JOHN TEMPLETON FOUNDATION
SUPPORTING SCIENCE~INVESTING IN THE BIG QUESTIONS

Why is not so easy to change Quantum Mechanics and one of the only possible changes is GRW

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Università degli Studi di Pavia

Is quantum theory exact? The endeavor for the theory beyond standard quantum mechanics.

Laboratori Nazionali di Frascati, April 29 2014

G. M. D'Ariano and P. Perinotti, arXiv:1306.1934

A. Bibeau-Delisle, A. Bisio, G. M. D'Ariano, P. Perinotti, A. Tosini, arXiv:1310.6760

A. Bisio, G. M. D'Ariano, A. Tosini, arXiv:1212.2839

TOC

Quantum Mechanics =
Quantum Theory+Mechanics

1. Information-theoretic Axioms for QT
   - Operational probabilistic theory (OPT) framework
2. QT → QFT → QM
3. Which modifications destroy the epistemological value of QT, and why GRW is compatible with (1)
4. Proposal: through (2) we can make GRW Lorentz covariant and for QFT

Problems with GRW

Lorentz covariance

Indistinguishable particles

QFT

A. Rimini (private comm.)

Historical background

- The experience in Quantum Information has led us to look at Quantum Theory (QT) under a completely new angle
- QT is a *theory of information*

Selected for a Viewpoint in *Physics*

PHYSICAL REVIEW A **84**, 012311 (2011)

Informational derivation of quantum theory

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We derive quantum theory from purely informational principles. Five elementary axioms—causality, perfect distinguishability, ideal compression, local distinguishability, and pure conditioning—define a broad class of theories of information processing that can be regarded as standard. One postulate—purification—singles out quantum theory within this class.

DOI: 10.1103/PhysRevA.84.012311 PACS number(s): 03.67.Ac, 03.65.Ta

Principles for Quantum Theory

P1. Causality

P2. Local discriminability

P3. Purification *

P4. Atomicity of composition

P5. Perfect distinguishability

P6. Lossless Compressibility

Book from CUP (by the end of 2014)

Principles for Quantum Theory

The *informational* framework

Logic ⊂ Probability ⊂ OPT

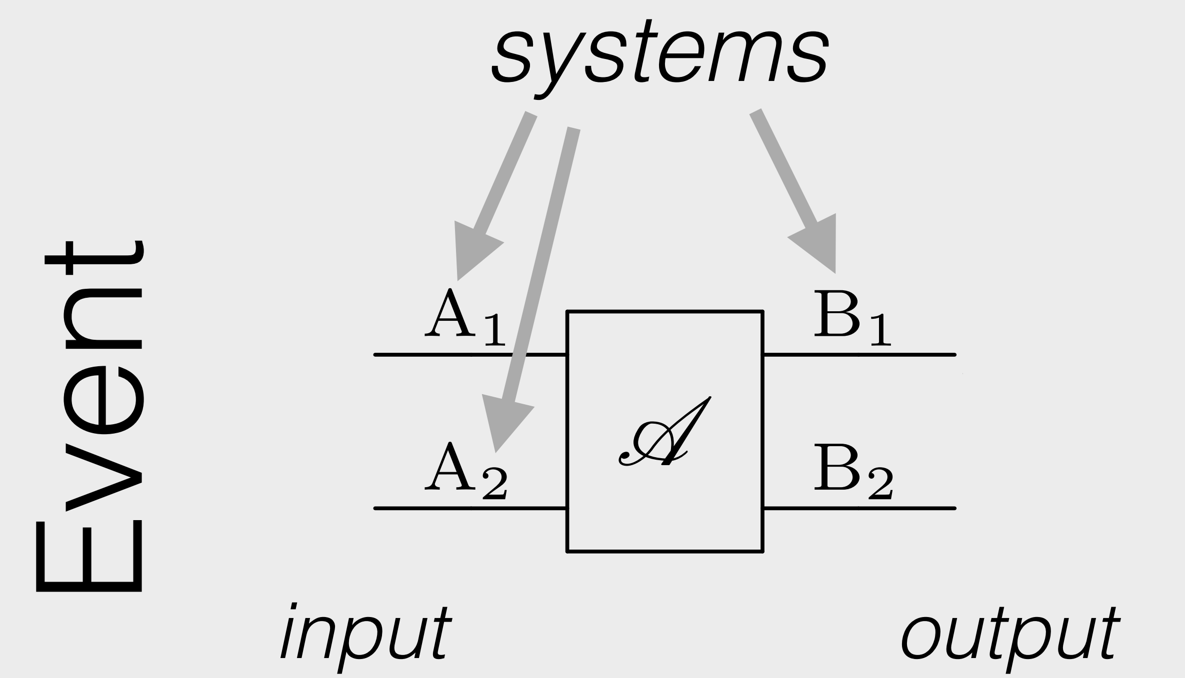
joint probabilities + connectivity

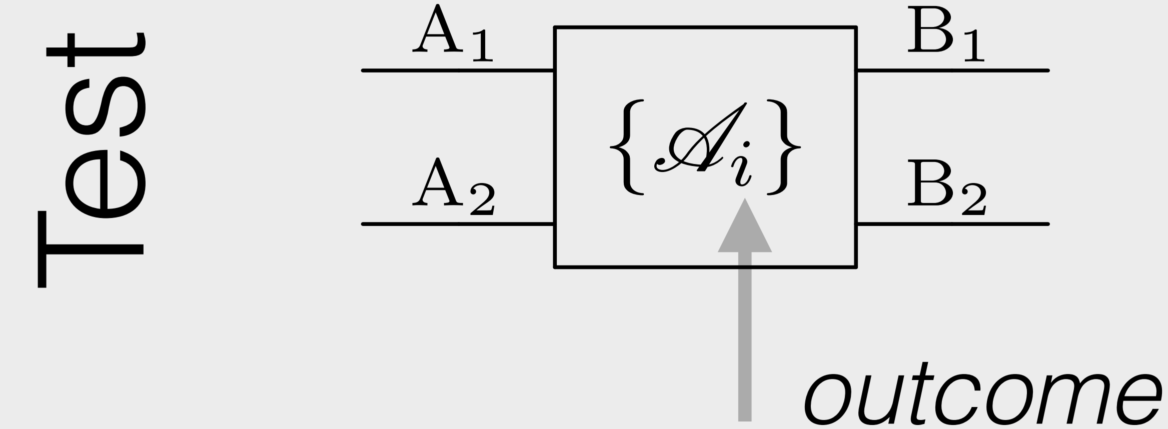
$$p(i, j, k, \ldots | \mathrm{circuit})$$

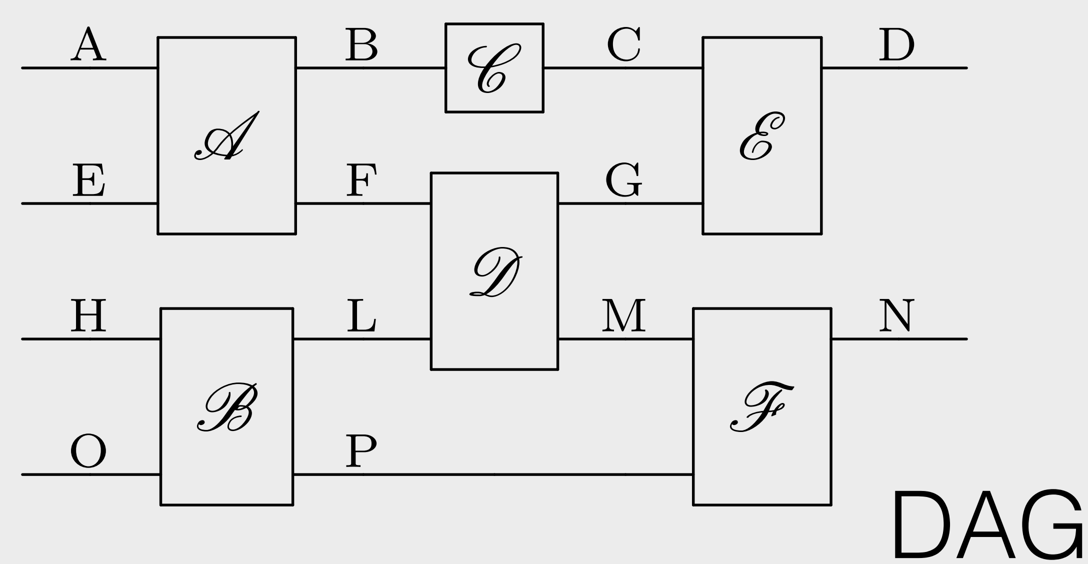
Marginal probability

$$\sum_{i,k,\ldots} p(i, j, k, \ldots | \mathrm{circuit}) =$$

$$p(j | \mathrm{circuit})$$







Principles for Quantum Theory

The *informational* framework

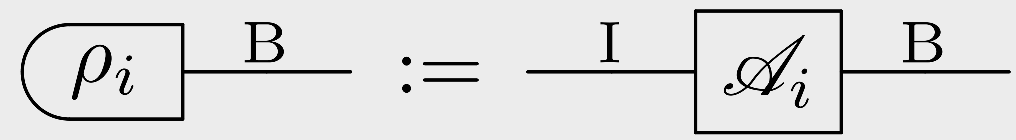
Logic ⊂ Probability ⊂ OPT

joint probabilities + connectivity

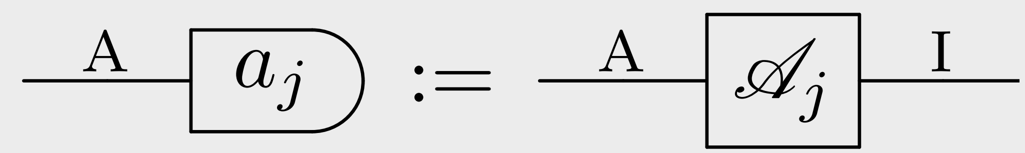
$$p(i, j, k, \dots | \text{circuit})$$

Leaf:

Maximal set of independent systems

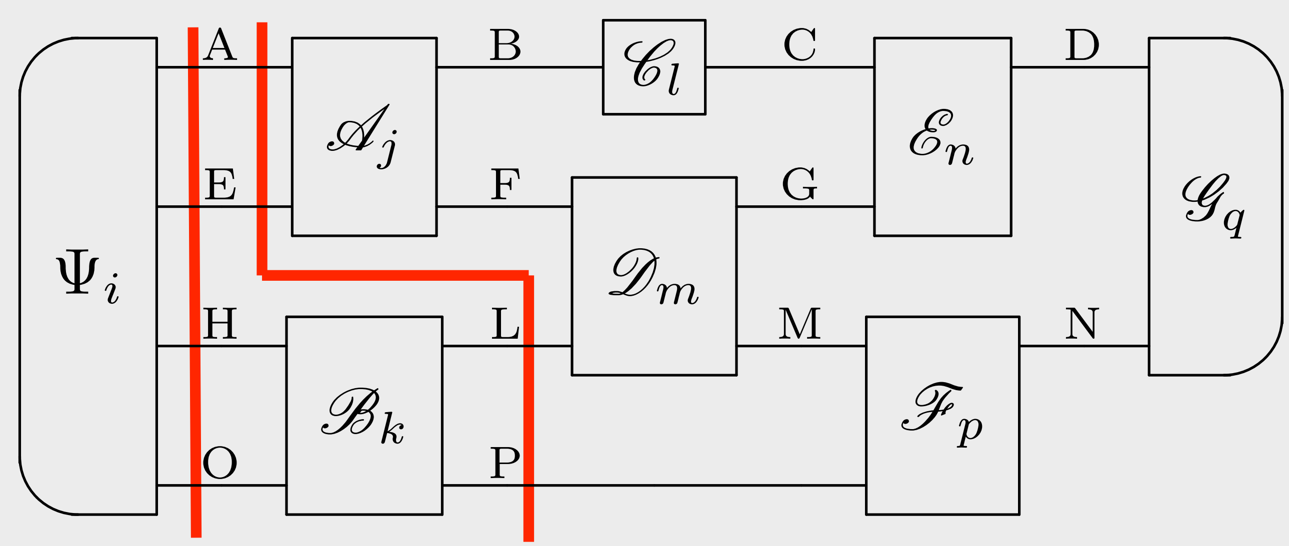


preparation



observation

$$p(i, j, k, l, m, n, p, q | \text{circuit})$$



Principles for Quantum Theory

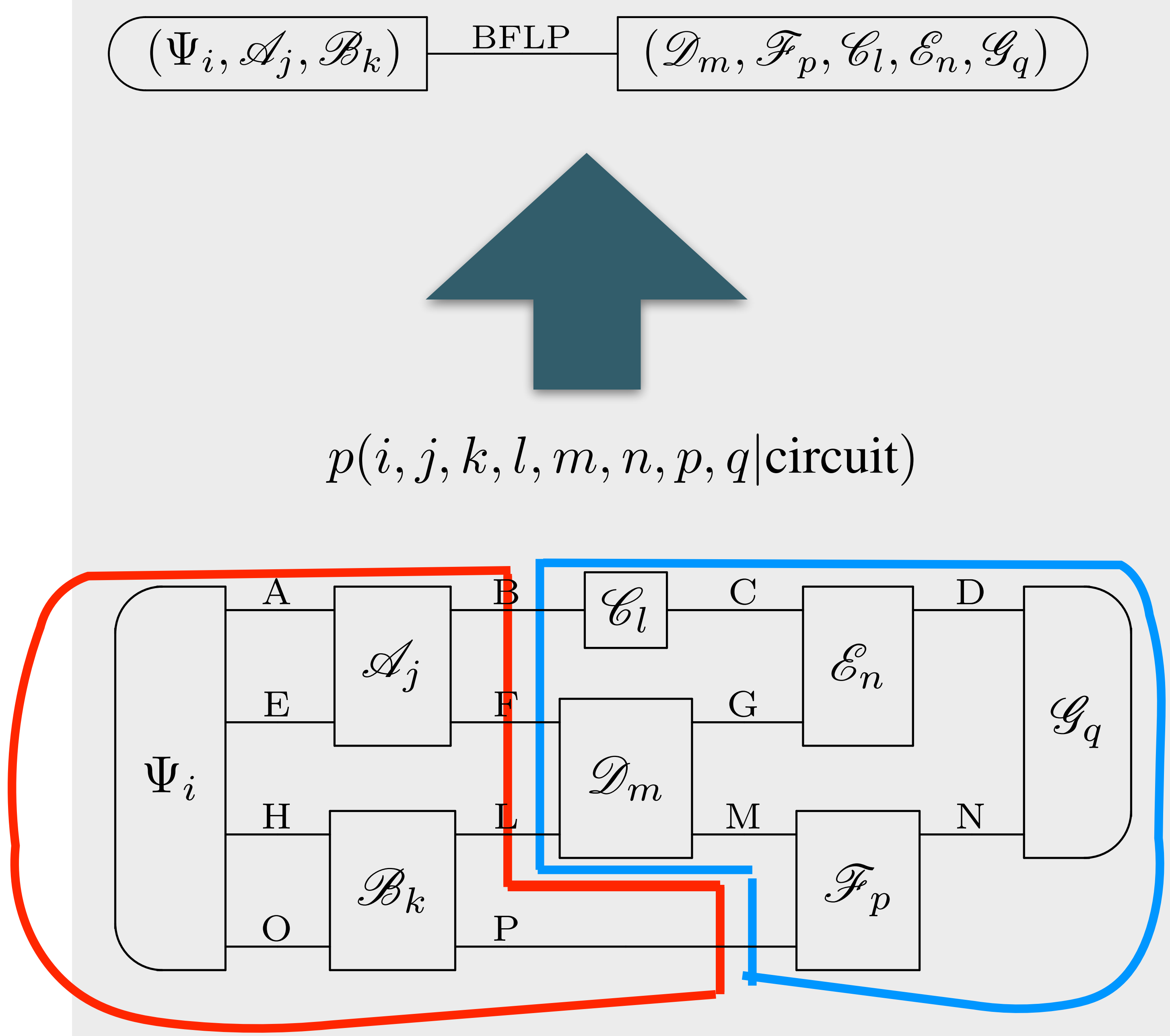
The *informational* framework

Logic ⊂ Probability ⊂ OPT

joint probabilities + connectivity

$$p(i, j, k, \ldots | \text{circuit})$$

Leaf:
Maximal set of
independent systems



Principles for Quantum Theory

The *informational* framework

Logic ⊂ Probability ⊂ OPT

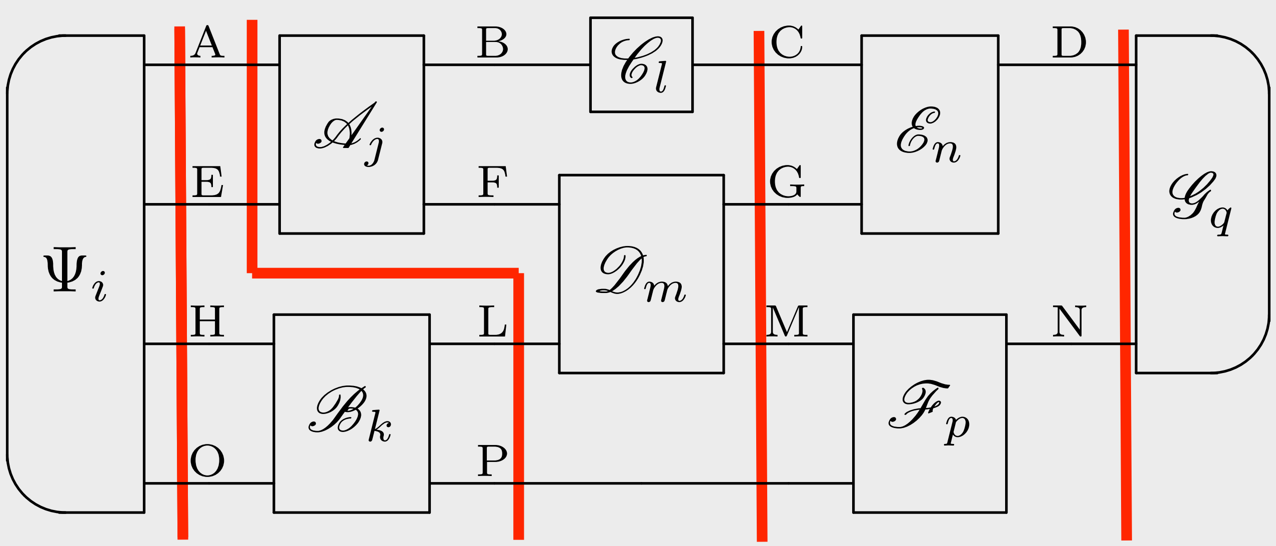
joint probabilities + connectivity

$$p(i, j, k, \ldots | \text{circuit})$$

Leaf:
Maximal set of independent systems

→ *Foliation*

$$p(i, j, k, l, m, n, p, q | \text{circuit})$$



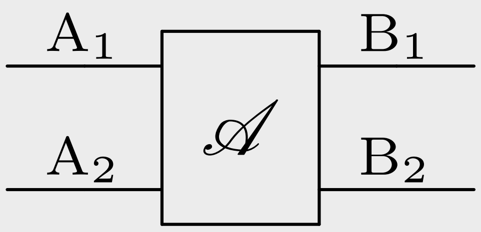
Principles for Quantum Theory

The *informational* framework

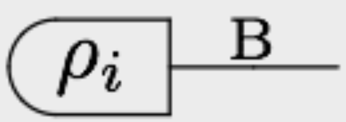
Logic ⊂ Probability ⊂ OPT

joint probabilities + connectivity

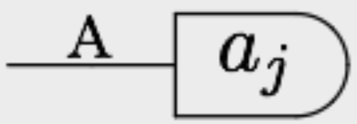
Probabilistic equivalence classes



transformation

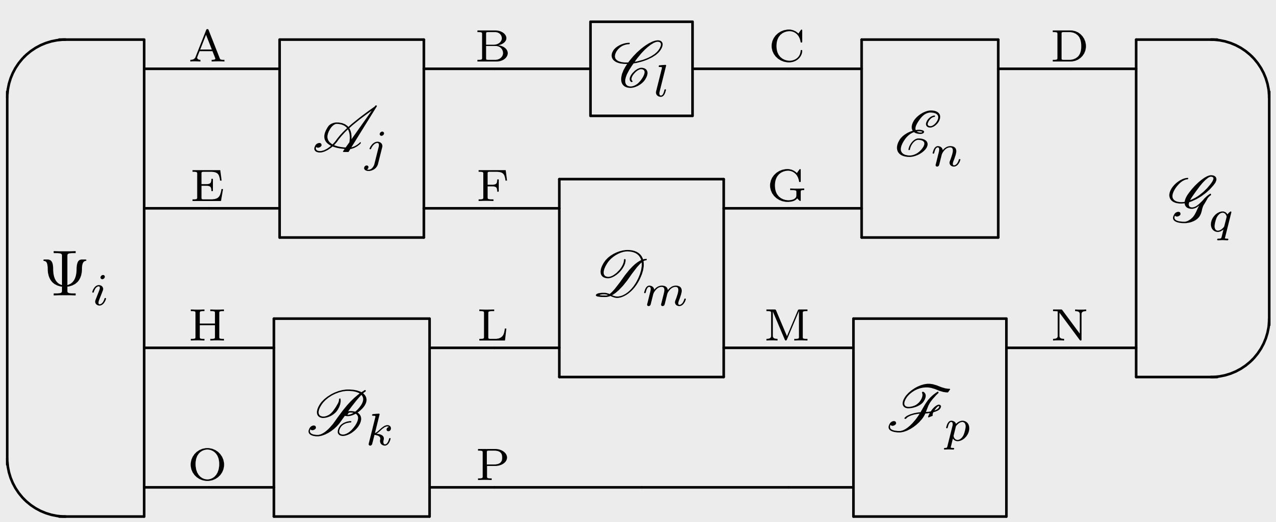


state



effect

$$p(i, j, k, l, m, n, p, q|\text{circuit})$$



Principles for Quantum Theory

P1. Causality → convexity

P2. Local discriminability

P3. Purification

P4. Atomicity of composition

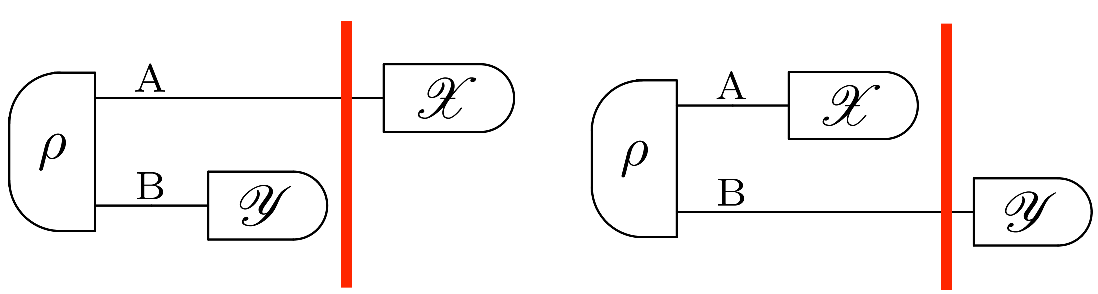
P5. Perfect distinguishability

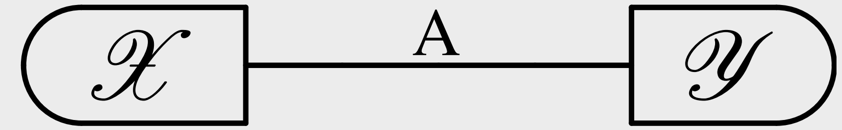
P6. Lossless Compressibility

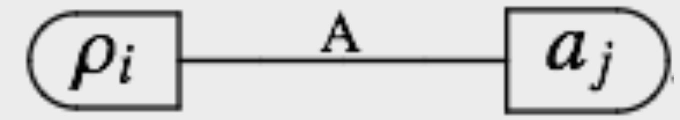
The probability of preparations is independent of the choice of observations

Control of experiment

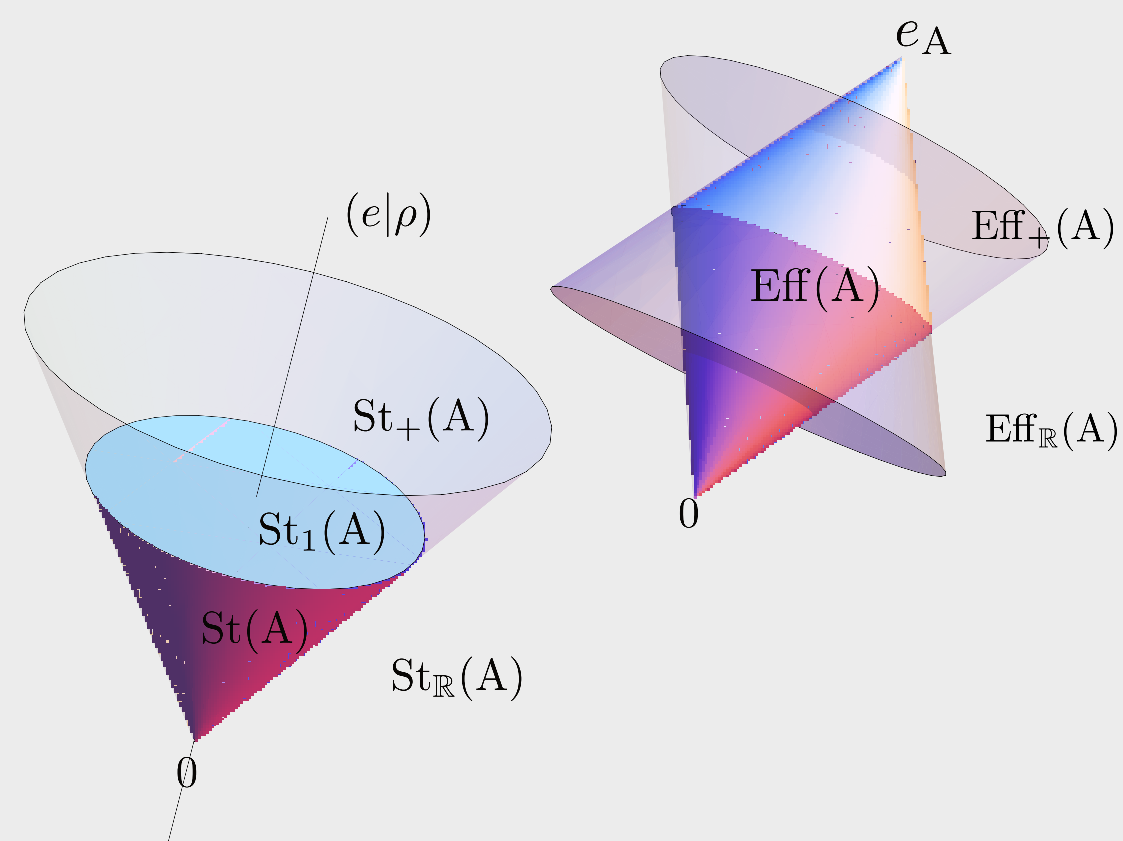
no signaling without interaction



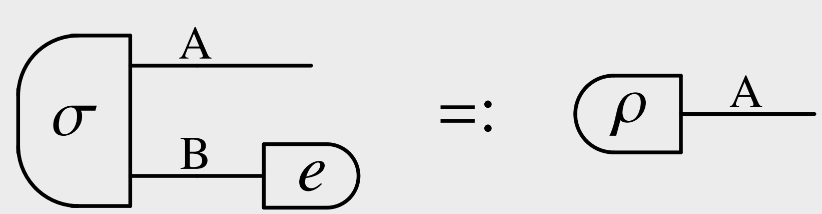


$p(i,j|\mathscr{X},\mathscr{Y}) :=$ 

$$p(i|\mathscr{X},\mathscr{Y}) = p(i|\mathscr{X},\mathscr{Y}') = p(i|\mathscr{X})$$



marginal state



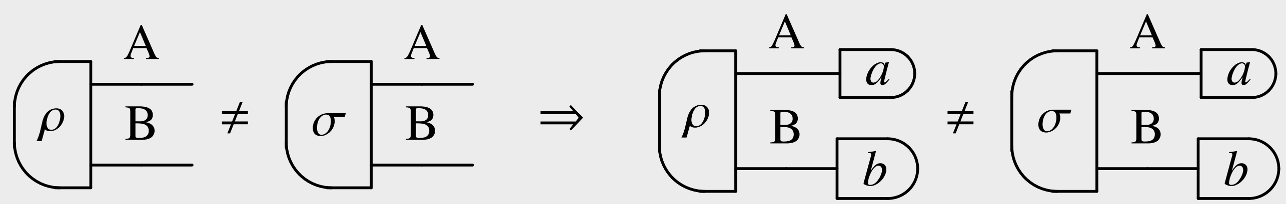
Principles for Quantum Theory

P1. Causality
P2. Local discriminability
P3. Purification
P4. Atomicity of composition
P5. Perfect distinguishability
P6. Lossless Compressibility

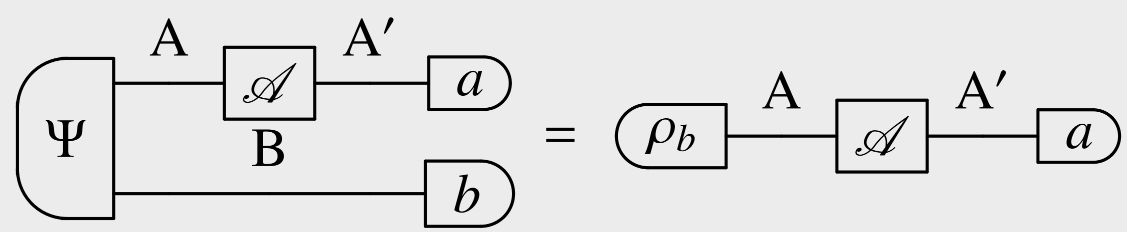
It is possible to discriminate any pair of states of composite systems using only local measurements.

Origin of the complex tensor product

Local testability of the physical law



Local characterization of transformations



Principles for Quantum Theory

P1. Causality

P2. Local discriminability

P3. Purification

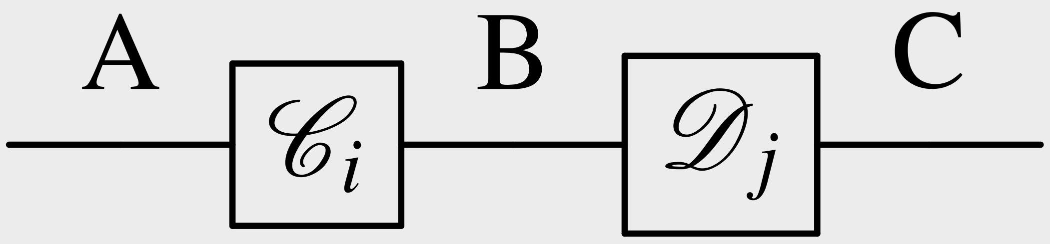
P4. Atomicity of composition

P5. Perfect distinguishability

P6. Lossless Compressibility

The composition of two atomic transformations is atomic

Complete information can be accessed on a step-by-step basis



Principles for Quantum Theory

P1. Causality
P2. Local discriminability
P3. Purification
P4. Atomicity of composition
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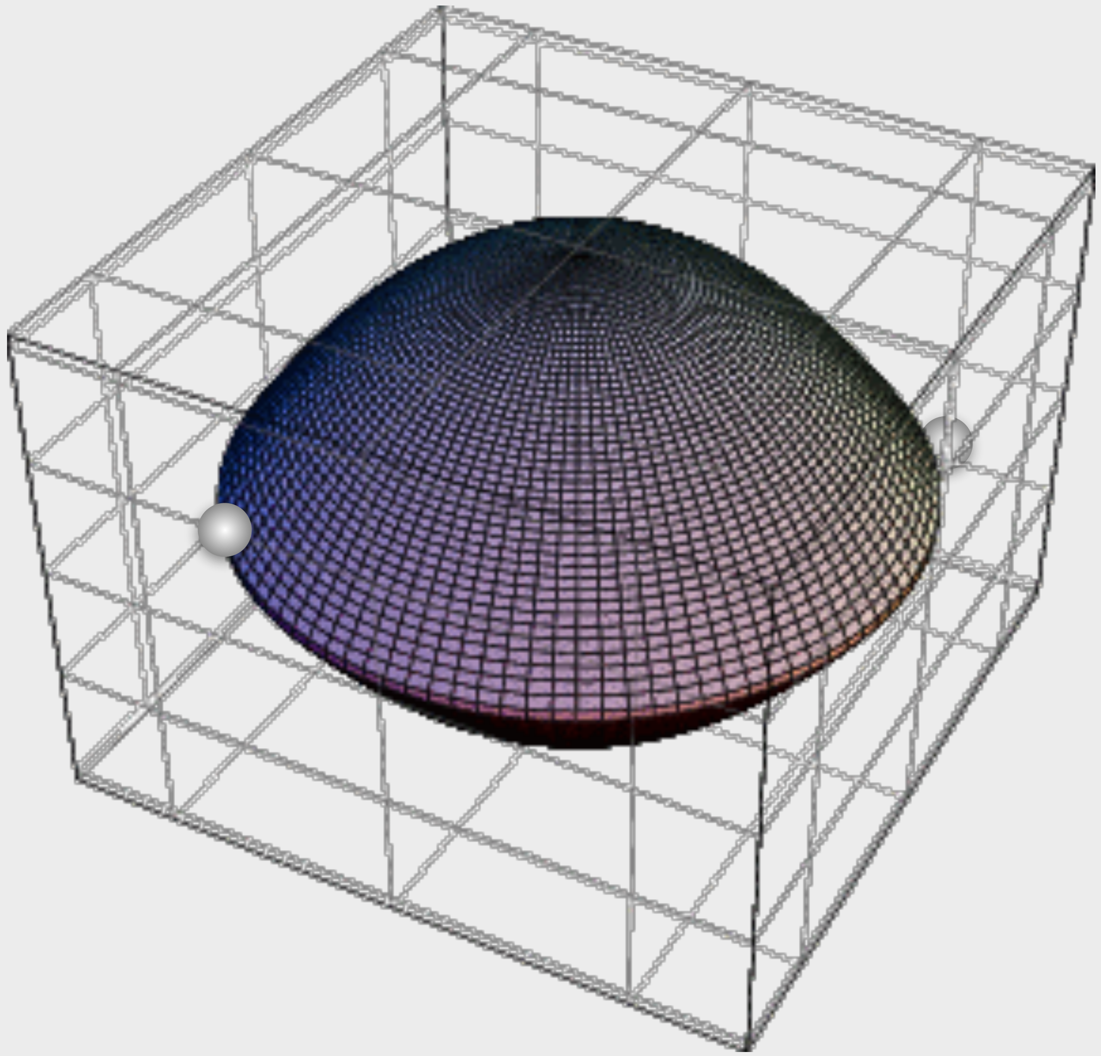
P6. Lossless Compressibility

Every state that is not completely mixed (i.e. on the boundary of the convex) can be perfectly distinguished from some other state.

Falsifiability of the theory



Principles for Quantum Theory

P1. Causality
P2. Local discriminability
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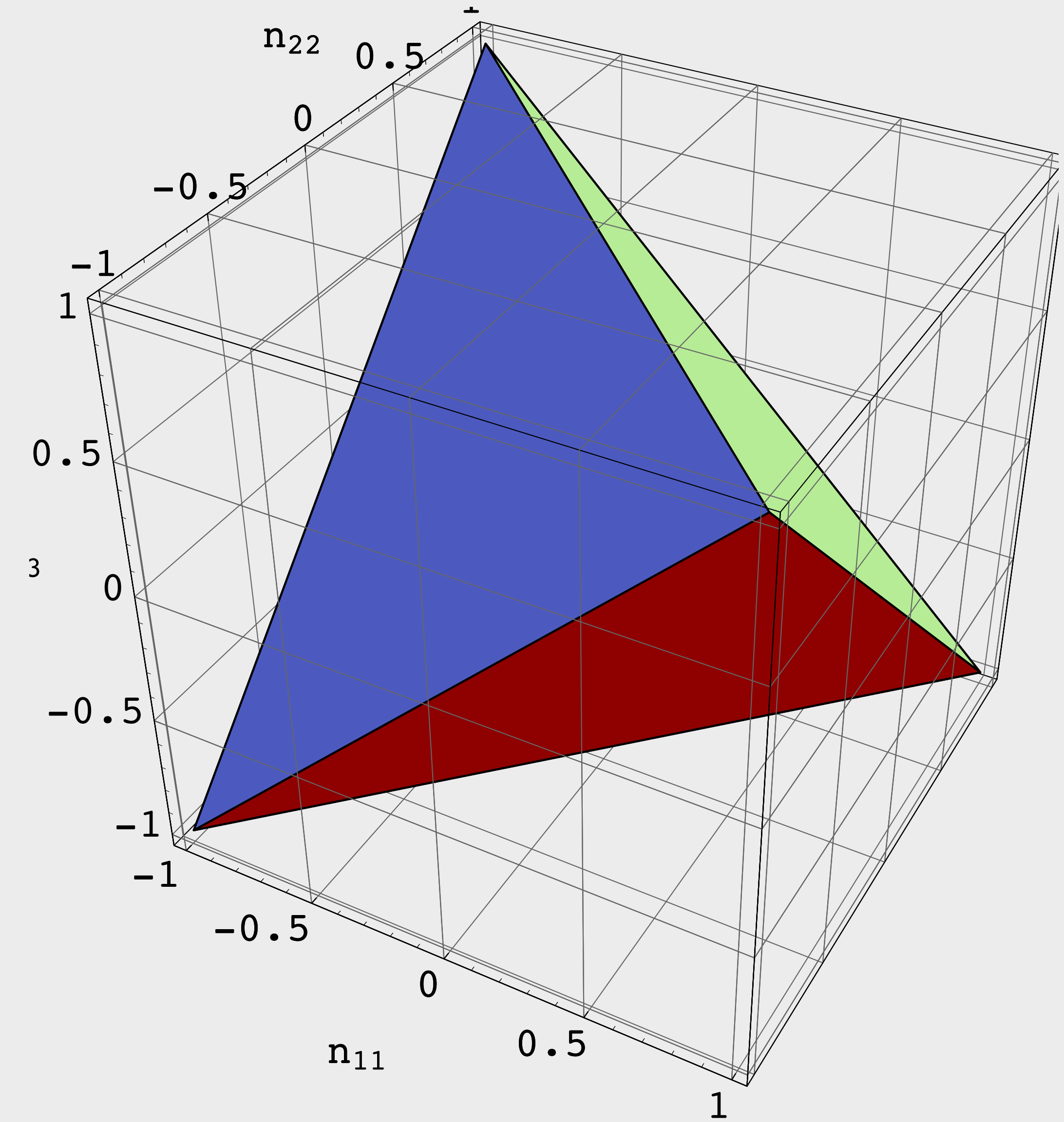
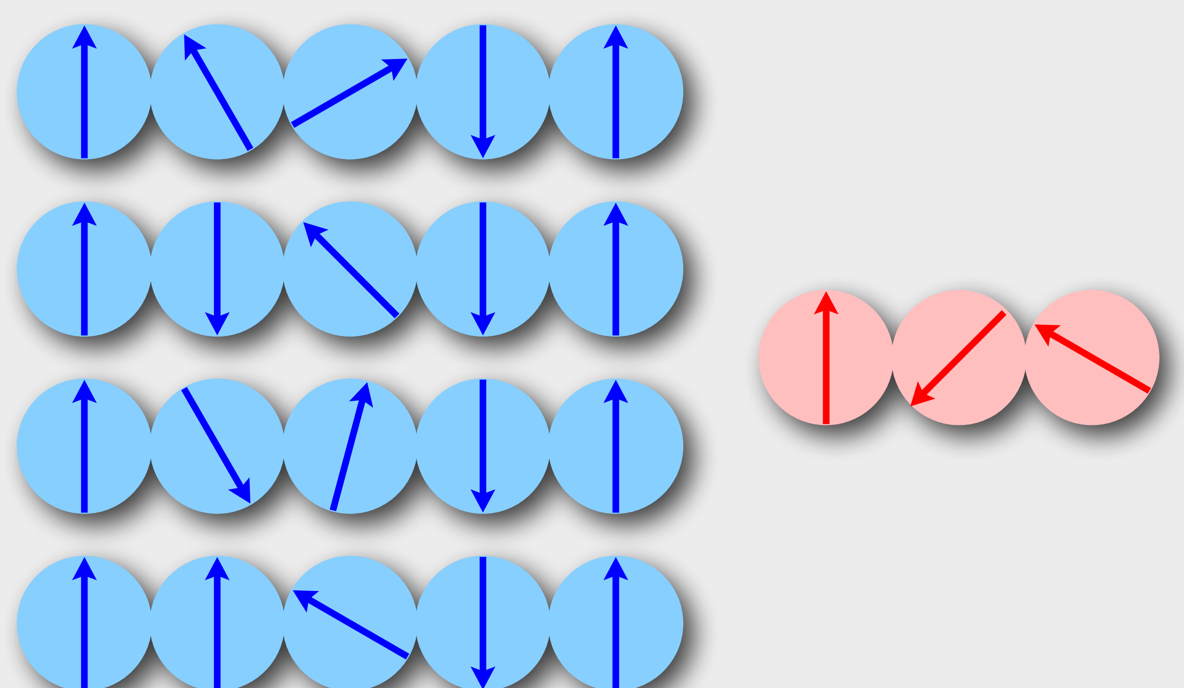
P6. Lossless Compressibility

For states that are not completely mixed there exists an ideal compression scheme

Any face of the convex set of states is the convex set of states of some other system

Encoding only unknown information





Principles for Quantum Theory

P1. Causality
P2. Local discriminability
P3. Purification
P4. Atomicity of composition
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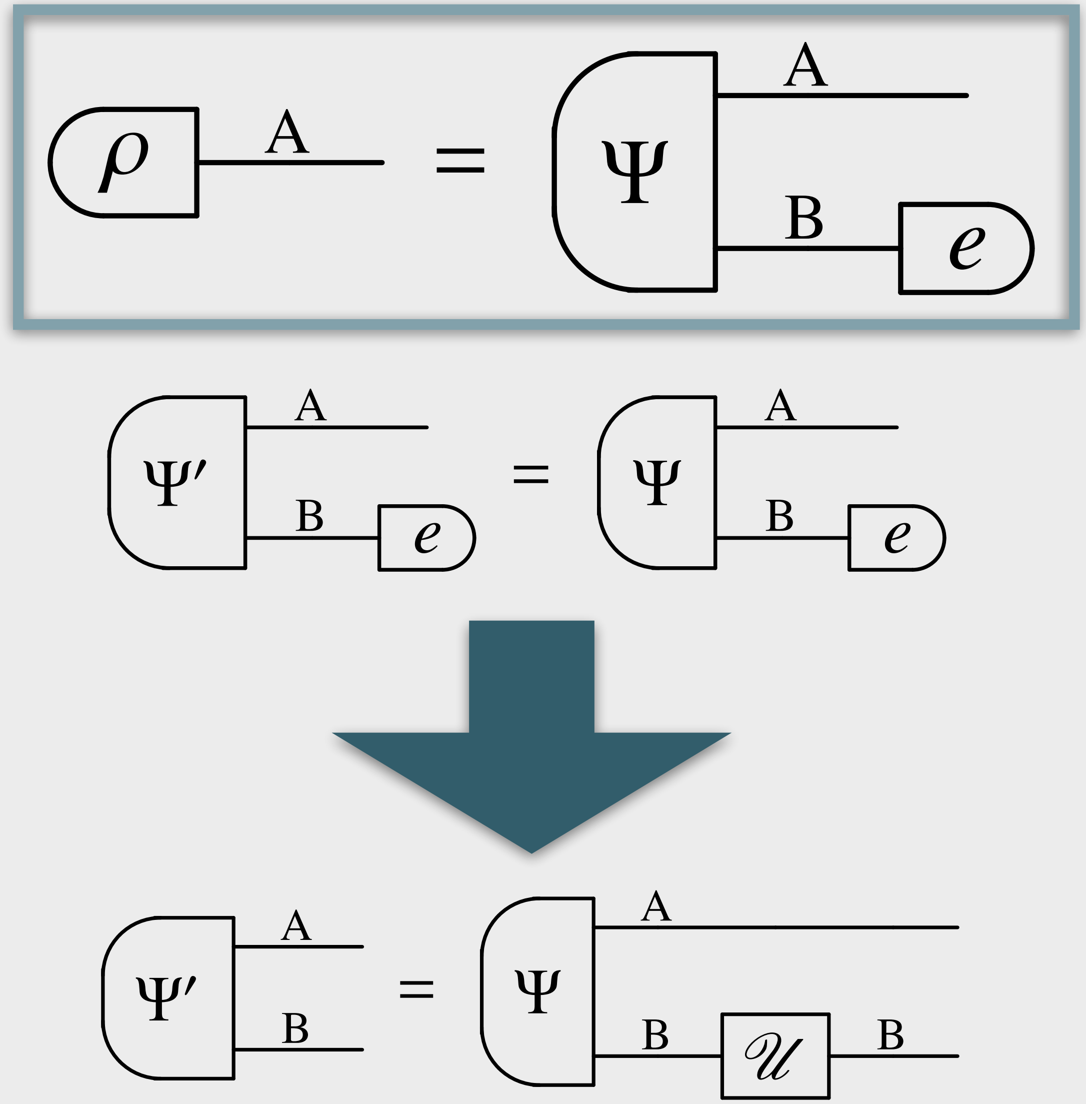
P6. Lossless Compressibility

Every state has a purification. For fixed purifying system, every two purifications of the same state are connected by a reversible transformation on the purifying system

Conservation of information. Reversibility.



Principles for Quantum Theory

P1. Causality
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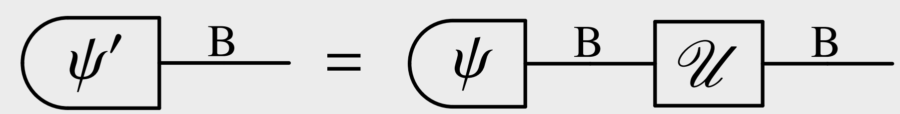
Every state has a purification. For fixed purifying system, every two purifications of the same state are connected by a reversible transformation on the purifying system

Conservation of information. Reversibility.

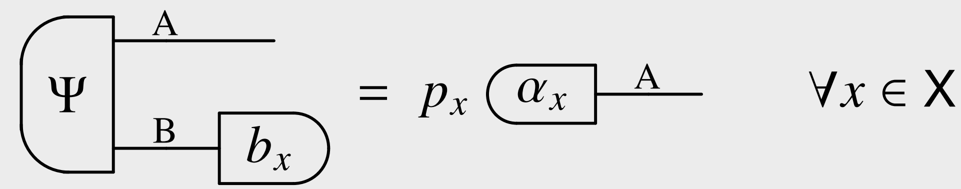
Consequences

1. ***Existence of entangled states:***
the purification of a mixed state is an entangled state;
the marginal of a pure entangled state is a mixed state;

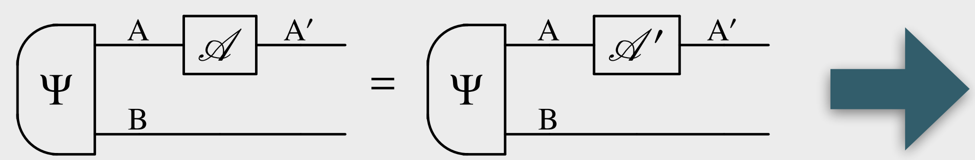
2. *Every two normalized pure states of the same system are connected by a reversible transformation*



3. ***Steering:*** Let $\boldsymbol{\Psi}$ purification of ρ. The for every ensemble decomposition $\rho=\sum_x p_x \alpha_x$ there exists a measurement $\{b_x\}$, such that



4. ***Process tomography (faithful state):***



$$\mathscr{A}\rho = \mathscr{A}'\rho \quad \forall\rho$$

5. ***No information without disturbance***

Principles for Quantum Theory

P1. Causality
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P3. Purification
P4. Atomicity of composition
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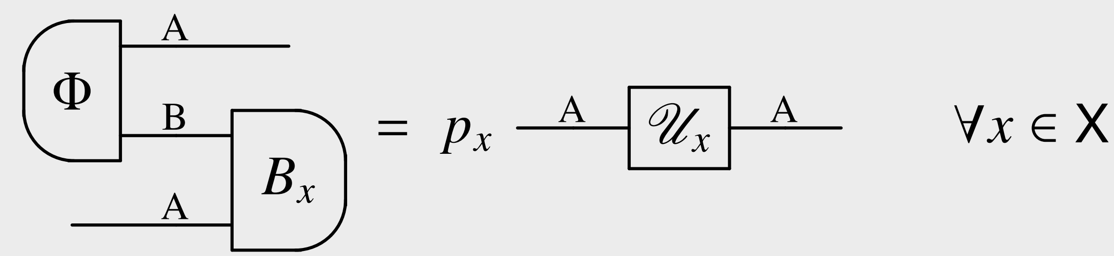
P6. Lossless Compressibility

Every state has a purification. For fixed purifying system, every two purifications of the same state are connected by a reversible transformation on the purifying system

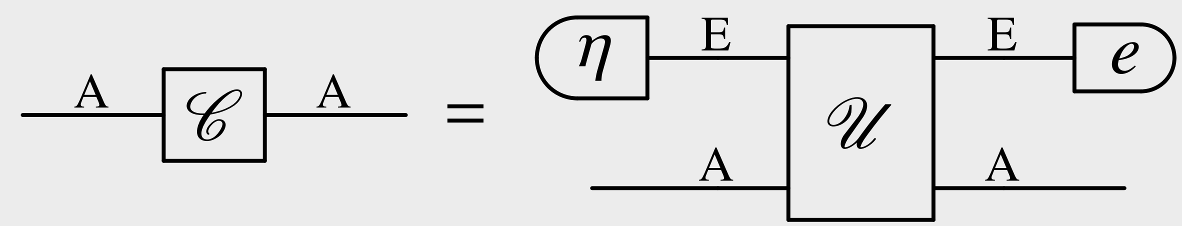
Conservation of information. Reversibility.

Consequences

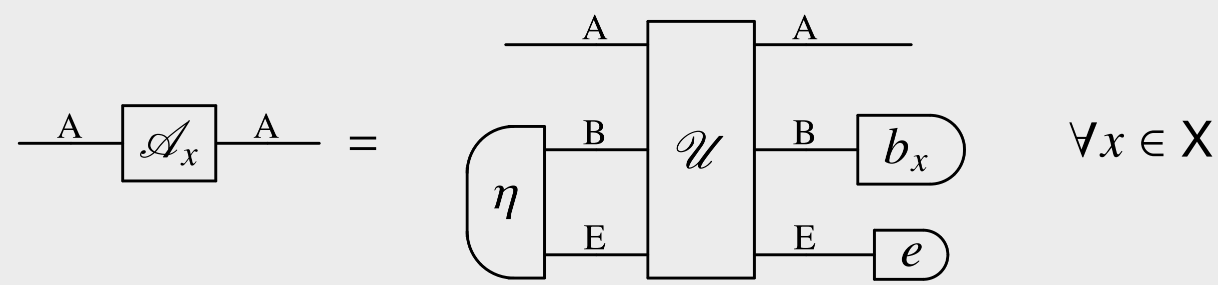
6. ***Teleportation***



7. ***Reversible dilation of "channels"***



8. ***Reversible dilation of "instruments"***



9. ***State-transformation cone isomorphism***

10. ***Rev. transform. for a system make a Lie group***

Moving to the *Mechanics*

- The Weyl, Dirac, and Maxwell equations are derived from information-theoretic principles only, <u>without assuming SR</u>
- Only denumerable quantum systems in interaction
- QCA to be regarded as a theory unifying scales from Planck to Fermi (no continuum limit!)

- QFT is recovered in the *relativistic limit* ($k \ll 1$)
- In the *ultra-relativistic limit* (Planck scale) Lorentz covariance is an approximate symmetry, and one has the *Doubly Special Relativity* of Amelino-Camelia/Smolin/Magueijo

Additional principles

Min algorithmic complexity of the processing

- linearity
- unitarity
- locality
- homogeneity
- transitivity
- isotropy
- minimal-dimension

→ Quantum Cellular Automaton (QCA)

GOOD FEATURES

1. **no SR assumed**: emergence of relativistic quantum field and space-time
2. **quantum *ab-initio***
3. no divergencies and all the problems from the continuum
4. no “violations” of causality
5. computable
6. dynamics stable (dispersive Schrödinger equation for narrow-band states valid at all scales)
7. solves the problem of localization in QFT
8. natural scenario for the *holographic principle*

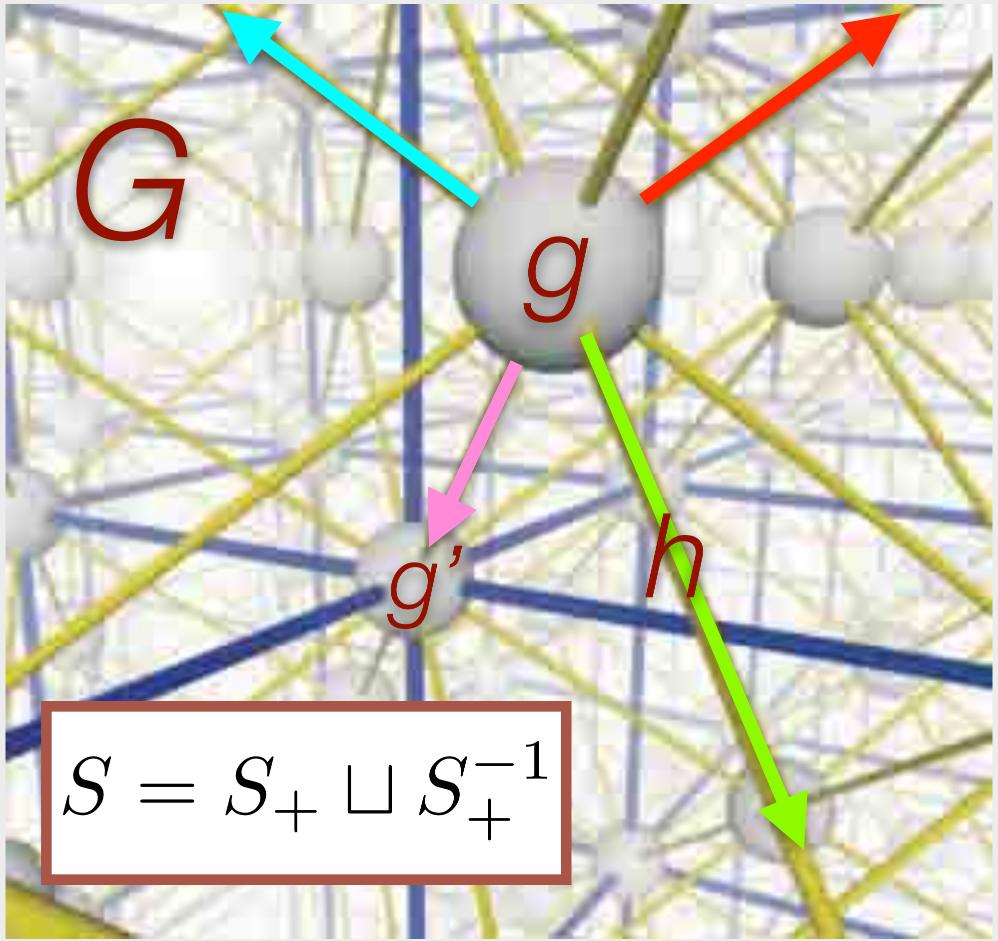
QFT from info-principles

the theoretical minimum

- System $\psi(g)$, ψ s-dimensional *field operator,* labeled by $g\in G$, $|G|\leq \aleph$

- minimal-dimension
  - $s>1$ ($s=1$ trivial evolution)
- linearity
  - Interactions described by transition matrices $A_{gg'}\in M_s(\mathbb{C})$ between systems $g\in G$: single evolution step $\psi(g) \to \psi(g) = \sum_{g'\in S_g} A_{gg'}\psi(g')$ $S_g\subseteq G$ set of systems interacting with g
- homogeneity
  - $\{A_{gg'}\}_{g'\in S_g}$ independent of g, Cayley graph $K(G,S_+)$
- transitivity
  - G group, $G=\langle h_1,h_2,\ldots,h_N, | r_1,r_2,\ldots,r_M\rangle$
  - $S_g = Sg$, $S:=\{h_1,h_2,\ldots,h_N\}$, $S=S_+ \cup S_-$, $S_-=S_+^{-1}$

Cayley graph $K(G,S_+)$



QFT from info-principles

the theoretical minimum

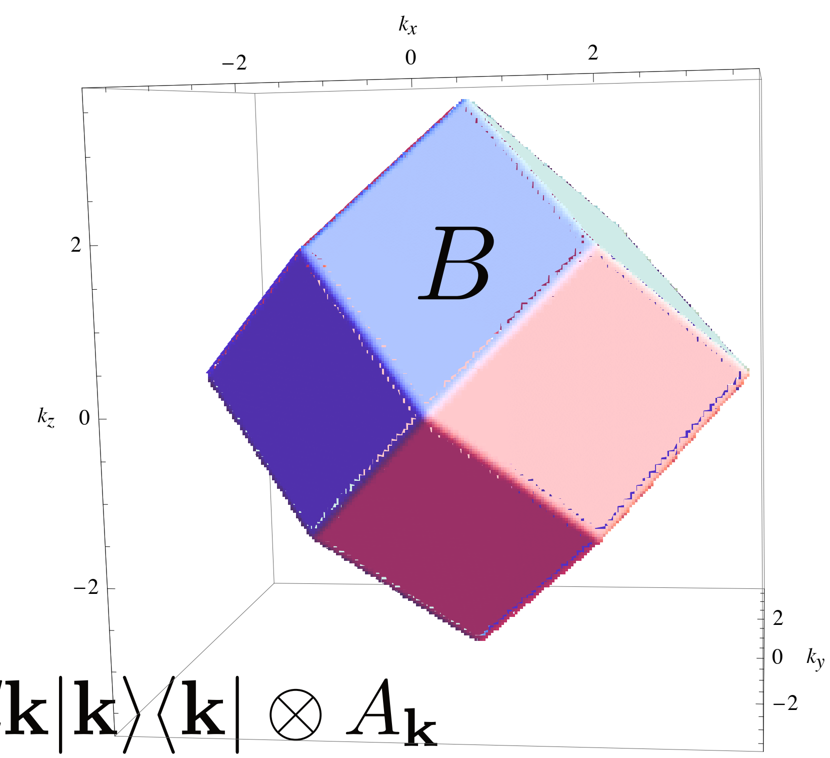
- System $\psi(g)$, ψ s-dimensional *field operator,* labeled by $g \in G$, $|G| \leq \aleph$

• minimal-dimension	• $s>1$ ($s=1$ trivial evolution)
• linearity	• Interactions described by transition matrices $A_{gg'} \in M_s(C)$ between systems $g \in G$: single evolution step $\psi(g) \to \psi(g) = \sum_{g' \in S_g} A_{gg'} \psi(g')$ $S_g \subseteq G$ set of systems interacting with g
• homogeneity • transitivity	• $\{A_{gg'}\}_{g' \in S_g}$ independent of g, Cayley graph $K(G,S_+)$ • G group, $G=\langle h_1,h_2,\ldots,h_N, \| r_1,r_2,\ldots,r_M\rangle$ • $S_g = Sg$, $S:=\{h_1,h_2,\ldots,h_N\}$, $S=S_+ \cup S_-$, $S_-=S_+^{-1}$
• locality	• $\|S\|<\infty \Leftrightarrow G$ finitely generated
• unitarity Problem: find $\{A_h\} \in M_s(C)$ such that $A^\dagger A=I$	$$A = \sum_{h \in S_+ \sqcup S_+^{-1}} T_h \otimes A_h$$ T_h unitary repr. of G on $l^2(G)$
• isotropy G finitely-generated group, $K(G,S_+)$ qi-isometrically embeds in R^3, G contains a free Abelian subgroup A of finite index, with $rank(A) \leq 3$ *(Misha Kapovich, priv. comm.)* unitary s-dimensional (projective) $\{L_l\}$ of L determines the statistics of ψ, if Fermion, Boson, Anyon	• $A_h \neq 0 \Leftrightarrow A_h^{-1} \neq 0$ • There exists a group L of permutations of S_+, <u>transitive</u> over S_+ that leaves $K(G,S_+)$ invariant • a nontrivial unitary s-dimensional (projective) representation $\{L_l\}$ of L such that: $$A = \sum_{h \in S} T_h \otimes A_h = \sum_{h \in S} T_{lh} \otimes L_l A_h L_l^\dagger$$

The Weyl QCA

☞ Minimal dimension for nontrivial unitary *A:* s=2

- Unitarity ⇒ the only possible *G* is the BCC!!
- ⇒ A_h are proportional to rank-one projectors
- Isotropy ⇒ Fermionic ψ (d=3)



$$A = \int_B d\mathbf{k} |\mathbf{k}\rangle\langle\mathbf{k}| \otimes A_{\mathbf{k}}$$

Two QCAs connected by CPT

$$\begin{aligned} A_{\mathbf{k}}^{\pm} = & -i\sigma_x(s_x c_y c_z \pm c_x s_y s_z) \\ & -i(\pm\sigma_y)(c_x s_y c_z \mp s_x c_y s_z) \\ & -i\sigma_z(c_x c_y s_z \pm s_x s_y c_z) \\ & +I(c_x c_y c_z \mp s_x s_y s_z) \end{aligned}$$

$$s_\alpha = \sin \frac{k_\alpha}{\sqrt{3}}$$

$$c_\alpha = \cos \frac{k_\alpha}{\sqrt{3}}$$

Dirac QCA $\oplus$

Local coupling: $A_{\mathbf{k}}$ coupled with inverse with off-diagonal identity block matrix

$$E^{\pm}_{\mathbf{k}} = \begin{pmatrix} nA^{\pm}_{\mathbf{k}} & imI \\ imI & nA^{\pm\dagger}_{\mathbf{k}} \end{pmatrix}$$

$$n^2 + m^2 = 1$$

$E^{\pm}_{\mathbf{k}}$ CPT-connected!

$$\omega^{E}_{\pm}(\mathbf{k}) = \cos^{-1}[n(c_x c_y c_z \pm s_x s_y s_z)]$$

Dirac in relativistic limit $k \ll 1$

m≤1: mass

n^{-1}: refraction index

Maxwell QCA $\otimes$

$$M_{\mathbf{k}} = A^{\dagger}_{\mathbf{k}} \otimes A_{\mathbf{k}}$$

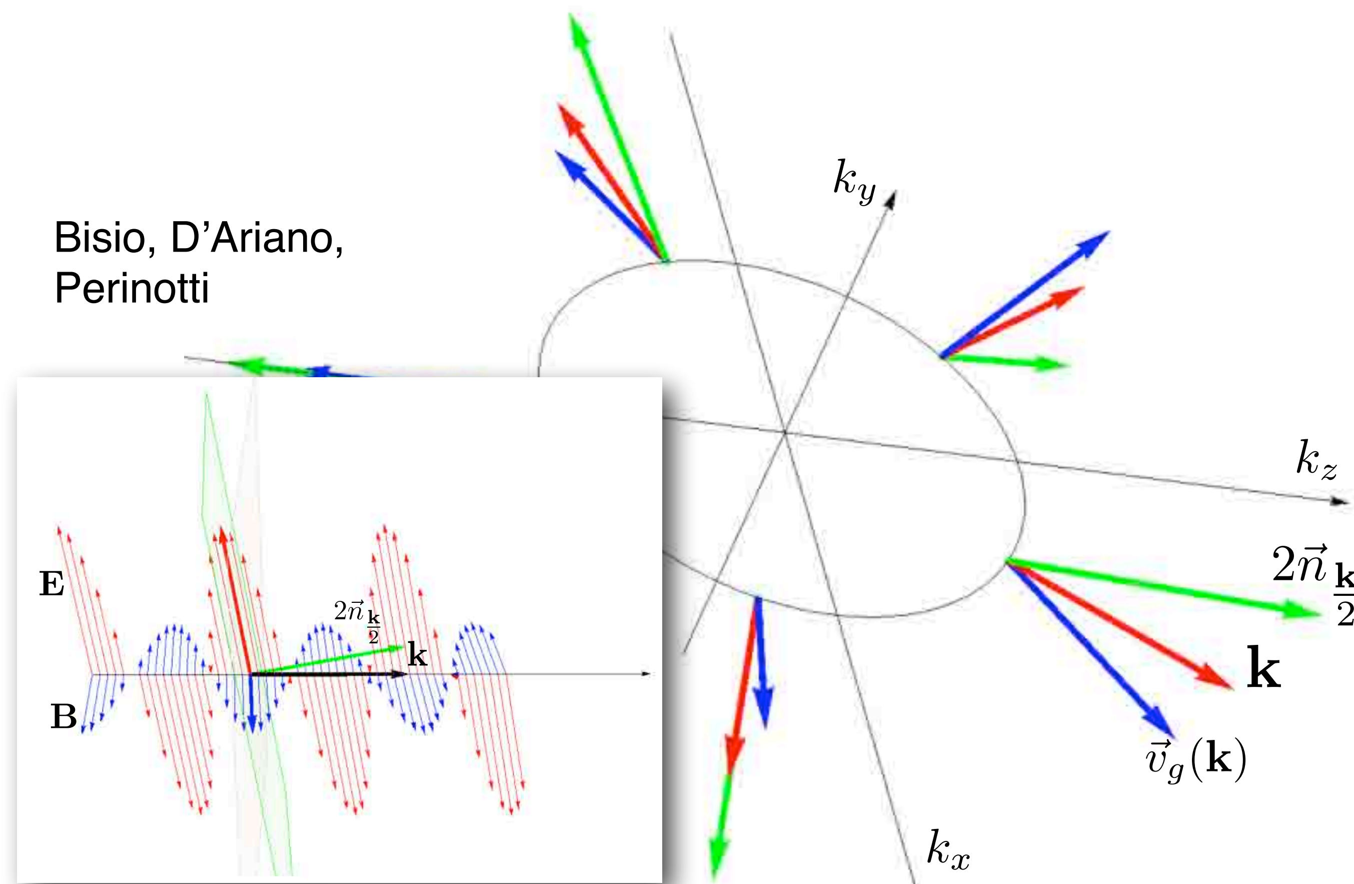
$$F^{\mu}(\mathbf{k}) = \int \frac{\mathrm{d}\,\mathbf{q}}{2\pi} f(\mathbf{q}) \tilde{\psi}(\tfrac{\mathbf{k}}{2} - \mathbf{q}) \sigma^{\mu} \varphi(\tfrac{\mathbf{k}}{2} + \mathbf{q})$$

Maxwell in relativistic limit $k \ll 1$

Boson: emergent from convolution of fermions

(De Broglie neutrino-theory of photon)

Bisio, D'Ariano, Perinotti



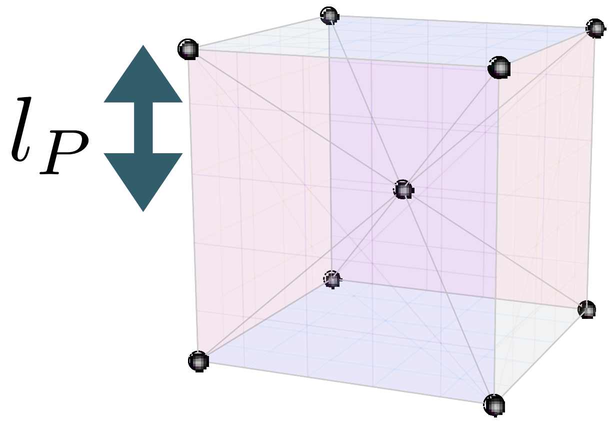
Universal constants of QCA theory

Conversion to dimensional units

l_P	t_P	m_P	*fundamental system* (Wilczek)
[L]	[T]	[M]	

t_P: automaton time-step

m_P: bound for particle mass



$$m_g = m\, m_P \qquad p = \frac{\hbar k}{\sqrt{3} l_P}$$

$$c := \frac{l_P}{t_P} \qquad \hbar = m_P l_P c \qquad G = \frac{l_P c^2}{m_P}$$

D'Ariano, Perinotti, arXiv:1306.1934

Dirac emerging from the QCA

fidelity with Dirac evolution for a narrowband packet in the relativistic limit $k \simeq m \ll 1$

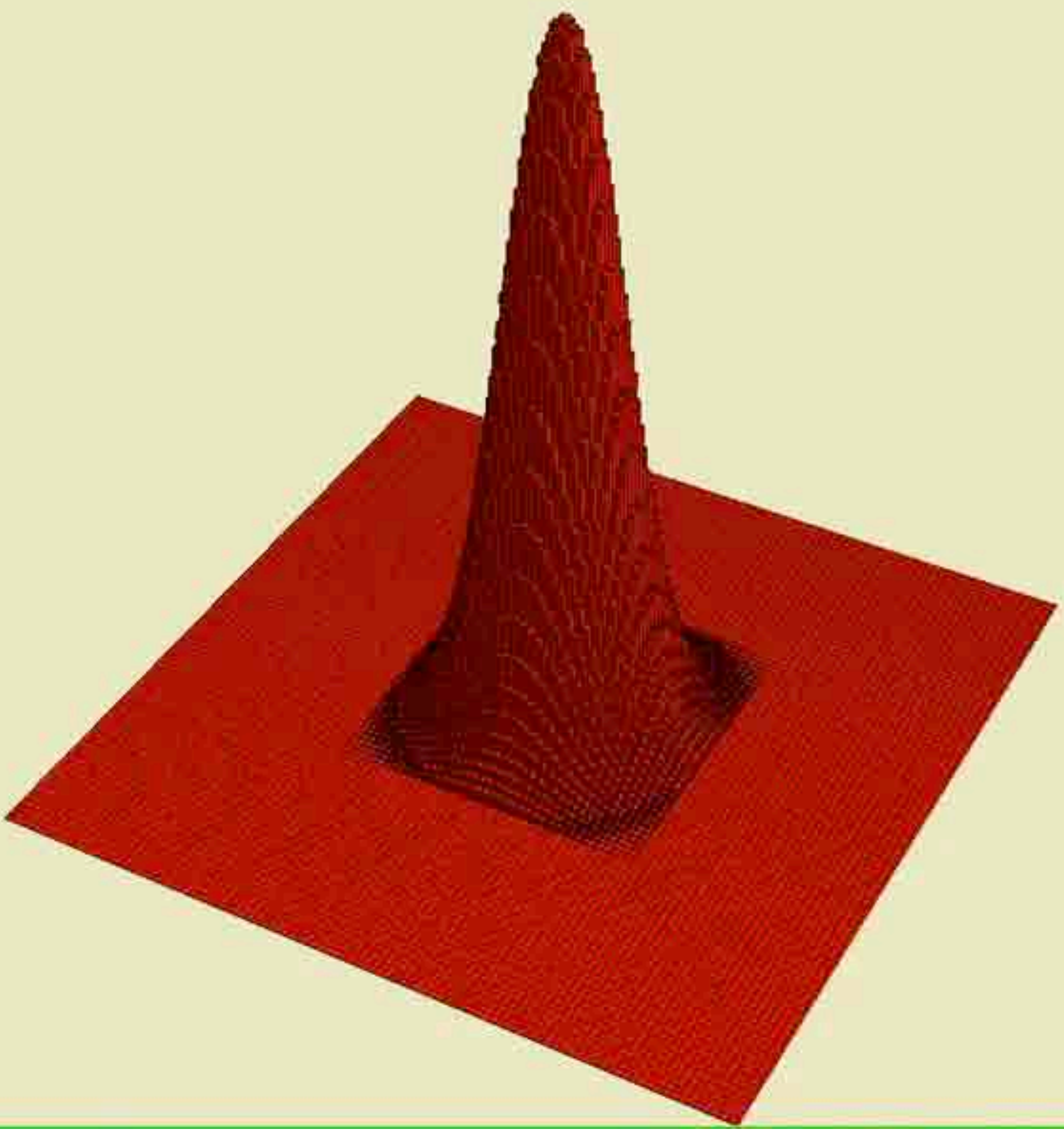
$$F = |\langle \exp\left[-iN\Delta(\mathbf{k})\right]\rangle| \qquad \omega^E(\mathbf{k}) = \sqrt{\mathbf{k}^2 + m^2}$$

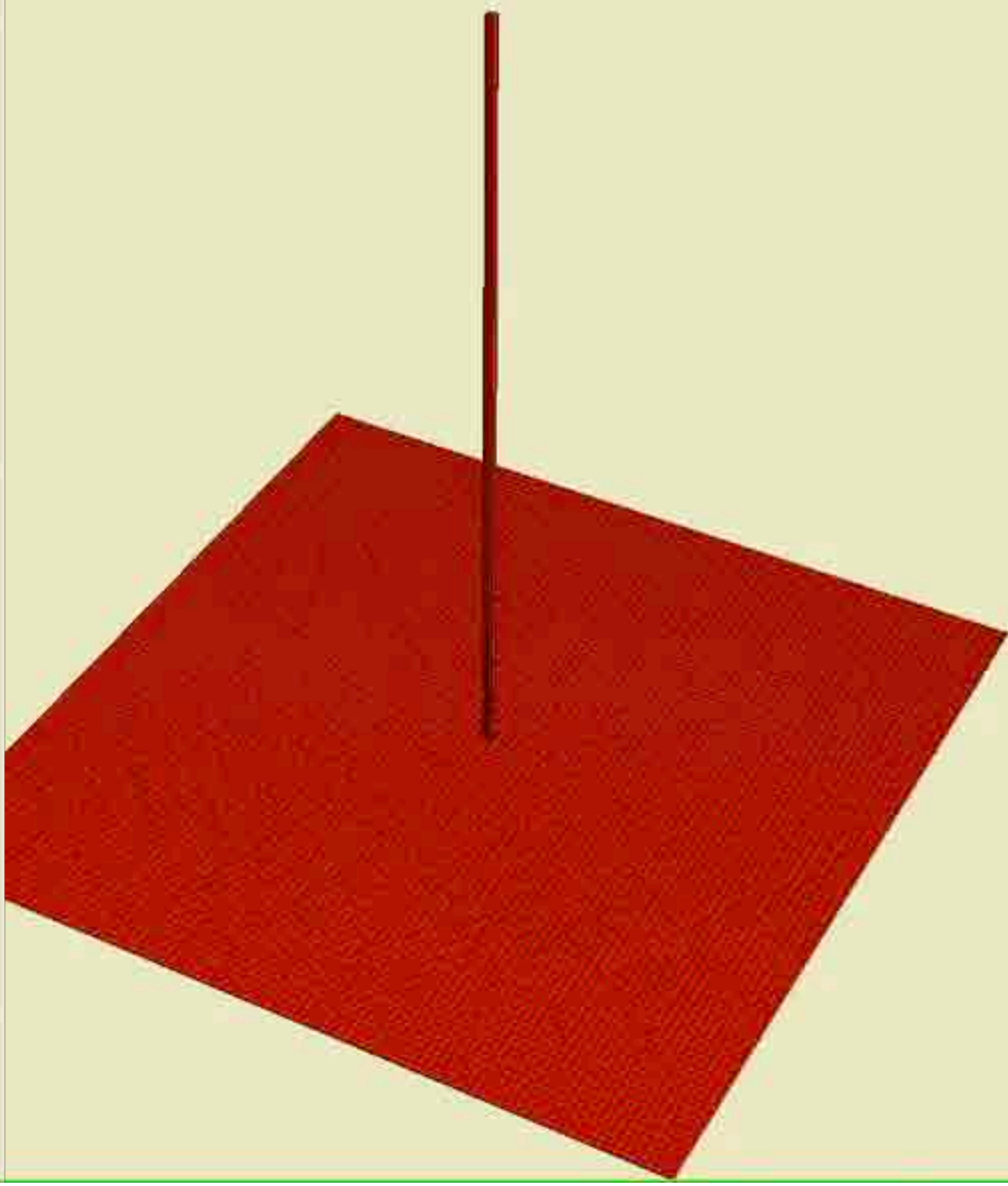
$$\Delta(\mathbf{k}) := (m^2 + \tfrac{k^2}{3})^{\frac{1}{2}} - \omega^E(\mathbf{k})$$

$$= \frac{\sqrt{3}k_x k_y k_z}{(m^2 + \frac{k^2}{3})^{\frac{1}{2}}} - \frac{3(k_x k_y k_z)^2}{(m^2 + \frac{k^2}{3})^{\frac{3}{2}}} + \tfrac{1}{24}(m^2 + \tfrac{k^2}{3})^{\frac{3}{2}} + \mathcal{O}(k^4 + N^{-1}k^2)$$

relativistic proton: $N \simeq m^{-3} = 2.2 * 10^{57} \Rightarrow t = 1.2 * 10^{14}\mathrm{s} = 3.7 * 10^6$ y

UHECRs: $k = 10^{-8} \gg m \Rightarrow N \simeq k^{-2} = 10^{16} \Rightarrow 5 * 10^{-28}$ s

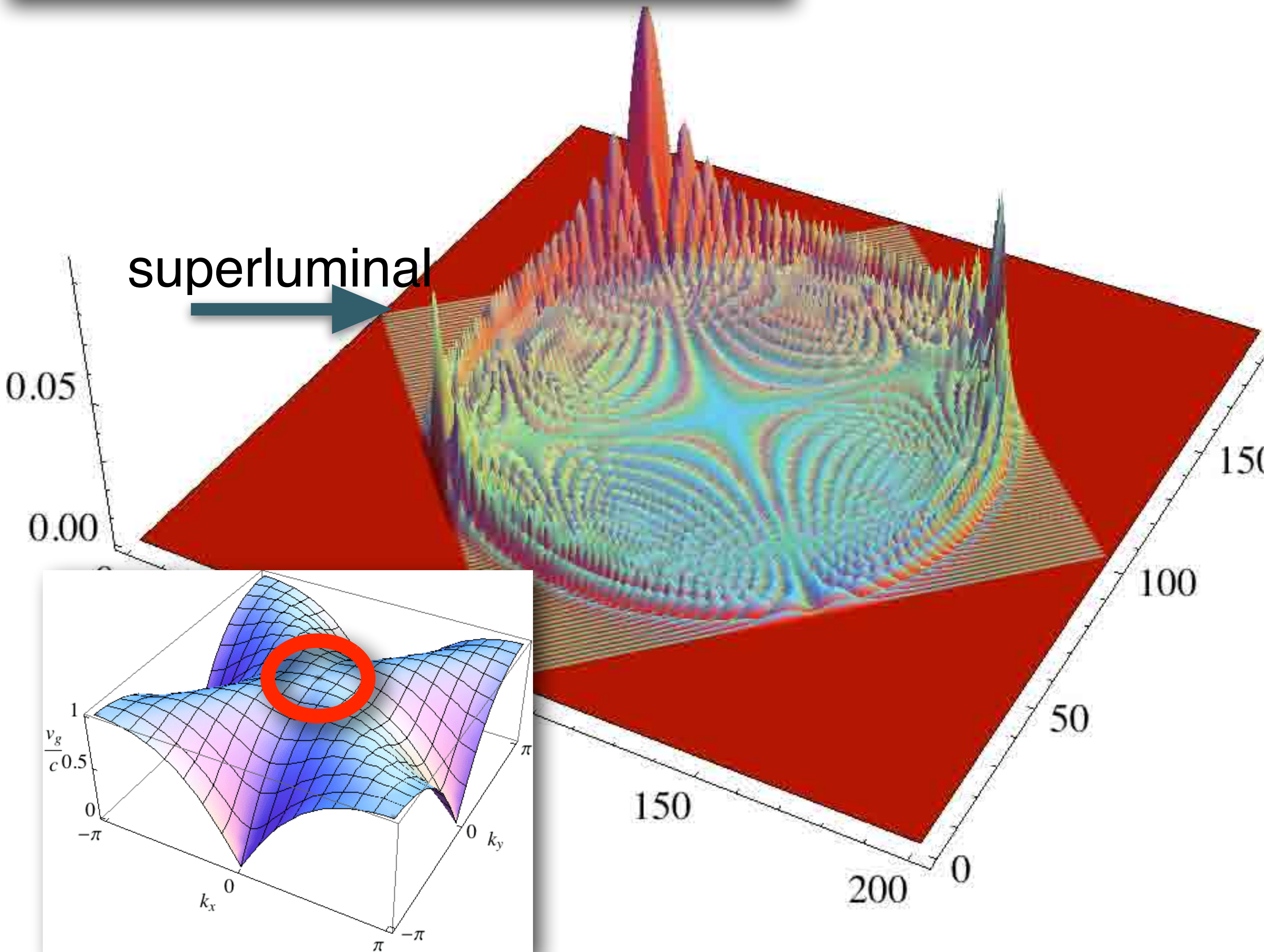




2d automaton

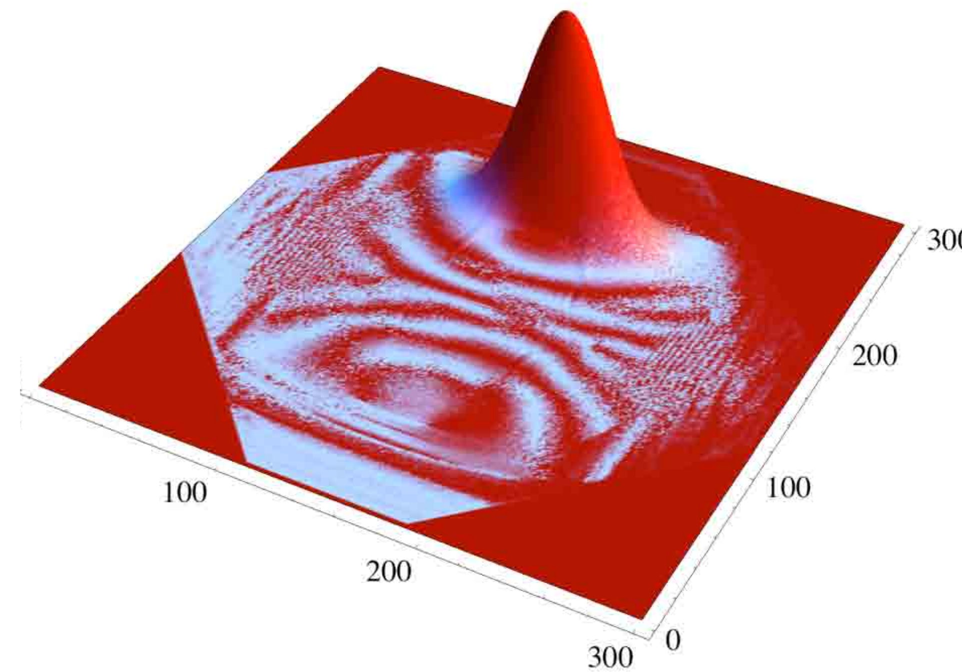
- Evolution of a *narrow-band particle-state*
- Evolution of a *localized state*

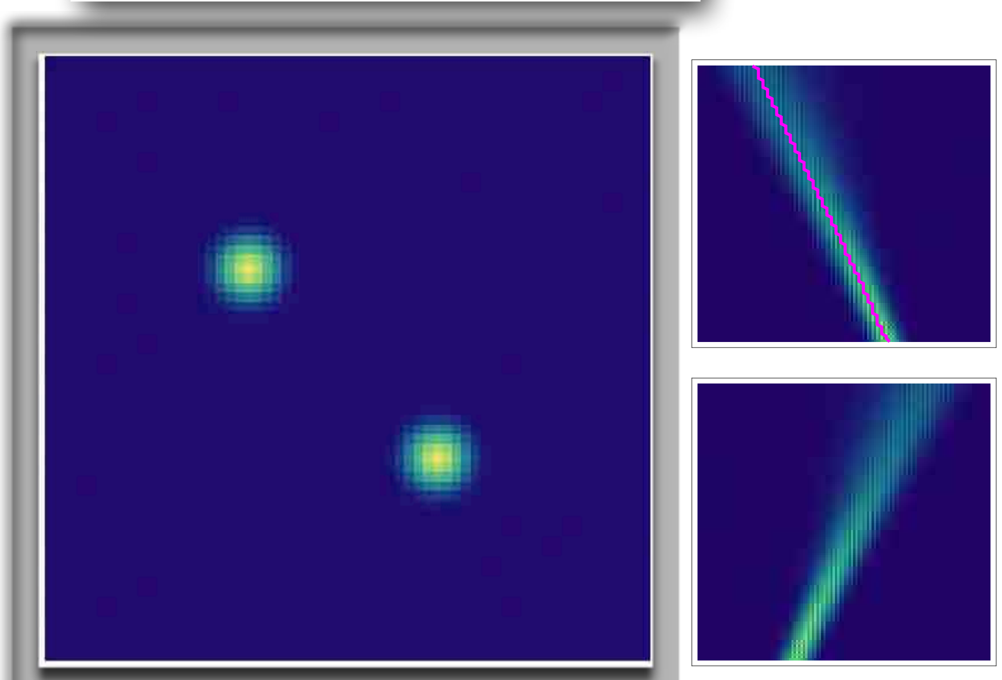
Dirac QCA

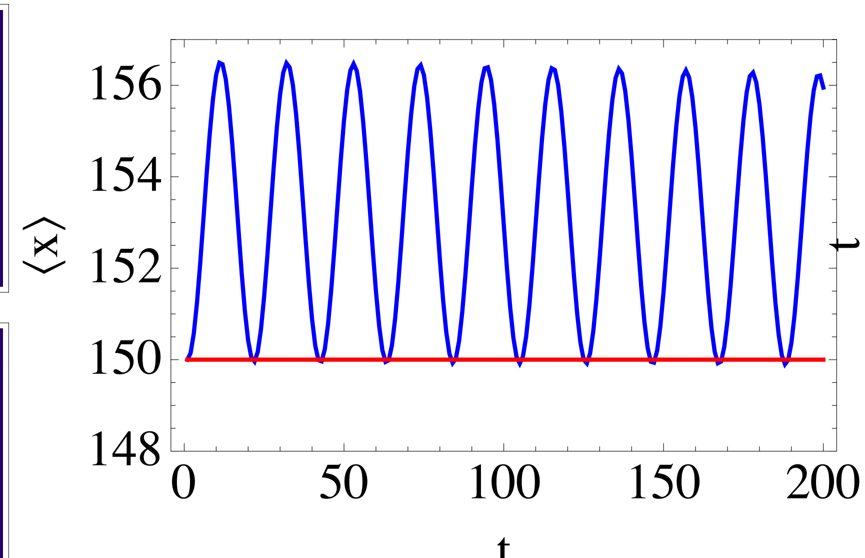


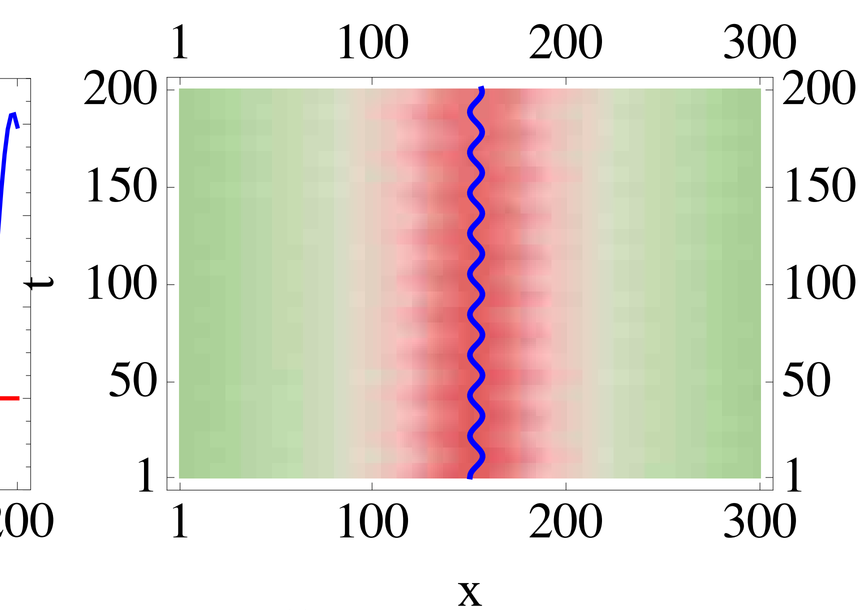
$\mathbf{k} = (\frac{\pi}{10}, 0)$, $m = .1$, $N = 120$

$\Delta_x^2 = 10^2$, $\Delta_y^2 = \frac{1}{2}\Delta_x^2$









Particle state: $k_0 = 0$, $m = 0.15$, $\sigma = 40$. Oscillation frequency $\nu = 0.048$

D'Ariano, Perinotti,
arXiv:1306.1934

The general dispersive Schrödinger equation

$$i\partial_t e^{-i\mathbf{k}_0\cdot\mathbf{x}+i\omega_0 t}\psi(\mathbf{k},t) = s[\omega(\mathbf{k})-\omega_0]e^{-i\mathbf{k}_0\cdot\mathbf{x}+i\omega_0 t}\psi(\mathbf{k},t)$$

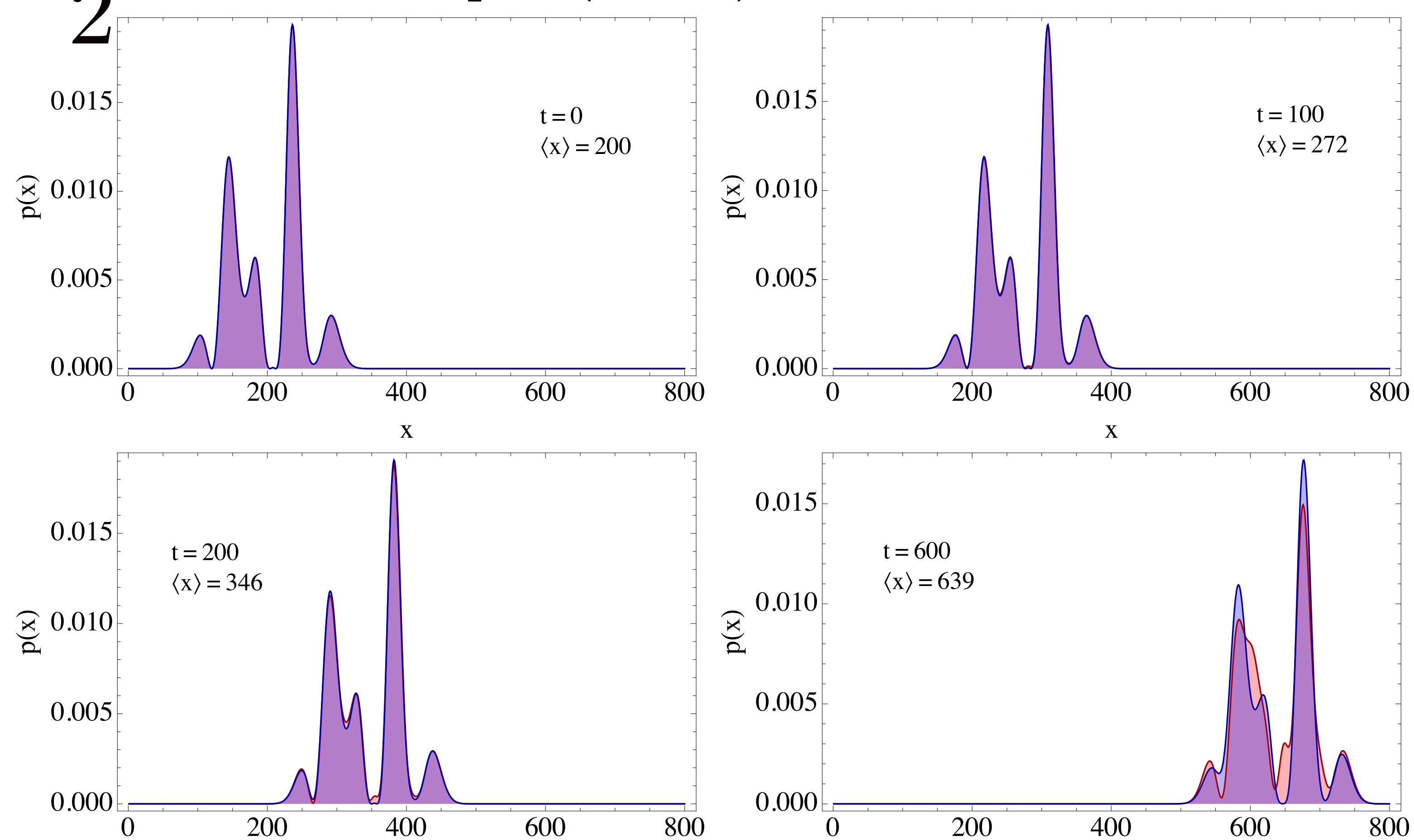
$$i\partial_t\tilde{\psi}(\mathbf{k},t) = s[\omega(\mathbf{k})-\omega_0]\tilde{\psi}(\mathbf{k},t)$$

$$i\partial_t\tilde{\psi}(\mathbf{x},t) = s[\mathbf{v}\cdot\boldsymbol{\nabla}+\frac{1}{2}\mathbf{D}\cdot\boldsymbol{\nabla}\boldsymbol{\nabla}]\tilde{\psi}(\mathbf{x},t)$$

$$\mathbf{v} = \left(\boldsymbol{\nabla}_{\mathbf{k}}\omega\right)(\mathbf{k}_0)$$

$$\mathbf{D} = \left(\boldsymbol{\nabla}_{\mathbf{k}}\boldsymbol{\nabla}_{\mathbf{k}}\omega\right)(\mathbf{k}_0)$$

$k_0=3\pi/10$. m=.6

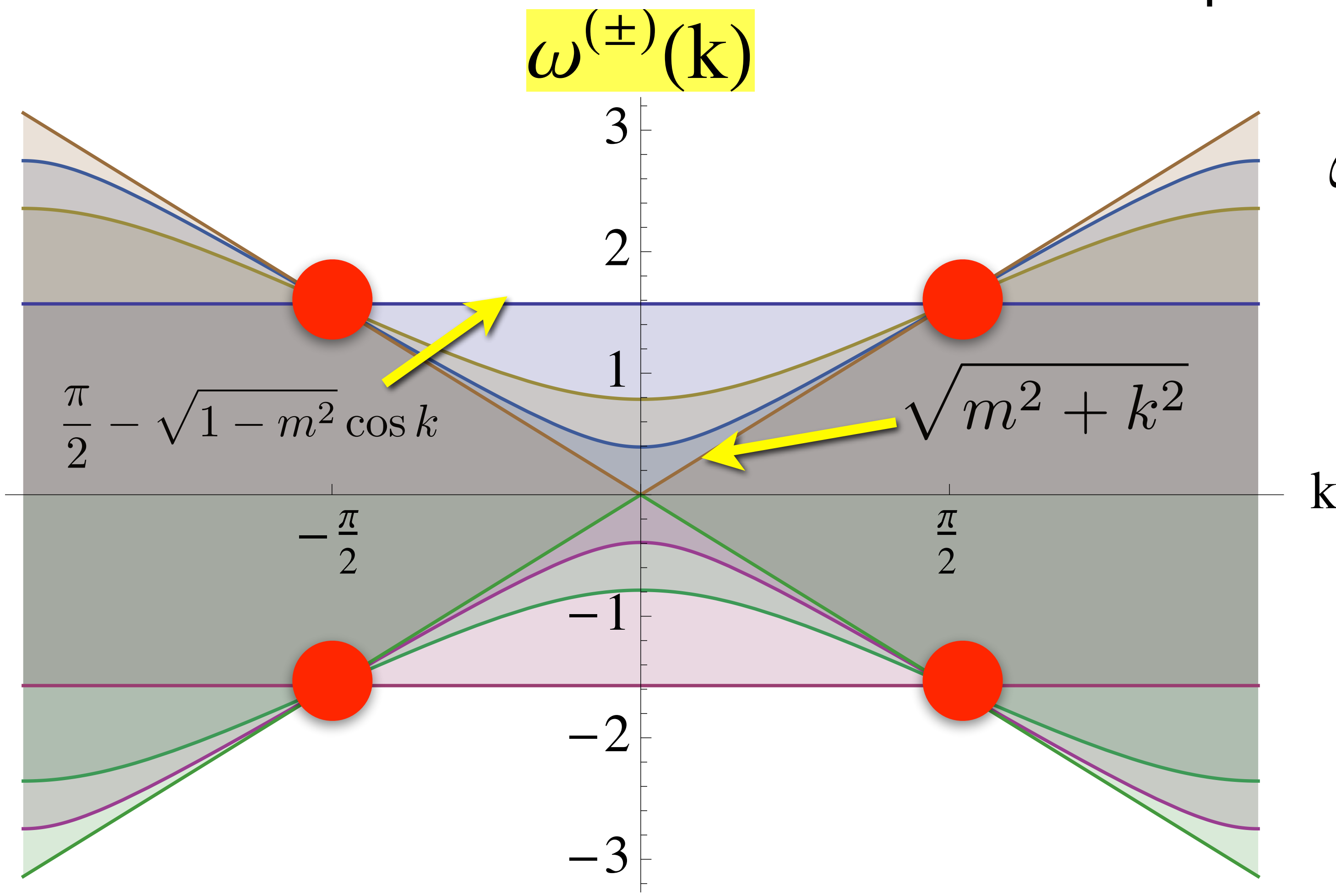


Bisio, D'Ariano, Tosini,
arXiv:1212.2839

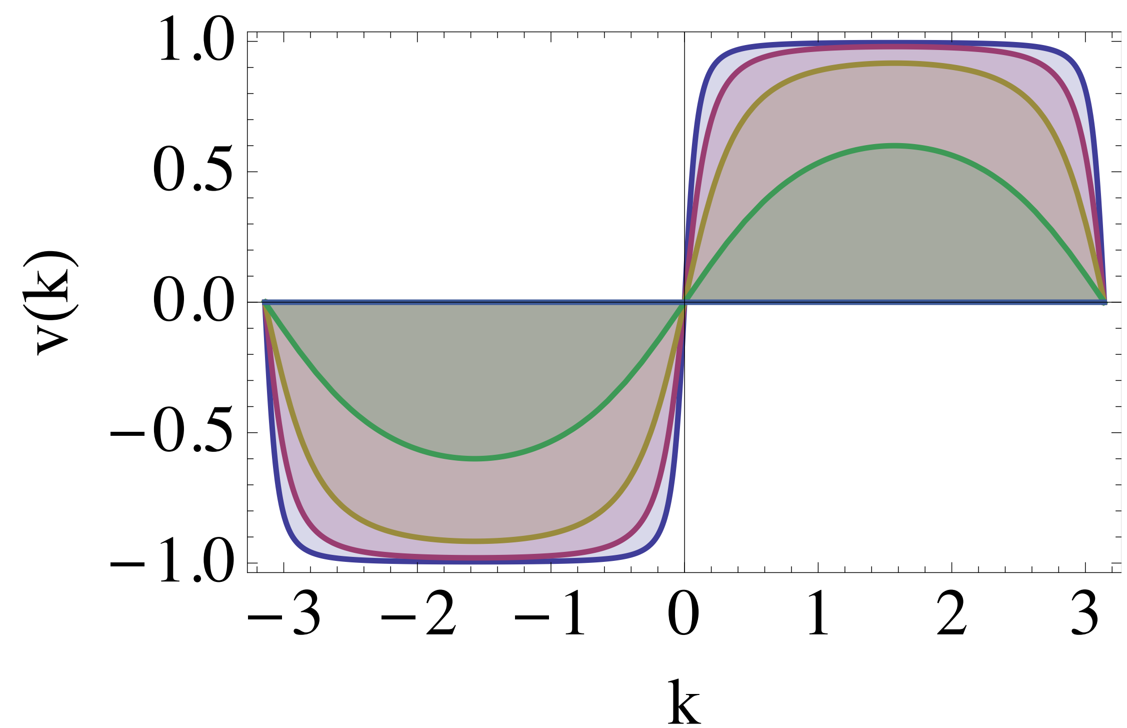
Bibeau-Delisle, Bisio, D'Ariano,
Perinotti, Tosini, arXiv:1310.6760

Planck-scale effects: Lorentz covariance distortion

Transformations that leave the dispersion relation invariant



$$\omega_E(k) := \pm\cos^{-1}(\sqrt{1-m^2}\cos k)$$



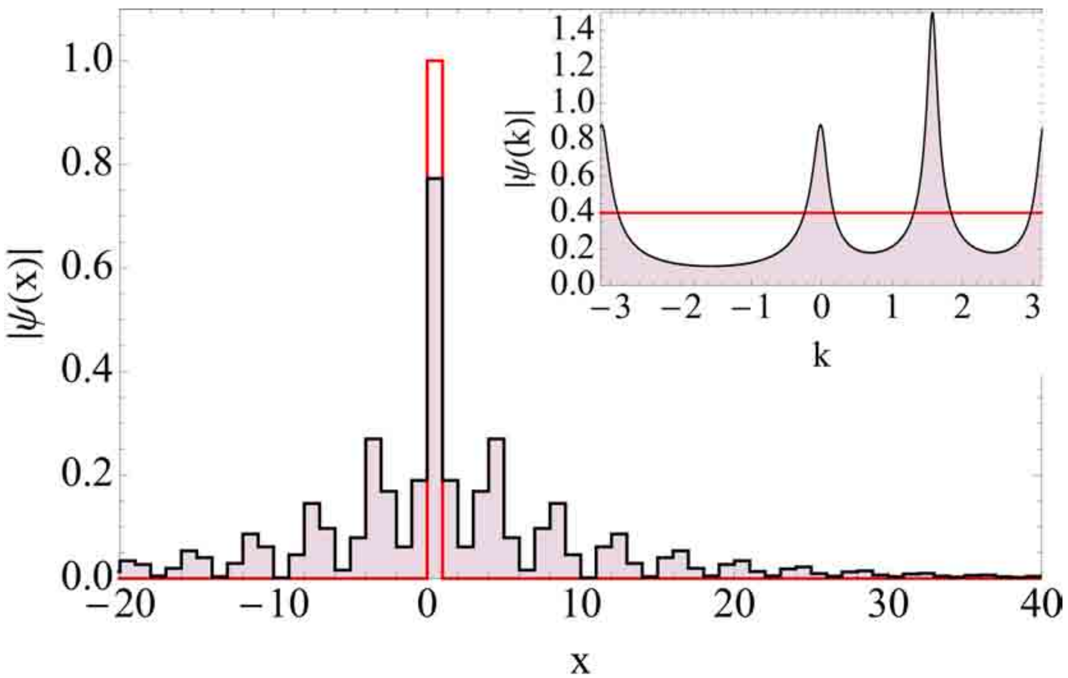
$$\omega' = \arcsin\left[\gamma\left(\sin\omega/\cos k - \beta\tan k\right)\cos k'\right]$$

$$k' = \arctan\left[\gamma\left(\tan k - \beta\sin\omega/\cos k\right)\right]$$

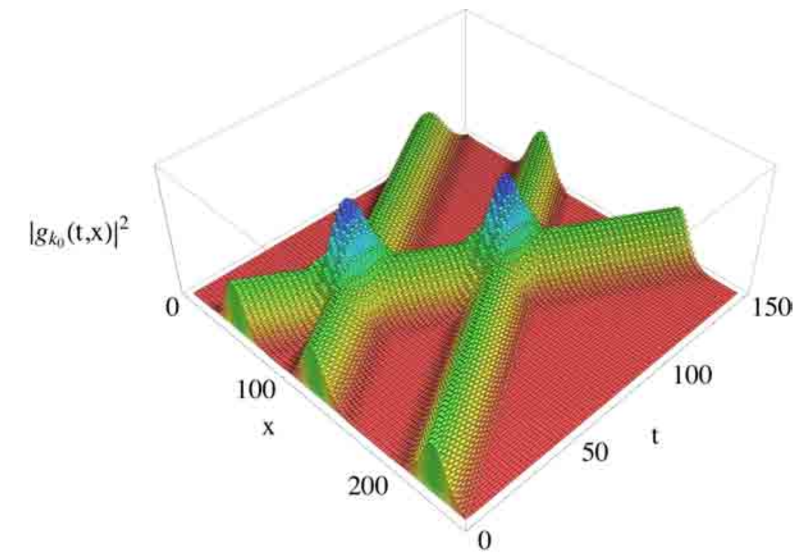
$$\gamma := (1-\beta^2)^{-1/2}$$

Bibeau-Delisle, Bisio, D'Ariano,
Perinotti, Tosini, arXiv:1310.6760

Planck-scale effects: Lorentz covariance distortion

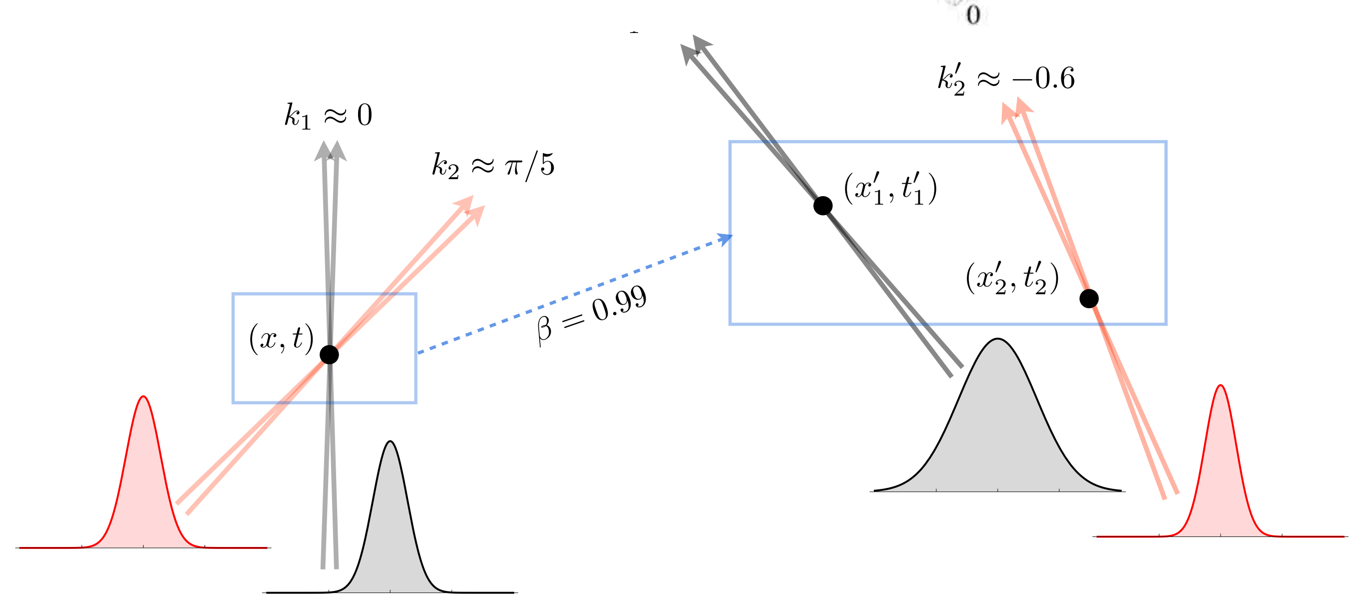


For narrow-band states we can linearize Lorentz transformations around k=k_0 and we get k-dependent Lorentz transformations



Delocalization under boost

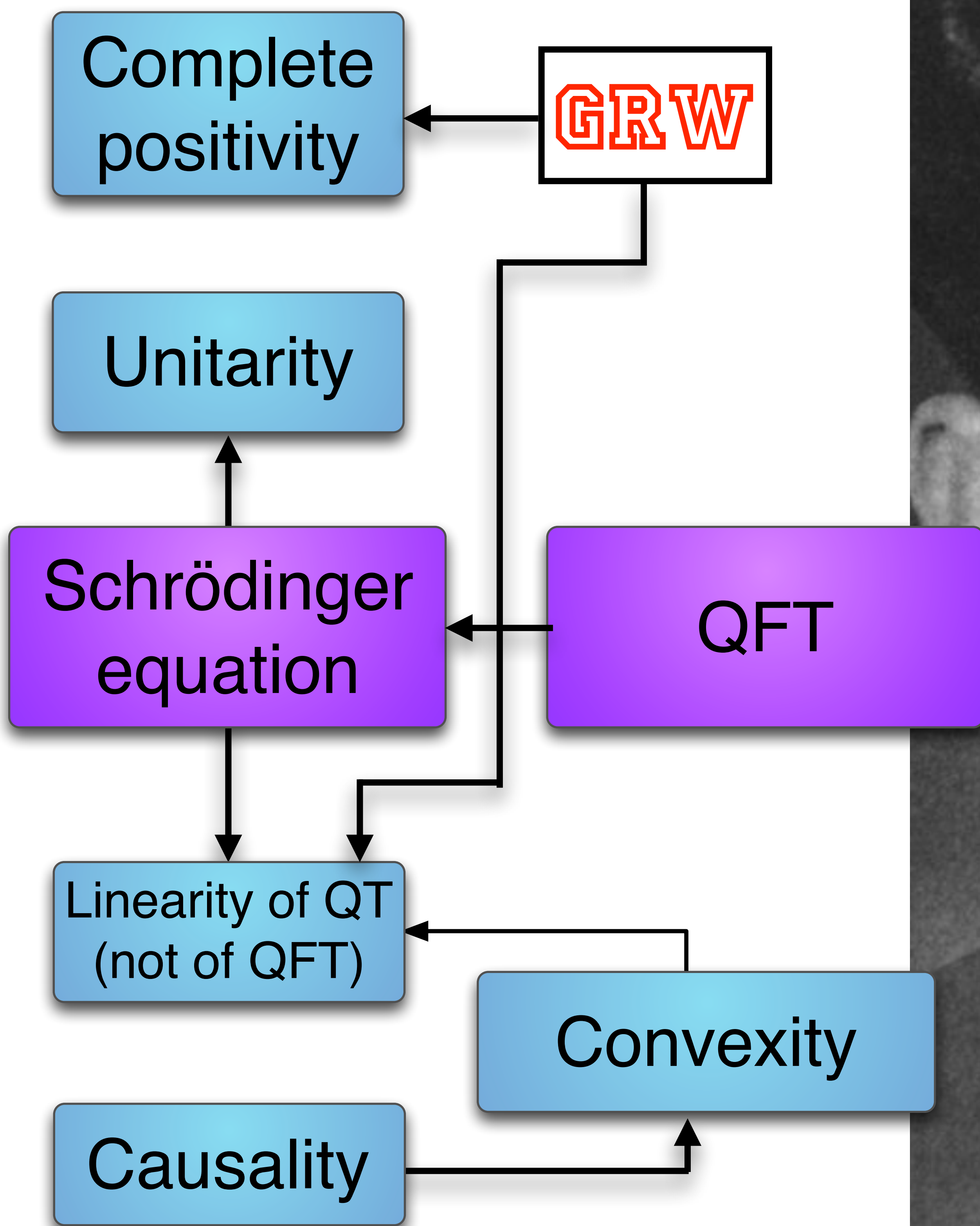
$$|\psi\rangle = \int \mathrm{d}k\mu(k)\hat{g}(k)|k\rangle \xrightarrow{L^D_\beta} \int \mathrm{d}k\mu(k)\,\hat{g}(k)|k'\rangle =$$
$$= \int \mathrm{d}k\mu(k')\,\hat{g}(k(k'))|k'\rangle$$



Relative locality

R. Schützhold and W. G. Unruh, J. Exp. Theor. Phys. Lett. **78** 431 (2003)
G. Amelino-Camelia, L. Freidel, J. Kowalski-Glikman, and L. Smolin, arXiv:1106.0313 (2011)

Modifications of QM

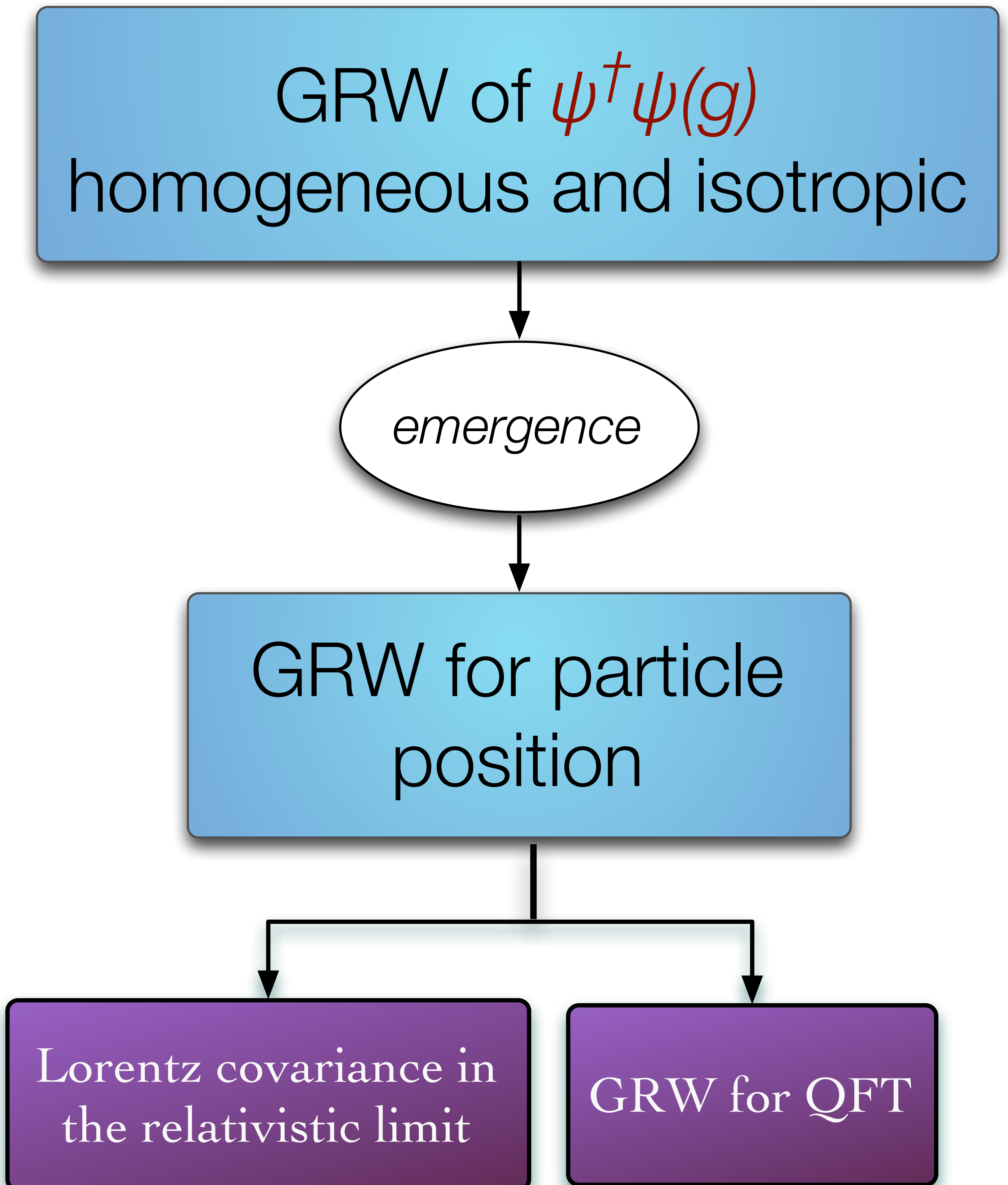


Problems with GRW

- Lorentz covariance
- Indistinguishable particles
- QFT

A. Rimini (private comm.)

A proposed solution



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A Quantum-Digital Universe **(ID: 43796)**

THANK YOU!

Paolo Perinotti

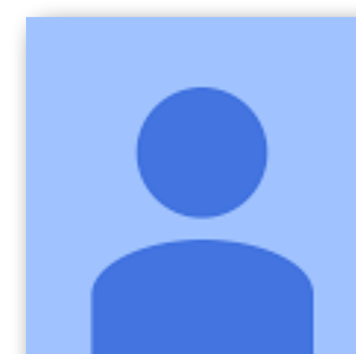
Alessandro Bisio

Alessandro Tosini

Alexandre Bibeau

Franco Manessi

Nicola Mosco

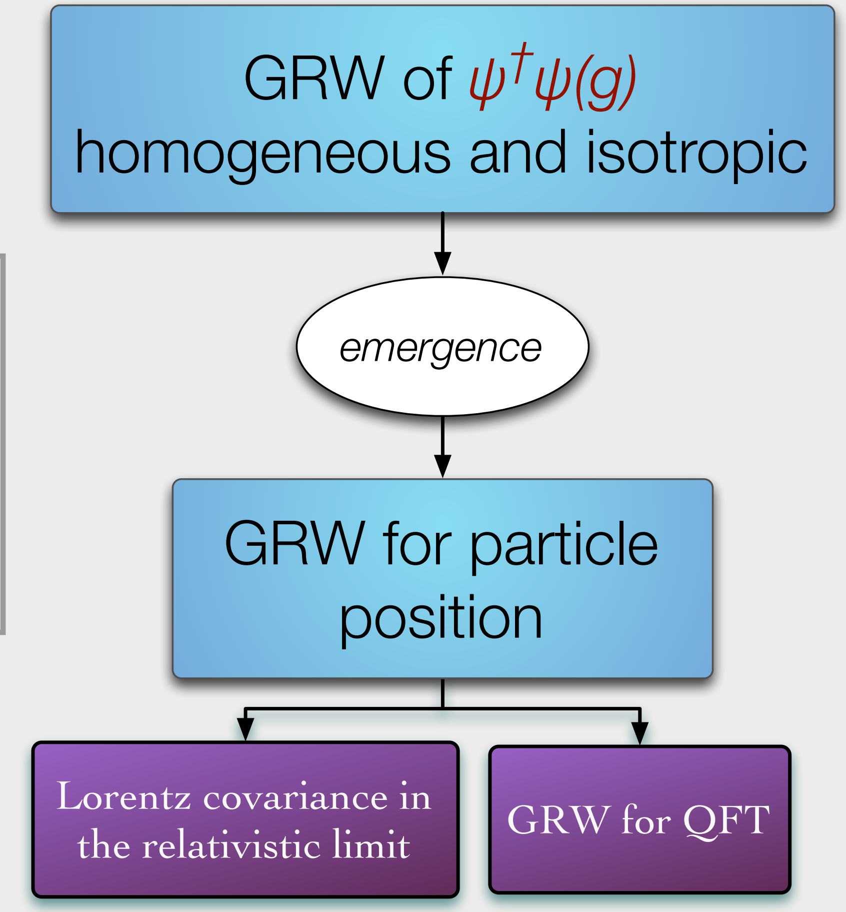


Marco Erba

Items for discussion

- QT is a *theory of information*
- The Weyl, Dirac, and Maxwell equations are derived from information-theoretic principles only, without assuming SR
- Only denumerable quantum systems in interaction
- QCA theory to be regarded as a theory unifying scales from Planck to Fermi (no continuum limit!)
- QFT is recovered in the *relativistic limit* (k≪1)
- In the *ultra-relativistic limit* (Planck scale) Lorentz covariance is an approximate symmetry, and one has the *Doubly Special Relativity* of Amelino-Camelia/Smolin/Magueijo

Proposed solution for GRW



GOOD FEATURES

1. **no SR assumed**: emergence of relativistic quantum field and space-time
2. ***quantum ab-initio***
3. no divergencies and all the problems from the continuum
4. no “violations” of causality
5. computable
6. dynamics stable (dispersive Schrödinger equation for narrow-band states valid at all scales)
7. solves the problem of localization in QFT
8. natural scenario for the *holographic principle*