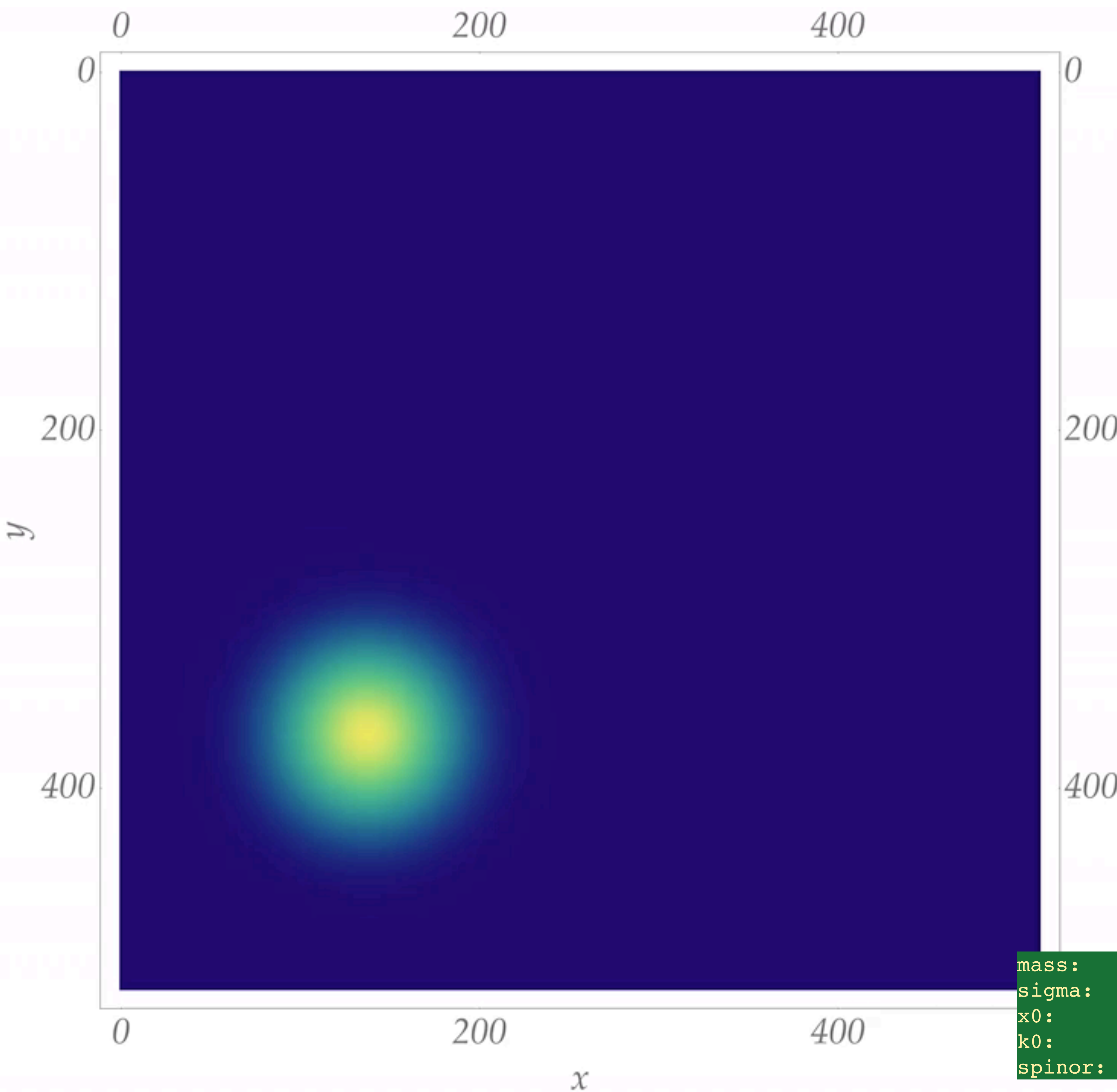
Free Quantum Field Theory: got it!





2d Dirac

- Evolution of a *narrow-band particle-state*
- Evolution of a *localized state*

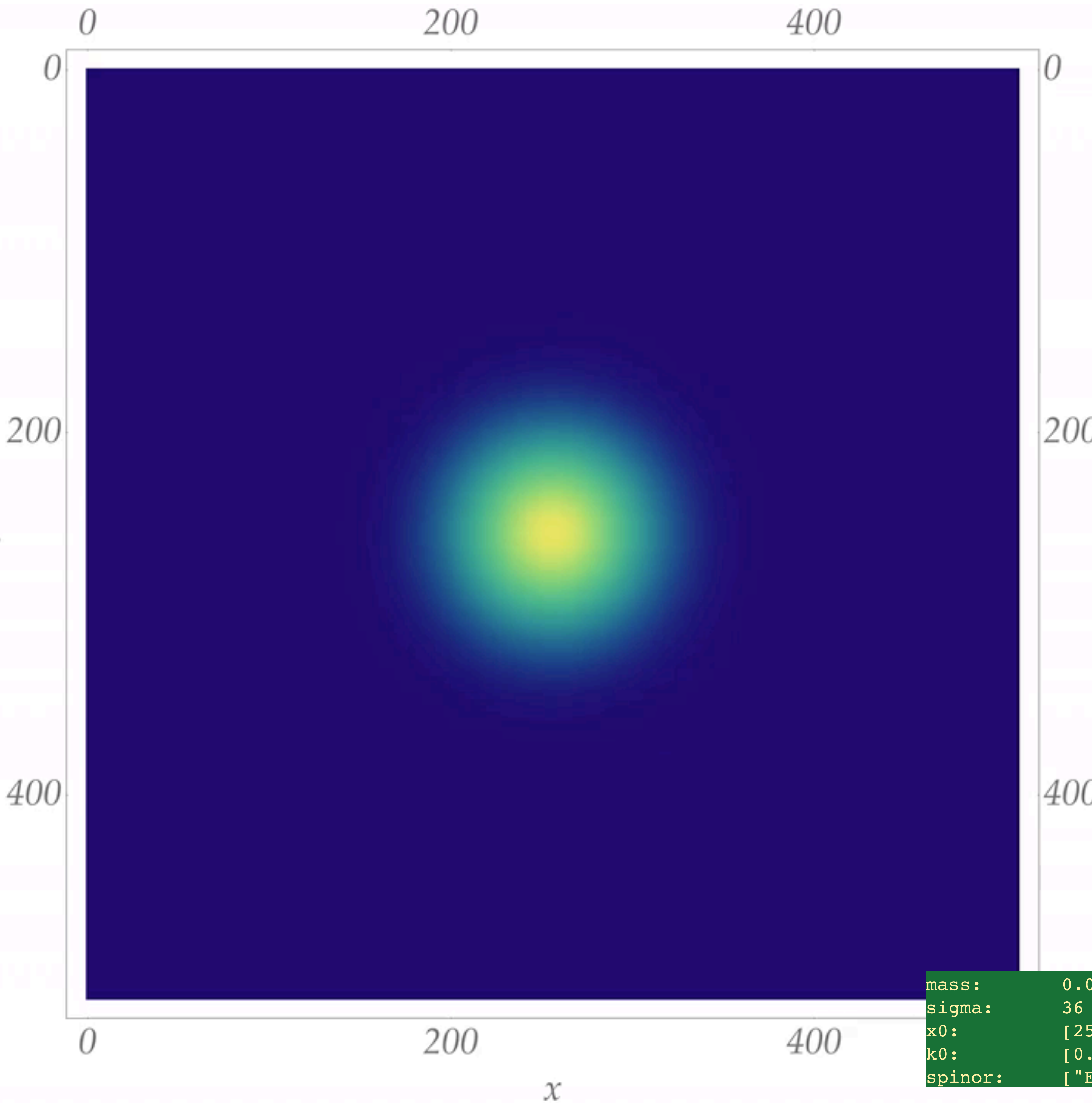


D'Ariano, Mosco, Perinotti, Tosini, arXiv:1603.06442

Dirac 3d

```
mass:      0.002
sigma:     32
x0:        [140,140,140]
k0:        [0.05,0.05,0.05]
spinor:    ["Exp[I k0.#]",0,0,0]
```

D'Ariano, Mosco, Perinotti, Tosini, arXiv:1603.06442



Dirac 3d

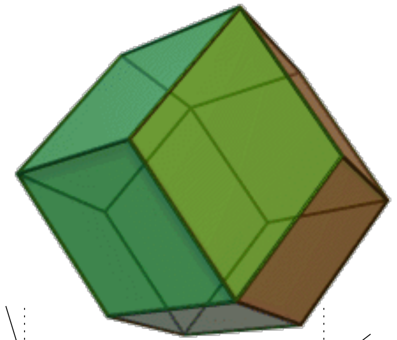
```
mass:       0.008
sigma:      36
x0:         [256,256,256]
k0:         [0.05,0.05,0.05]
spinor:     ["Exp[I k0.#]",0,0,"Exp[I k0.#]"]
```

The LTM standards of the theory

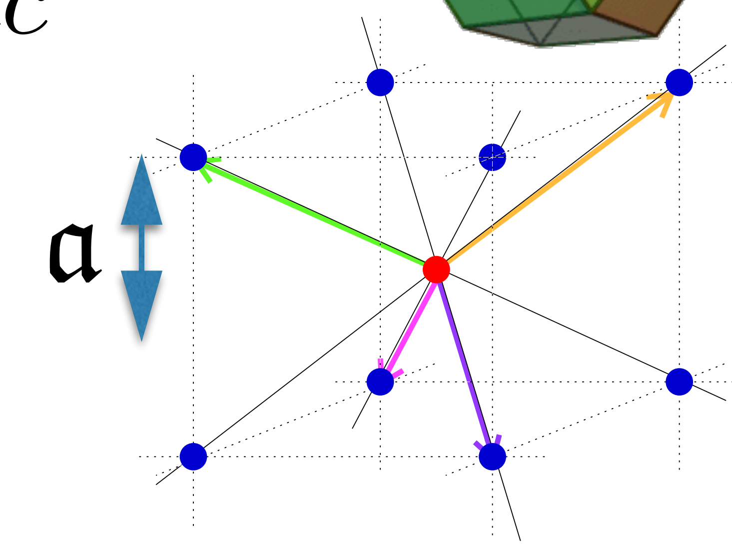
Dimensionless variables

$$x = \frac{x_{[m]}}{\mathfrak{a}} \in \mathbb{Z}, \quad t = \frac{t_{[sec]}}{\mathfrak{t}} \in \mathbb{Z}, \quad m = \frac{m_{[kg]}}{\mathfrak{m}} \in [0,1]$$

$$k/k_{max} \qquad \omega/\omega_{max}$$

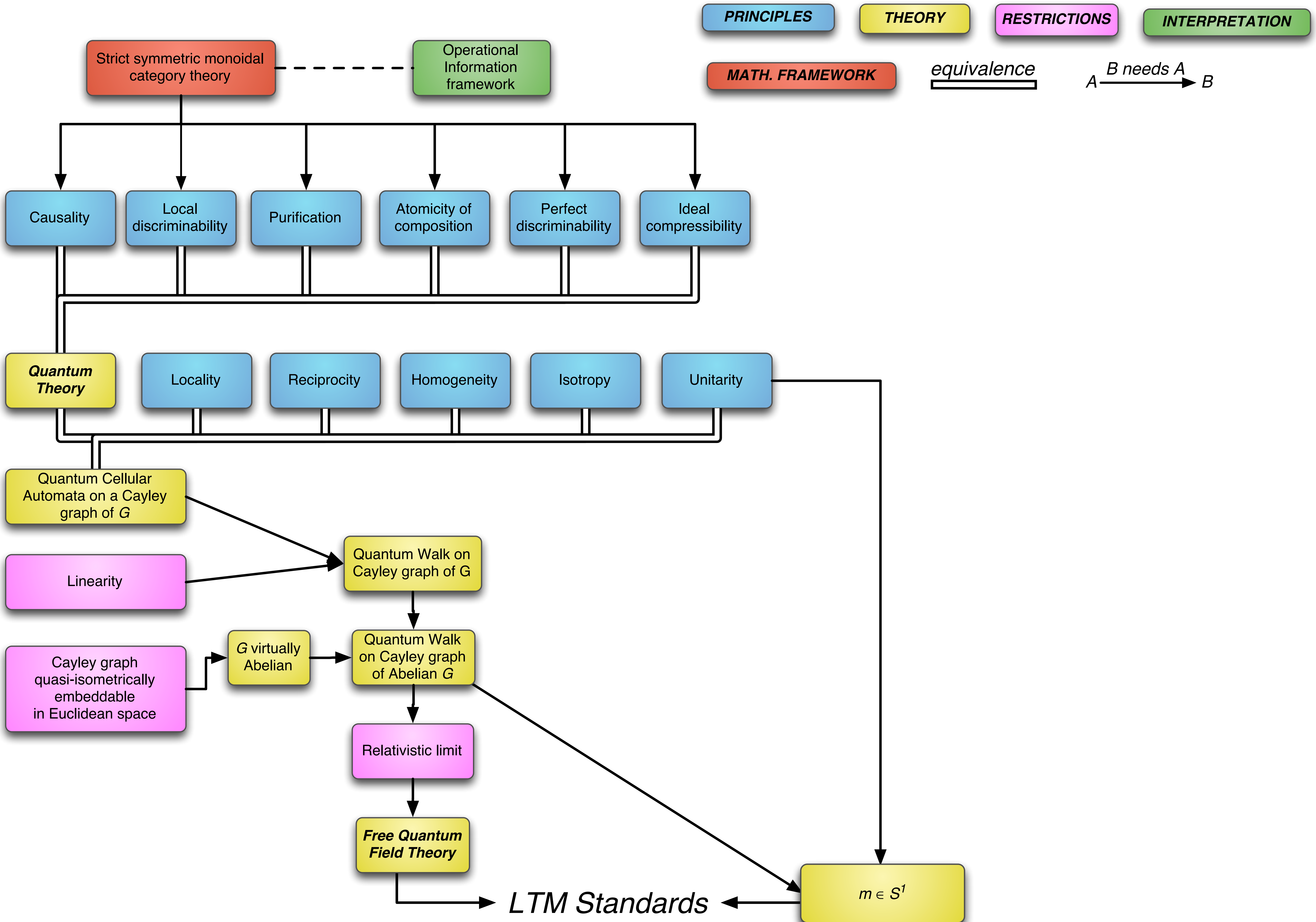


Relativistic limit: $\quad c = \mathfrak{a}/\mathfrak{t} \qquad \hbar = \mathfrak{m}\mathfrak{a}c$



Measure $\mathfrak{a}$ (k_{max}) from light-refraction-index

$$c^{\mp}(k) = c\left(1 \pm \frac{k}{\sqrt{3}k_{max}}\right)$$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B needs A
A
B
Strict symmetric monoidal category theory
Operational Information framework
Causality
Local discriminability
Purification
Atomicity of composition
Perfect discriminability
Ideal compressibility
Quantum Theory
Locality
Reciprocity
Homogeneity
Isotropy
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Quantum Cellular Automata on a Cayley graph of G
Linearity
Quantum Walk on Cayley graph of G
Cayley graph quasi-isometrically embeddable in Euclidean space
G virtually Abelian
Quantum Walk on Cayley graph of Abelian G
Relativistic limit
Free Quantum Field Theory
LTM Standards
m ∈ S^1

Bisio, D'Ariano, Perinotti,
unpublished

Special Relativity recovered … and more

Relativity principle: invariance of the physical law under change of inertial reference frame
→ invariance of eigenvalue equation under change of representation.

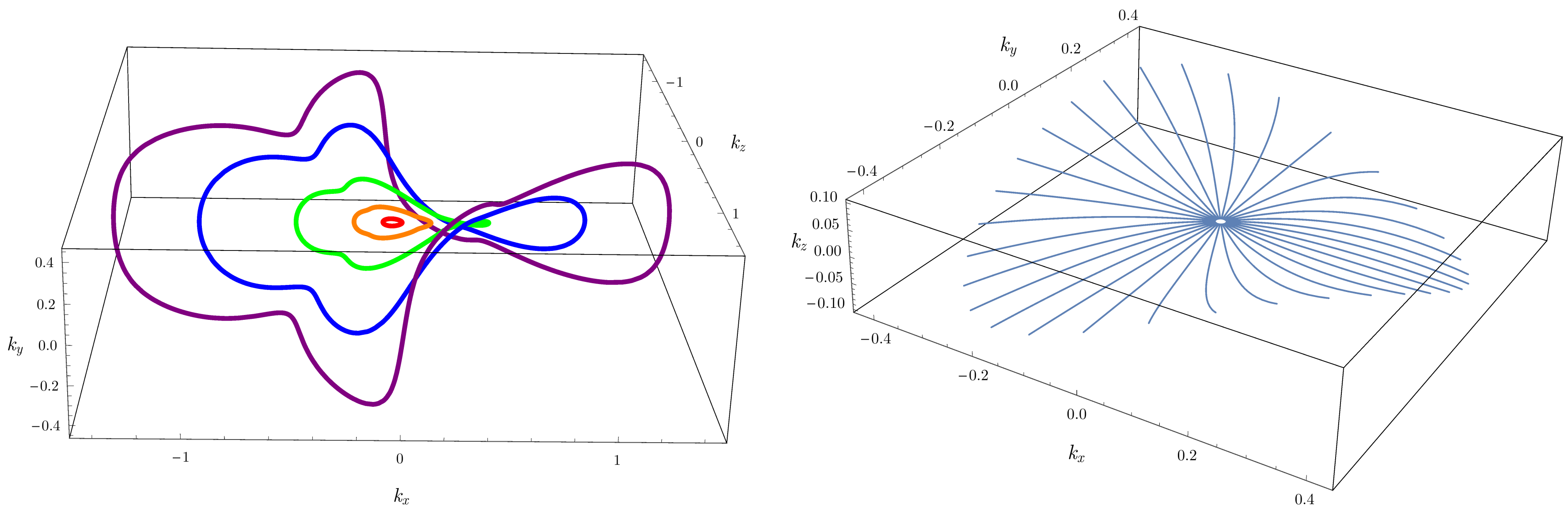


FIG. 2: The distortion effects of the Lorentz group for the discrete Planck-scale theory represented by the quantum walk in Eq. (6). Left figure: the orbit of the wavevectors $\mathbf{k} = (k_x, 0, 0)$, with $k_x \in \{.05, .2, .5, 1, 1.7\}$ under the rotation around the z axis. Right figure: the orbit of wavevectors with $|\mathbf{k}| = 0.01$ for various directions in the (k_x, k_y) plane under the boosts with $\boldsymbol{\beta}$ parallel to $\mathbf{k}$ and $|\boldsymbol{\beta}| \in [0, \tanh 4]$.

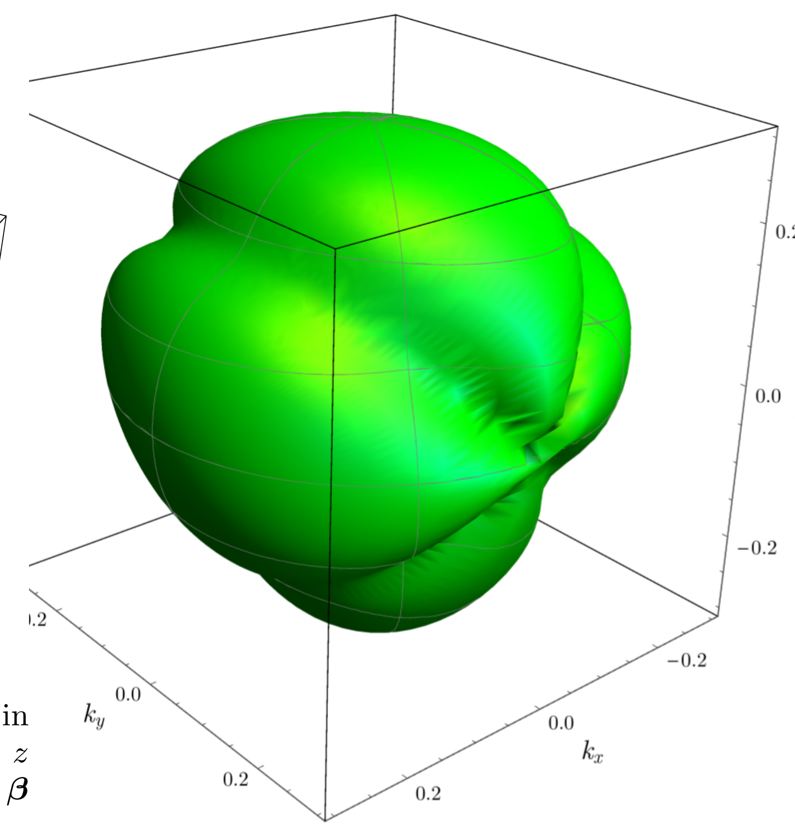


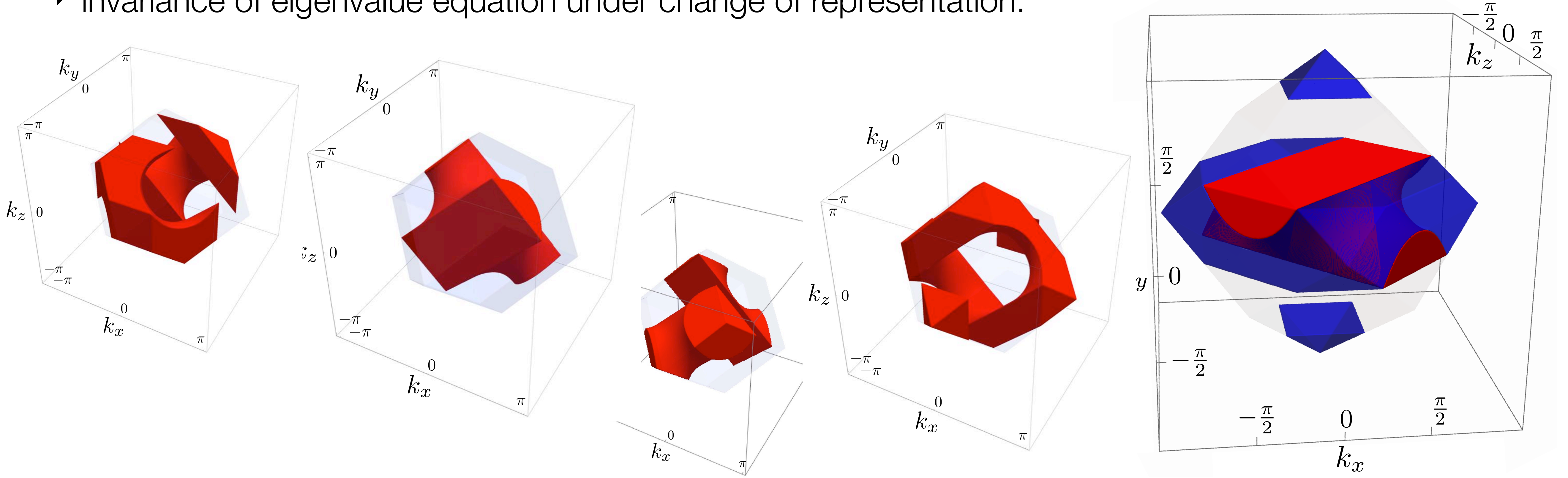
FIG. 3: The green surface represents the orbit of the wavevector $\mathbf{k} = (0.3, 0, 0)$ under the full rotation group $SO(3)$.

- Deformed Poincaré group
- Lorentz transformations are perfectly recovered for *k≪1.*
- For *k~1*:
  - *Double Special Relativity* (Camelia-Smolin).
  - *Relative locality* (in addition to relativity of simultaneity)

Bisio, D'Ariano, Perinotti, unpublished

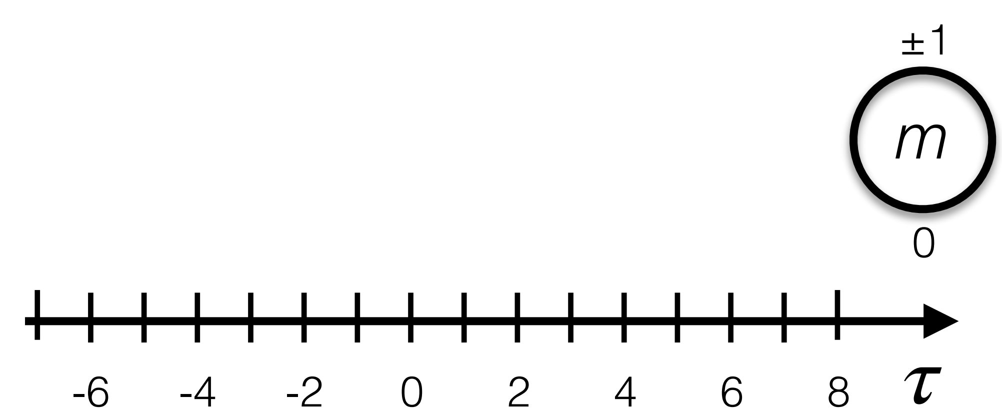
Special Relativity recovered … and more

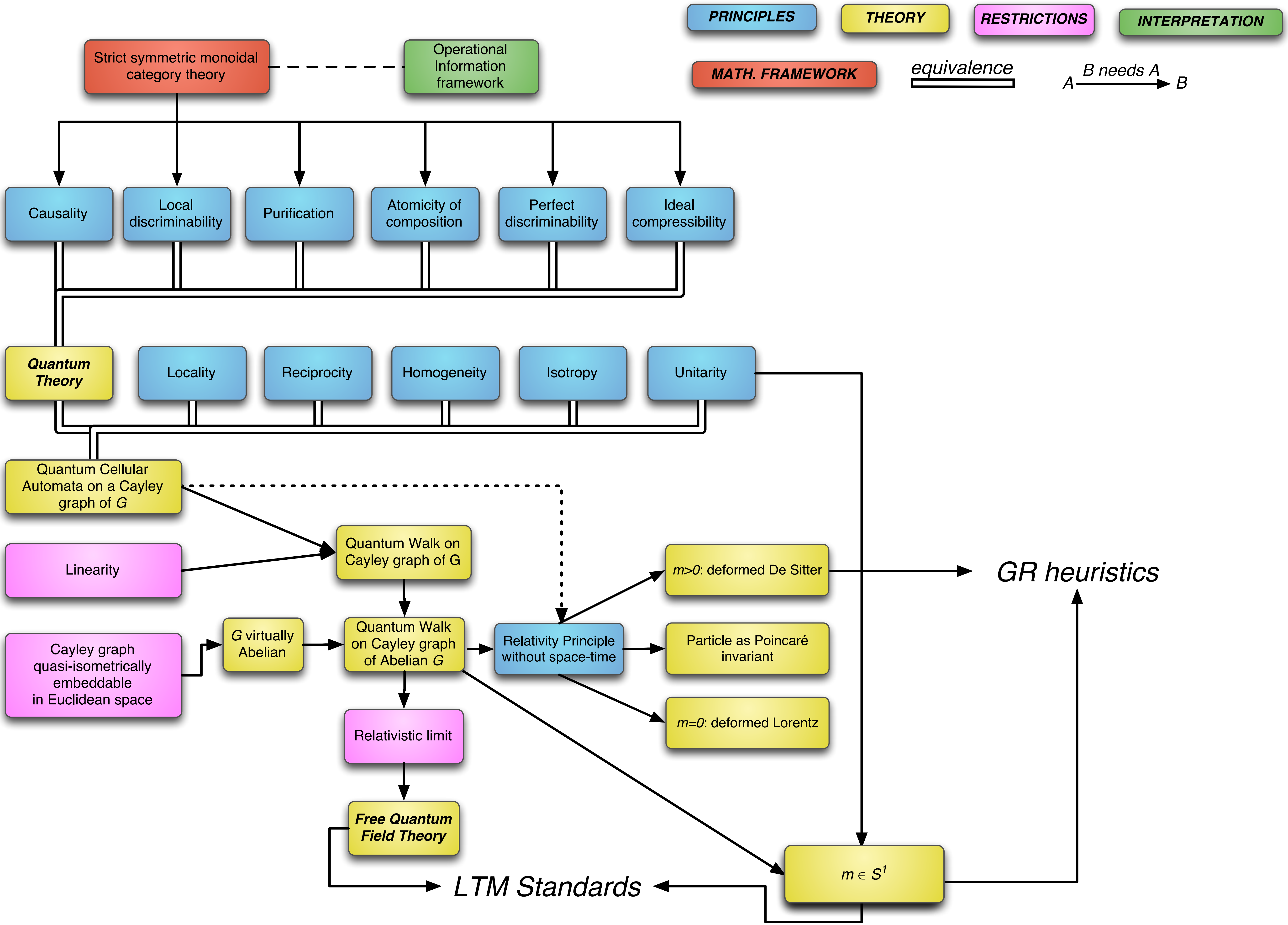
Relativity principle: invariance of the physical law under change of inertial reference frame
→ invariance of eigenvalue equation under change of representation.



- The Brillouin zone separates into four Poincaré-invariant regions diffeomorphic to balls, corresponding to four different particles.
- $m \neq 0$ De Sitter $SO(1,4)$
- mass m and proper-time τ are conjugated

$$H(q_\alpha, p_\alpha, \tau, m) = \sum_\alpha p_\alpha \dot{q}_\alpha + c^2 m \dot{\tau} - L$$

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m>0: deformed De Sitter
Particle as Poincaré invariant
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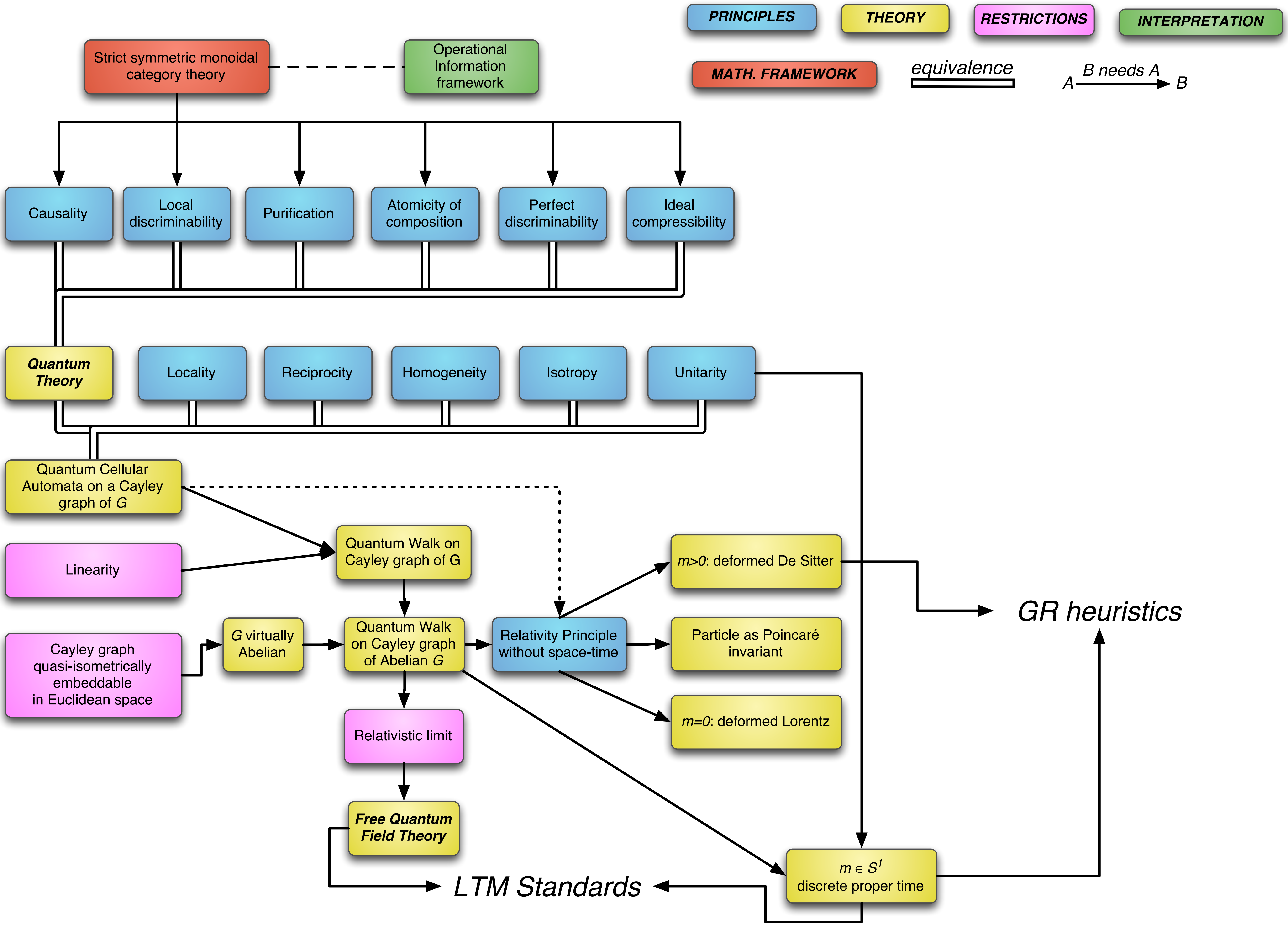
GR heuristics
Relativistic limit
Free Quantum Field Theory
LTM Standards
$m \in S^1$

This is more or less what I wanted to say

Thank you for your attention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Relativity Principle without space-time
m>0: deformed De Sitter
Particle as Poincaré invariant
m=0: deformed Lorentz
GR heuristics
Relativistic limit
Free Quantum Field Theory
LTM Standards
m ∈ S1 discrete proper time