Engineering Novel Quantum Information Processing Devices

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QUIT quantum information theory group

Quantum Optics II
Cozume, Mexico
5-10 December 2004

Essential literature

quantum information theory group

Convex structures of POVM's and Channels

- G. M. D'Ariano, P. Lo Presti, P. Perinotti, *Classical randomness in quantum measurements,* Phys. Rev. A (submitted), (quant-ph0408115)
- G. Chiribella, G. M. D'Ariano, P. Perinotti (unpublished)

Quantum calibration

- G. M. D'Ariano and P. Lo Presti, Phys. Rev. Lett. **91** 047902 (2003)
- G. M. D'Ariano, P. Lo Presti, and L. Maccone, *Quantum Calibration of Measuring Apparatuses,* Phys. Rev. Lett. (in press) (quant-ph/0408116)

Programmability of measurements

- G. M. D'Ariano, P. Perinotti, *Efficient universal programmable quantum measurements,* Phys. Rev. Lett. (submitted) (quant-ph-0410169)
- G. M. D'Ariano and P. Perinotti, *On the realization of Bell observables,* Phys. Lett A **329** 188 -192 (2004)

Transmission of reference frames

- G. Chiribella, G. M. D'Ariano, P. Perinotti, and M. Sacchi, *Efficient use of quantum resources for the transmission of a reference frame,* Phys. Rev. Lett. **93** 180503 (2004)

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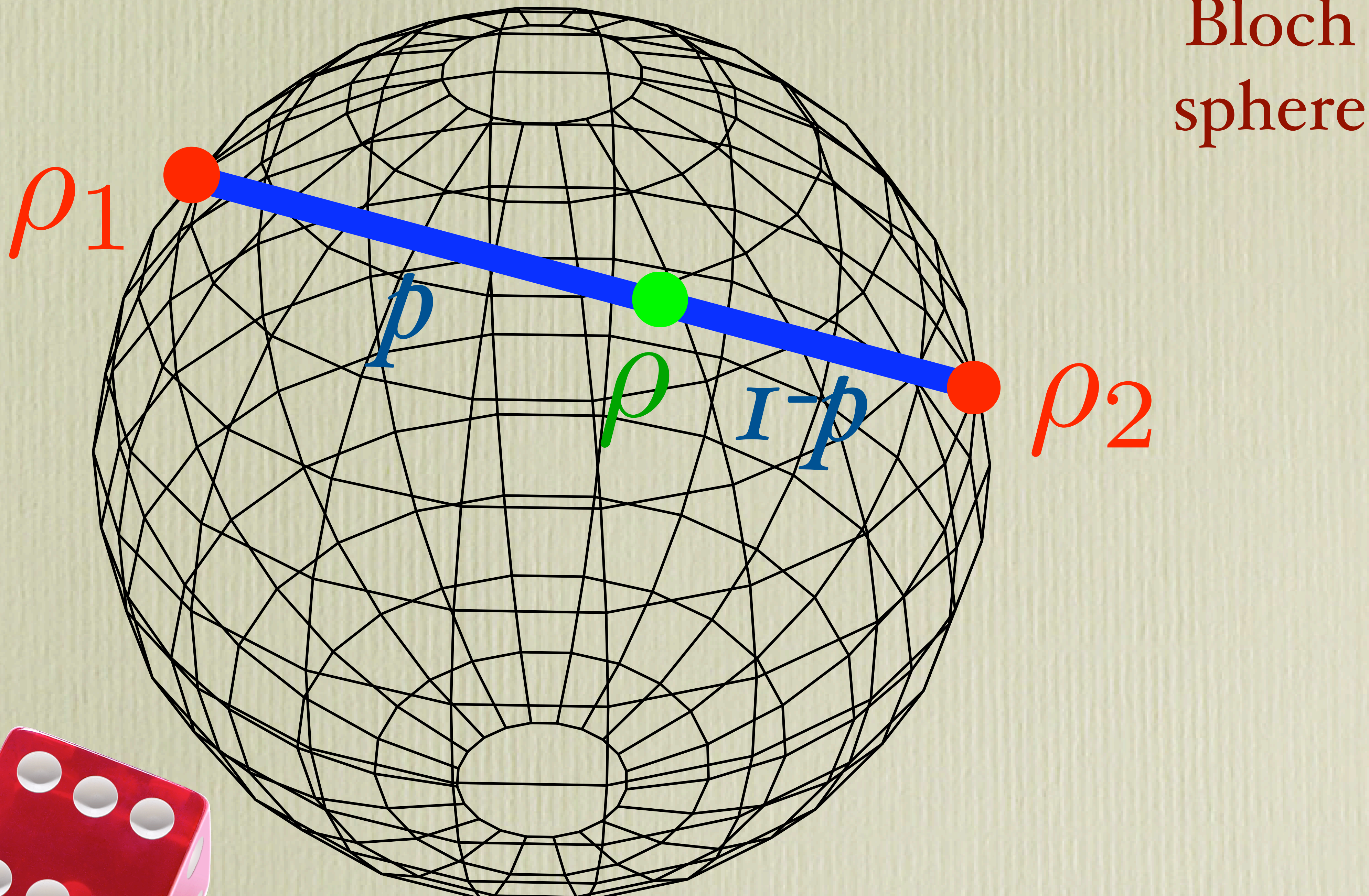
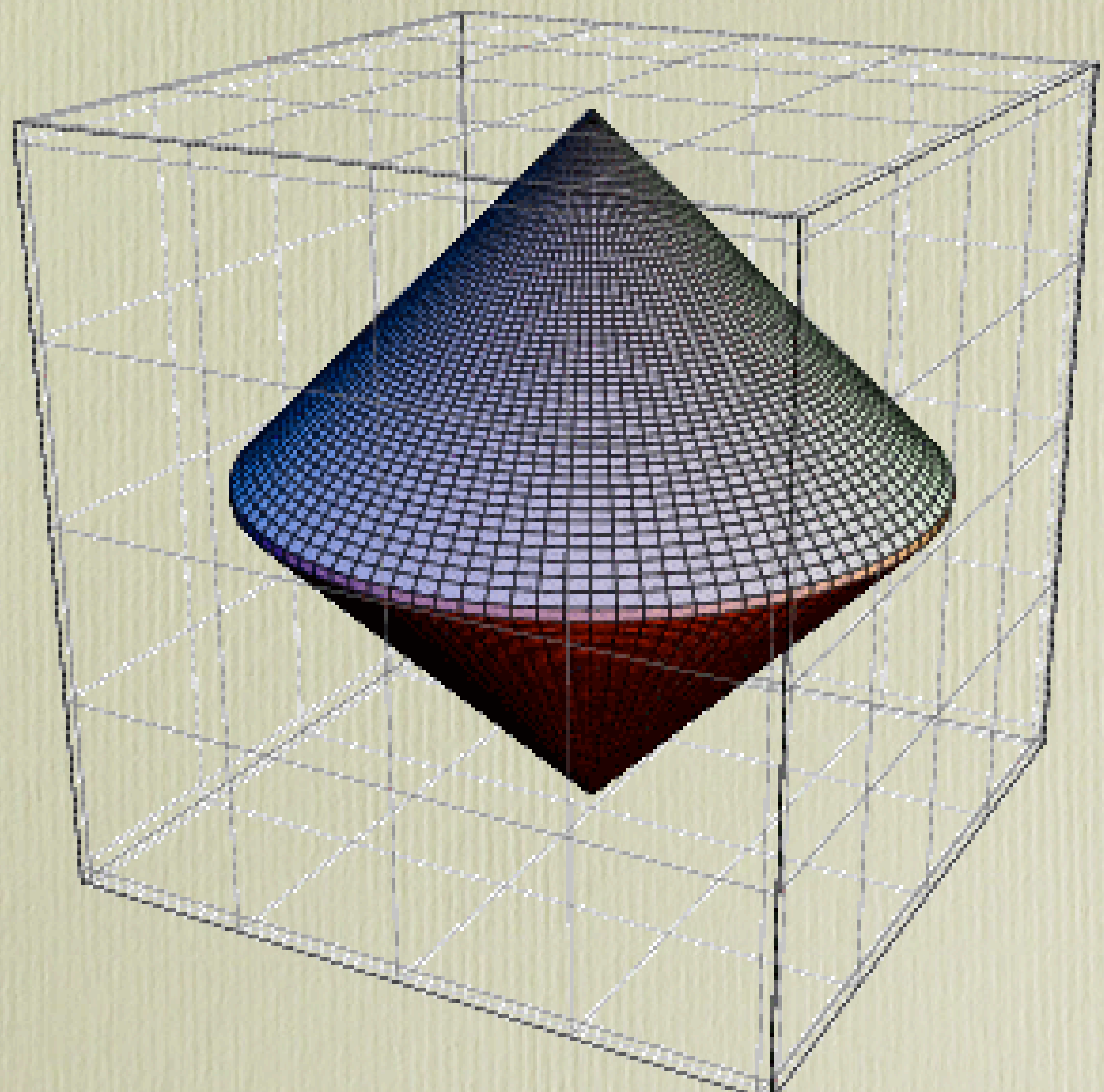
Convex set of states

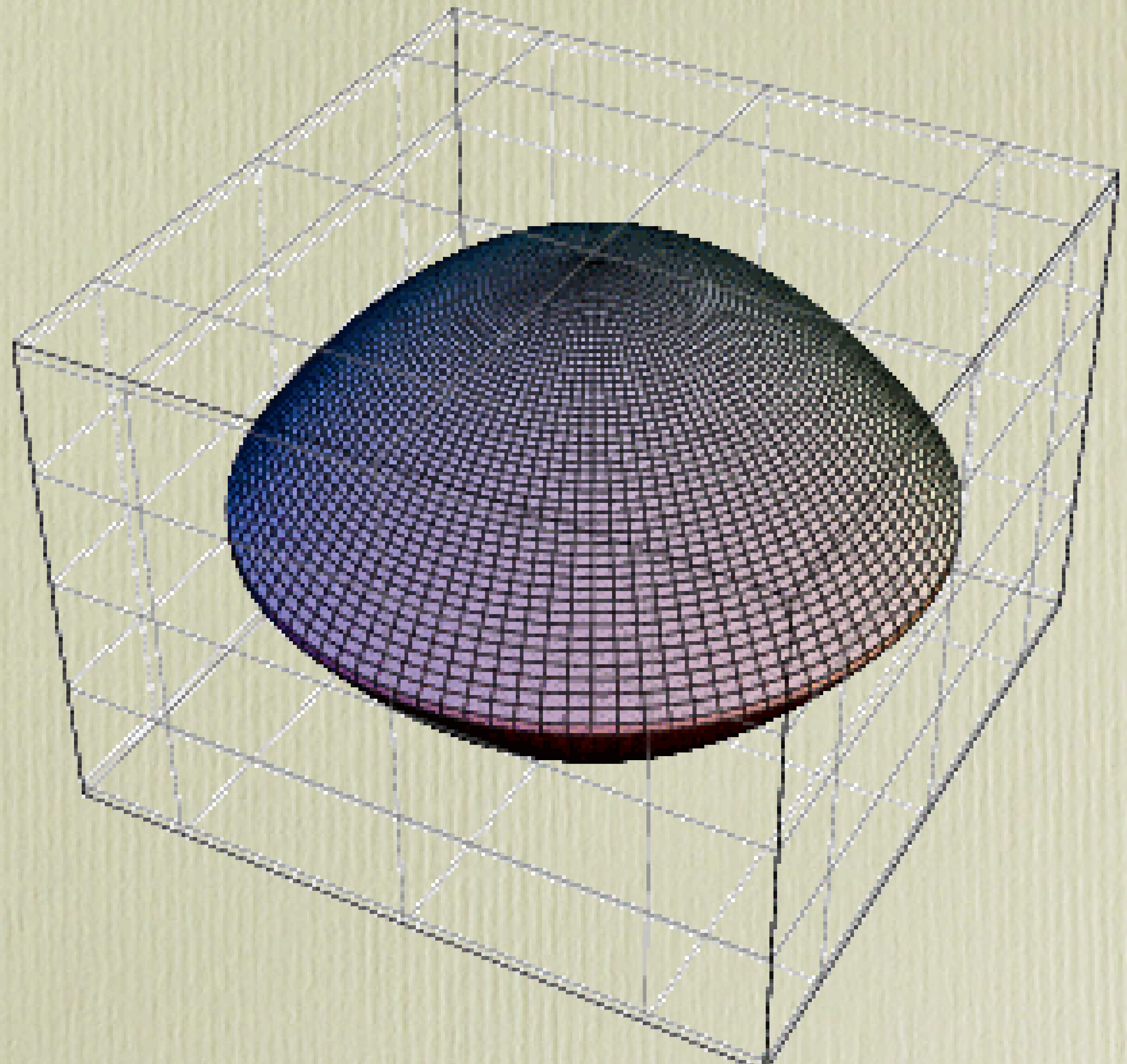


$$\rho = p\rho_1 + (1-p)\rho_2$$

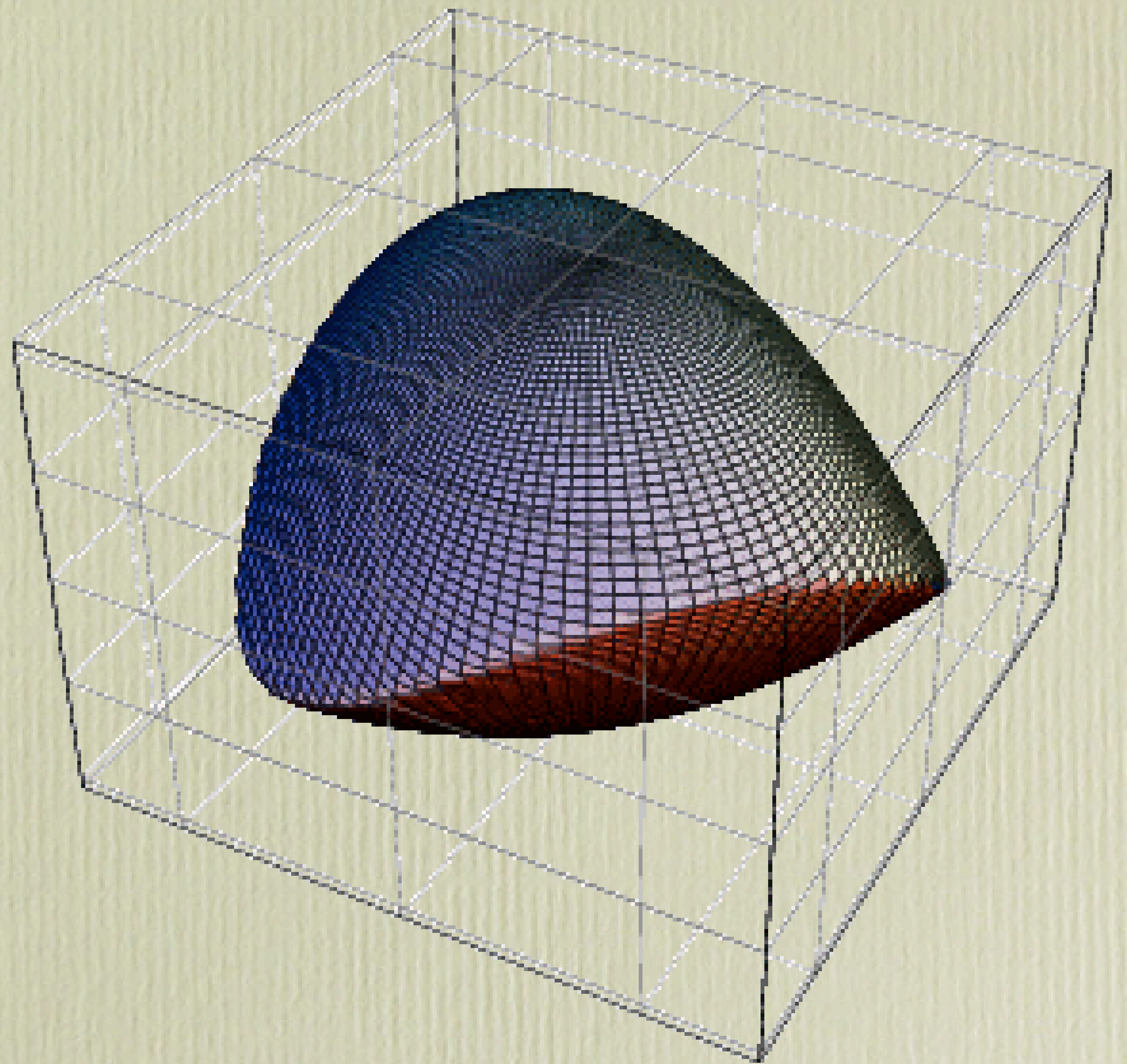
Convex set of states



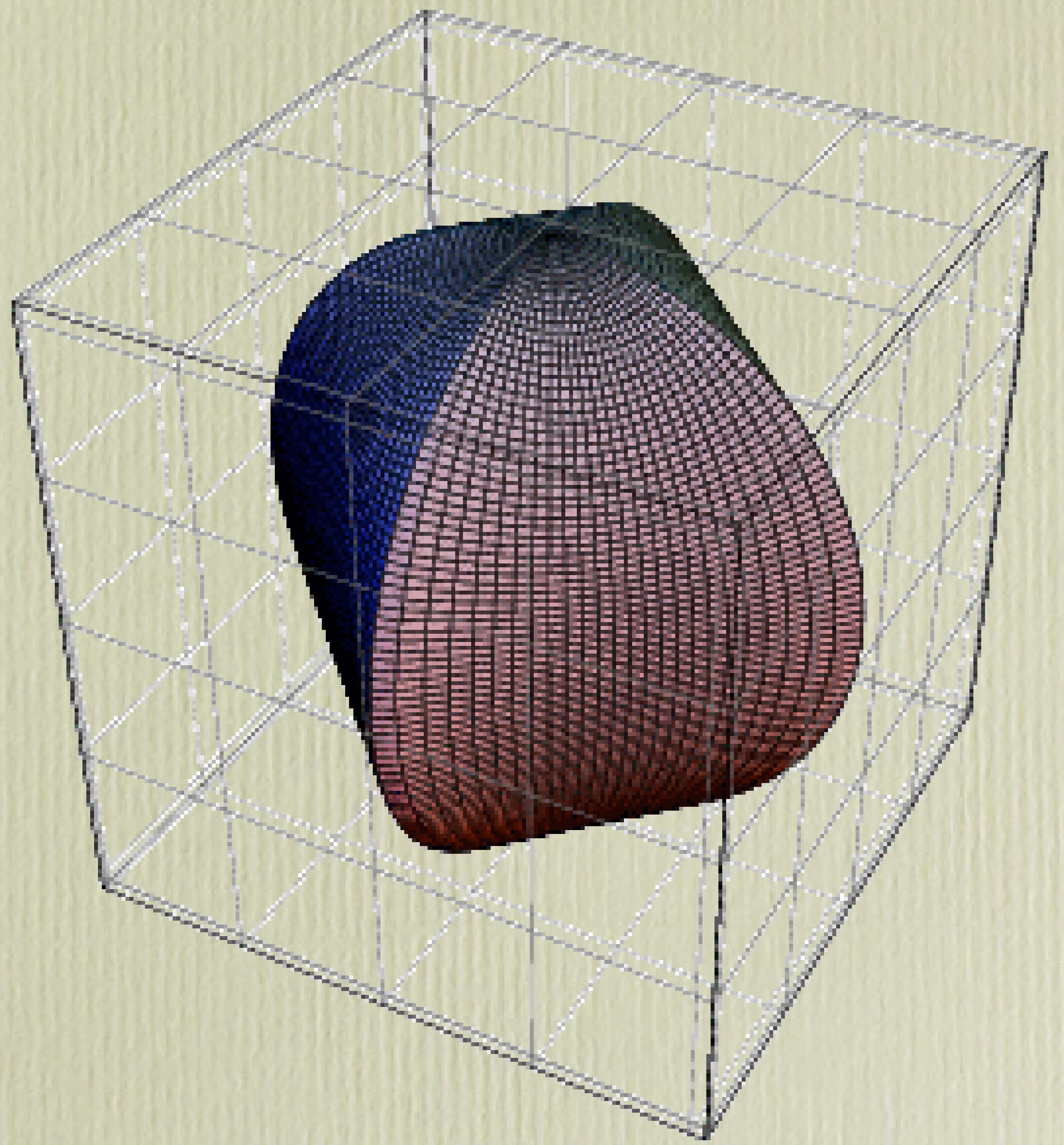
Convex set of states



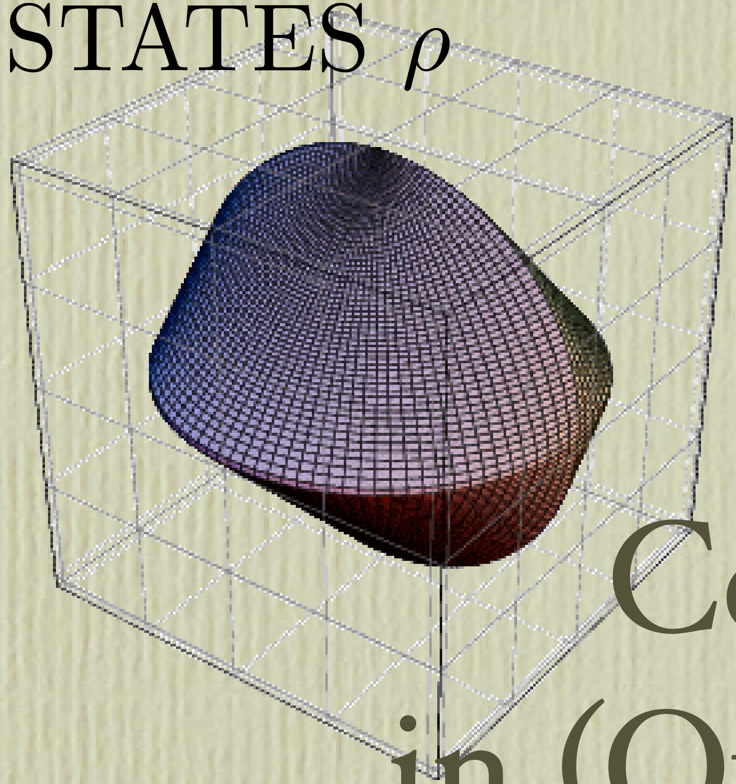
Convex set of states

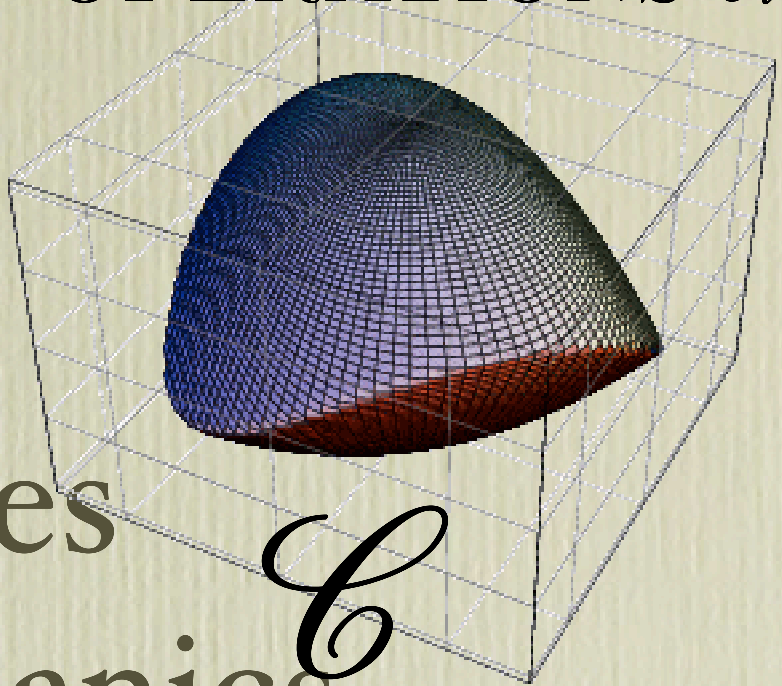


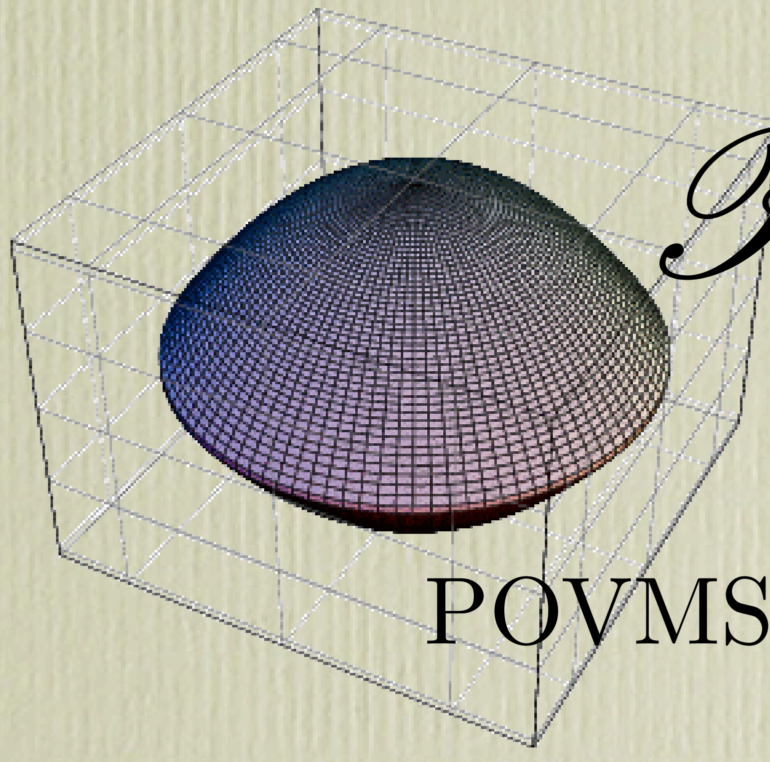
Convex set of states

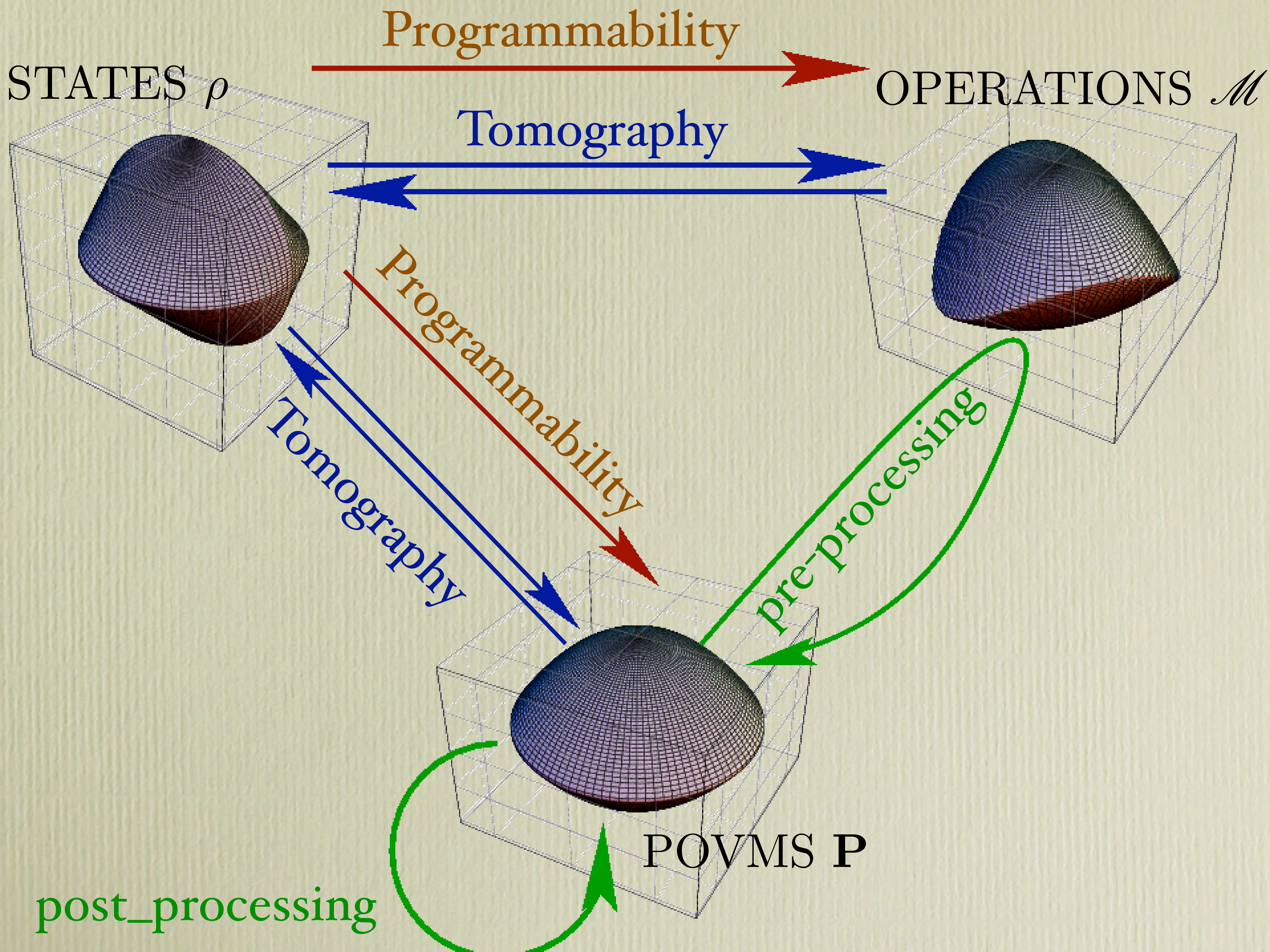


Convex structures in (Quantum) Mechanics







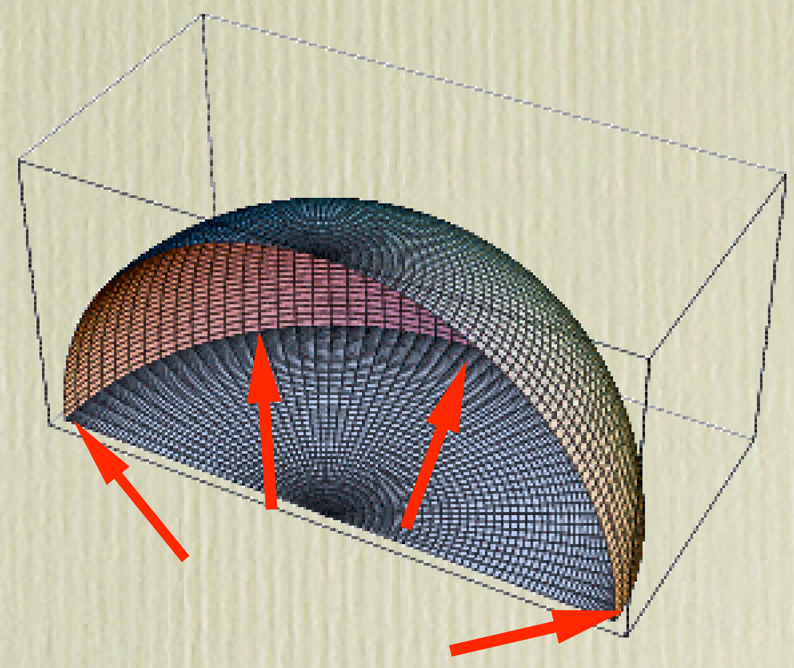

Programmability
STATES ρ
OPERATIONS $\mathscr{M}$
Tomography
Programmability
Tomography
pre-processing
POVMS $\mathbf{P}$
post_processing

Why to study convex structures?

Optimization problems:

Minimize a cost-function that is concave over the convex set

Minimum on the set of extremal points



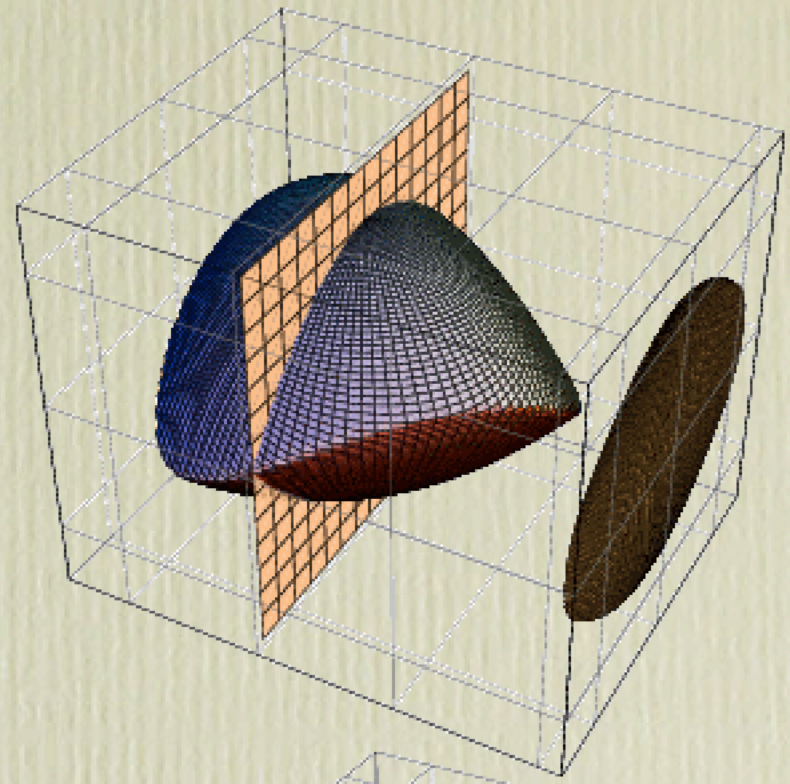
Why to study convex structures?

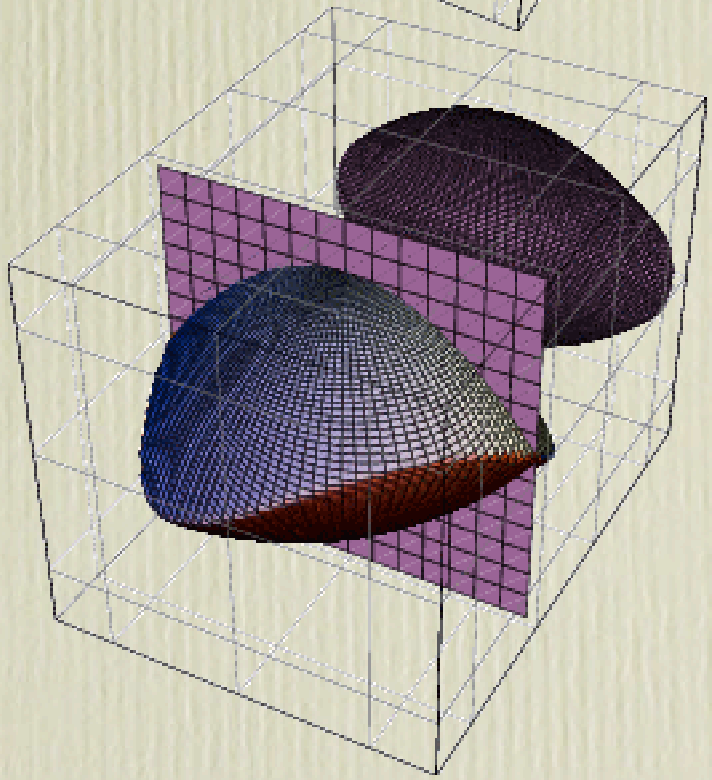
Linear Constraints

corresponds to plane sections of the convex

The border of the section is the section of the border

Extremals of the section belong to the original border





Notation

- Bipartite states $|\Psi\rangle\rangle \in \mathsf{H} \otimes \mathsf{K} \iff$ operators $\Psi \in \mathsf{HS}(\mathsf{K}, \mathsf{H})$

$$|\Psi\rangle\rangle = \sum_{nm} \Psi_{nm} |n\rangle \otimes |m\rangle.$$

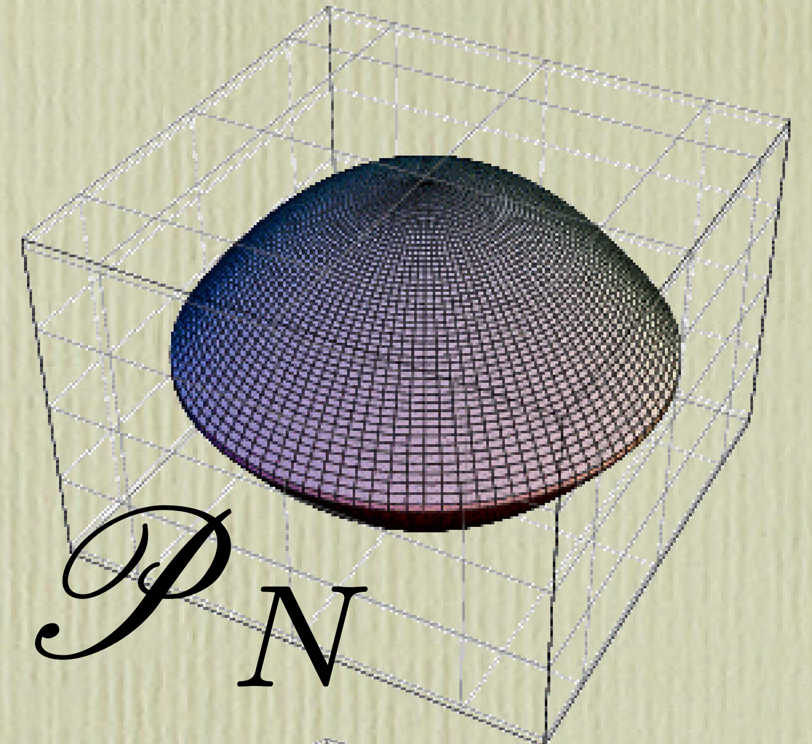
- Matrix notation (for fixed reference basis in the Hilbert spaces)

$$A \otimes B|C\rangle\rangle = |AC\,B^{\intercal}\rangle\rangle,$$

$$\langle\langle A|B\rangle\rangle \equiv \mathrm{Tr}[A^{\dagger}B].$$

$$|I\rangle\rangle = \sum_{n} |n\rangle \otimes |n\rangle$$

POVM's

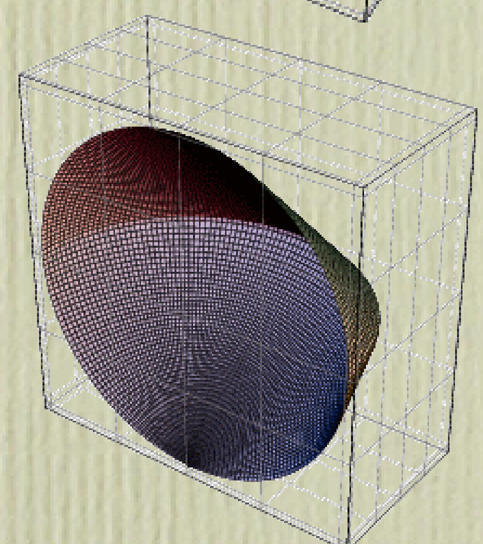


Hilbert space H, $d = \dim(\mathsf{H})$

$\mathscr{P}_N$ convex set of POVM's on H with N outcomes

$\mathbf{P} \in \mathscr{P}_N$, $\mathbf{P} = \{P_1, \ldots, P_N\}$

$\{|v_n^{(e)}\rangle\}$: eigenvectors of P_e



Border of the convex

$$b(\mathbf{P}) = r(\mathbf{P}) - l(\mathbf{P})$$

where

$$r(\mathbf{P}) = \sum_e \mathrm{rank}(P_e)^2,$$

$$l(\mathbf{P}) = \dim[\mathsf{Span}\{|v_m^{(e)}\rangle\langle v_n^{(e)}|\}_{nme}]$$

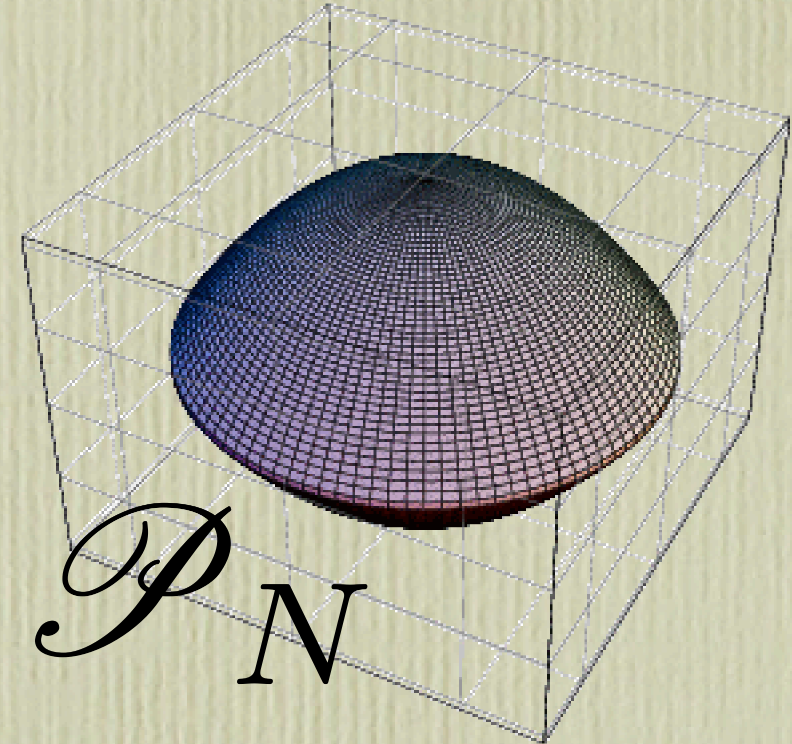
$b(\mathbf{P})$: dimension of the "face"

border $\partial\mathscr{P}_N$ of $\mathscr{P}_N$:

$$b(\mathbf{P}) < d^2(N-1)$$

G. M. D'Ariano, P. Lo Presti, and P. Perinotti,
Phys. Rev. A (submitted) (quant-ph0408115)

POVM's



Extremal POVM's

A POVM $\mathbf{P} = \{P_e\}_{e \in \mathsf{E}}$ is extremal iff the supports $\mathsf{Supp}(P_e)$ are weakly independent for all $e \in \mathsf{E}$.

We call a generic set of orthogonal projections $\{Z_e\}_{e\in\mathsf{E}}$ weakly independent if for any set of operators $\{T_e\}_{e\in\mathsf{E}}$ on H one has

$$\sum_{e\in\mathsf{E}} Z_e T_e Z_e = 0 \quad \Rightarrow \quad Z_e T_e Z_e = 0,\ \forall e \in \mathsf{E}.$$

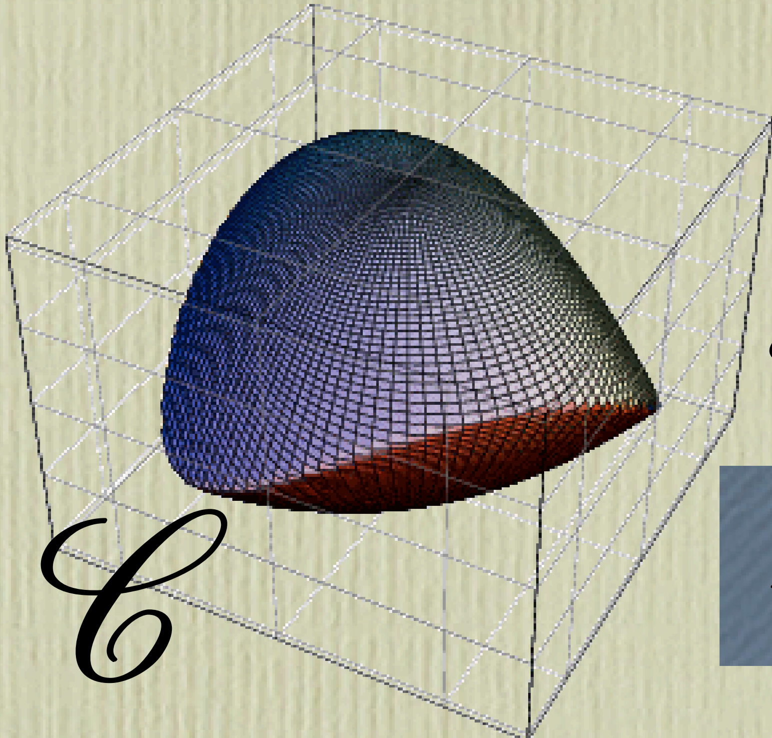
Extremal POVM's are not necessarily rank-one!

There are extremal POVM's only for $N \leq d^2$ outcomes.

For $N = d^2$ outcomes there exists always an extremal POVM, which is rank-one and informationally complete.

G. M. D'Ariano, P. Lo Presti, and P. Perinotti, Phys. Rev. A (submitted) (quant-ph0408115)

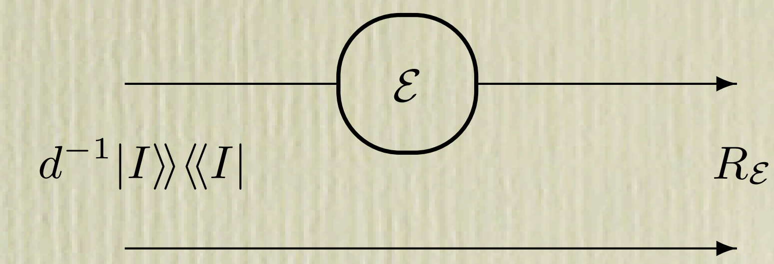
Channels



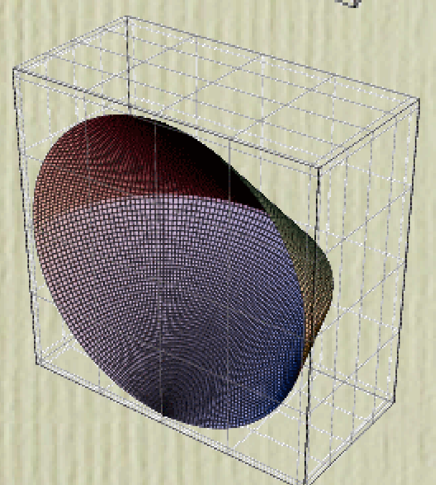
$\mathscr{C}$

$\mathscr{C}$ convex set of channels $\mathscr{E}$ from $\mathcal{S}(\mathsf{H})$ to $\mathcal{S}(\mathsf{K})$

$$R_{\mathscr{E}} = \mathscr{E} \otimes \mathscr{I}(|I\rangle\rangle\langle\langle I|)$$



$\mathscr{E} = \sum_n E_n \rho E_n^\dagger$ canonical Kraus



Border of the convex

$$b(\mathscr{E}) = r(\mathscr{E}) - l(\mathscr{E})$$

$b(\mathscr{E})$: dimension of the "face"

border $\partial\mathscr{C}$ of $\mathscr{C}$:

$$b(\mathscr{E}) < \dim(\mathsf{H})^2(\dim(\mathsf{K})^2 - 1)$$

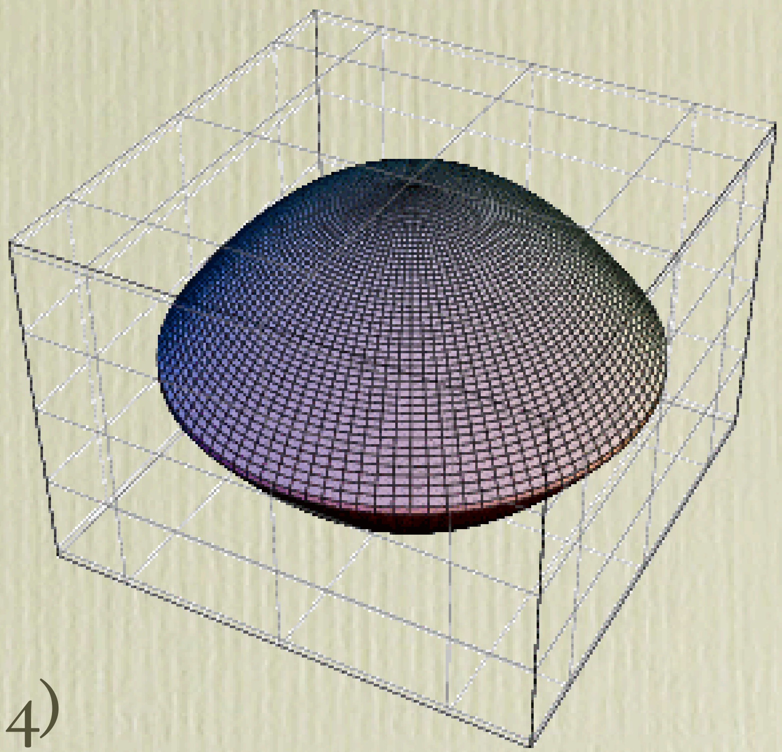
where

$$r(\mathscr{E}) = \mathrm{rank}(R_{\mathscr{E}})^2,$$

$$l(\mathscr{E}) = \dim(\mathsf{Span}\{E_i^\dagger E_j\})$$

G. Chiribella, G. M. D'Ariano, and P. Perinotti, (unpublished)

Convex of covariant POVM's



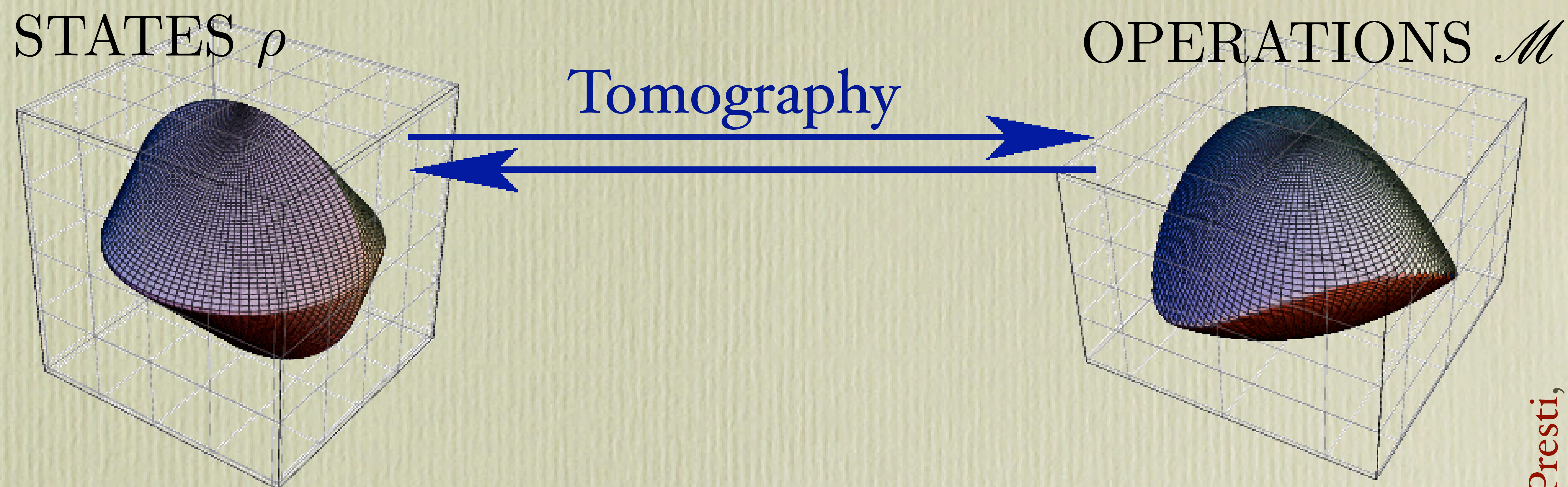
G. M. D'Ariano, *Extremal covariant Quantum Operations and POVM's*, J. Math. Phys. **45** 3620-3635 (2004)

G. Chiribella and G. M. D'Ariano, *Extremal covariant positive operator measures*, J. Math. Phys. **45** 4435-4447 (2004)

Extremal POVM's are not necessarily rank-one!

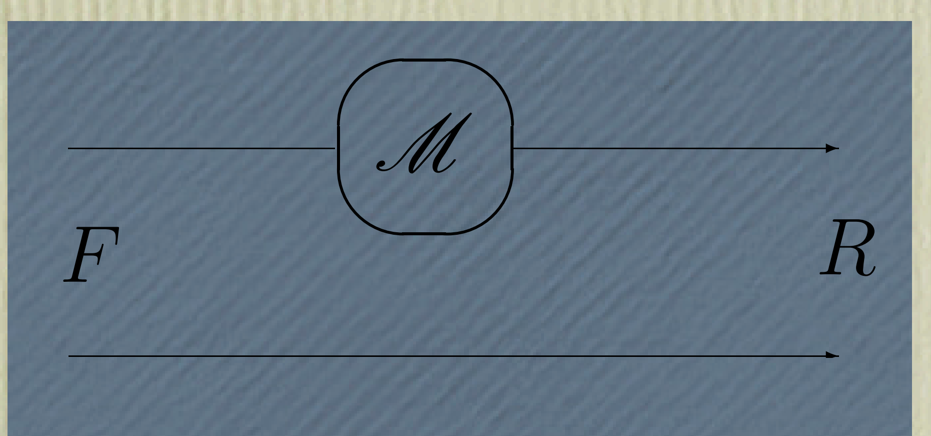
For some (reducible) representations rank-one POVM are forbidden!

Tomography of operations



$$R \Longleftrightarrow \mathscr{M}$$

$$R = \mathscr{M} \otimes \mathscr{I}(F)$$



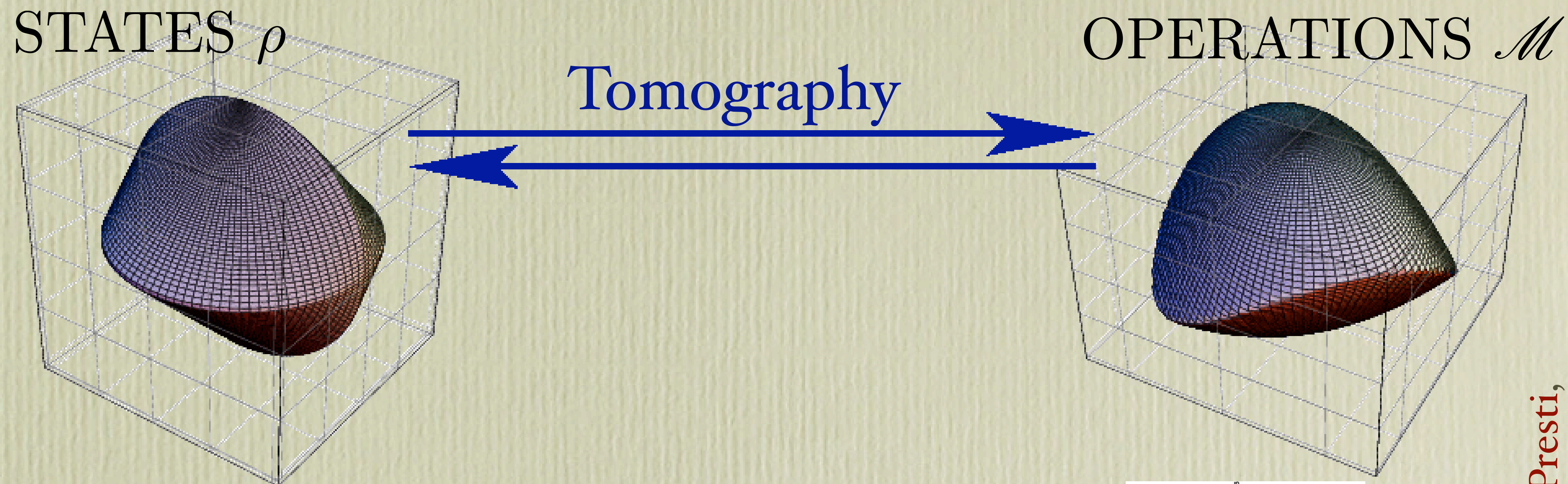
$$\mathscr{M}(\rho) = \mathrm{Tr}_2[(I \otimes \rho^{\intercal})\mathscr{I} \otimes \mathscr{F}^{-1}(R)]$$

$$\mathscr{F}(\rho) = \mathrm{Tr}_2[(I \otimes \rho^{\intercal})F]$$

F : faithful state

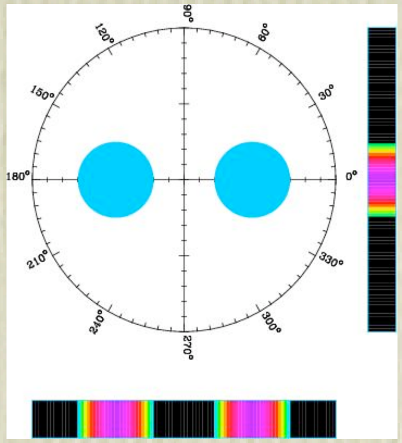
G. M. D'Ariano and P. Lo Presti, Phys. Rev. Lett. **91** 047902-(2003)

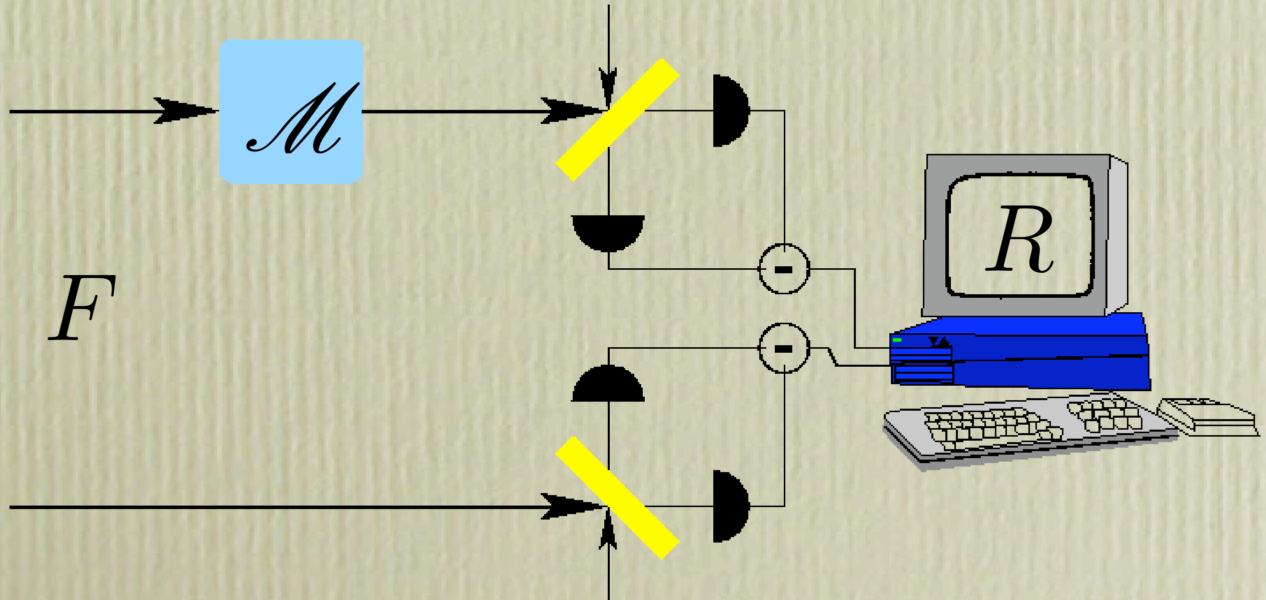
Tomography of operations

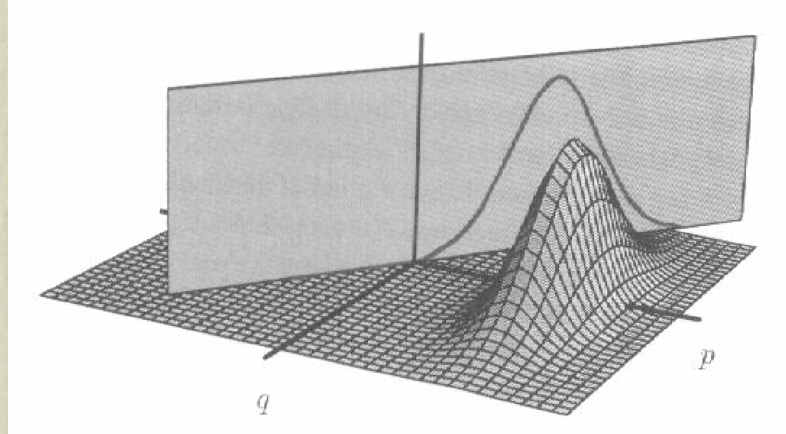


$$R \Longleftrightarrow \mathscr{M}$$

$$R = \mathscr{M} \otimes \mathscr{I}(F)$$





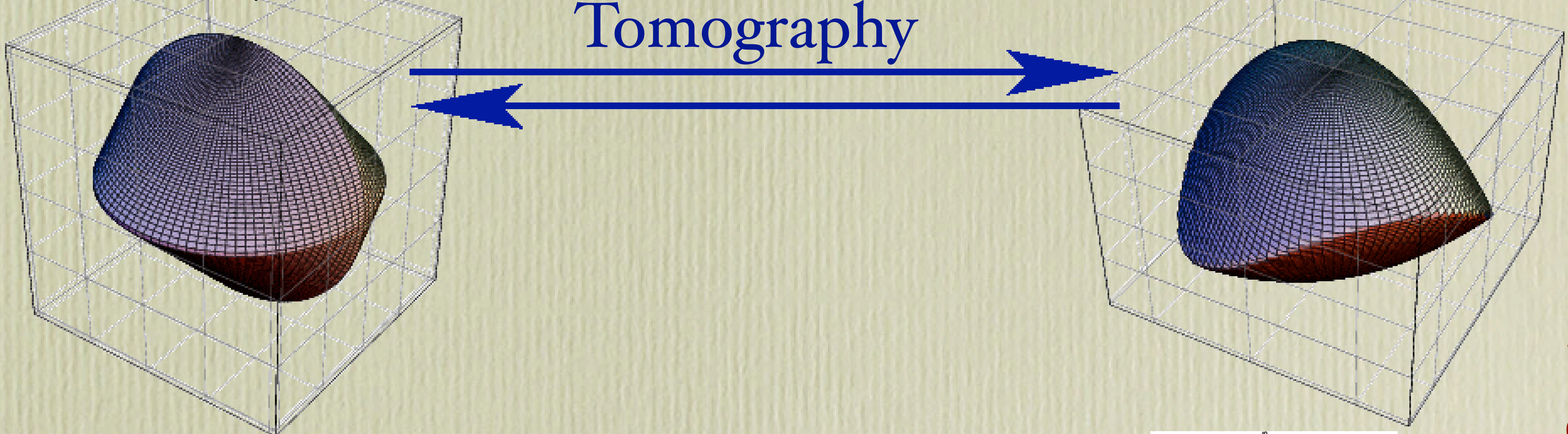


G. M. D'Ariano and P. Lo Presti, Phys. Rev. Lett. **91** 047902-(2003)

Tomography of operations

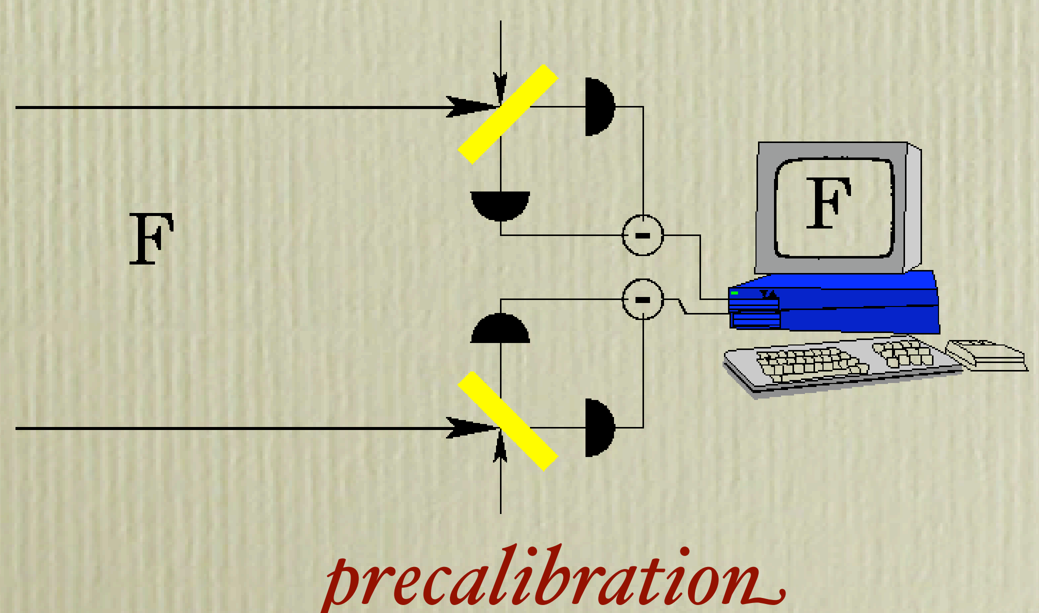
STATES ρ

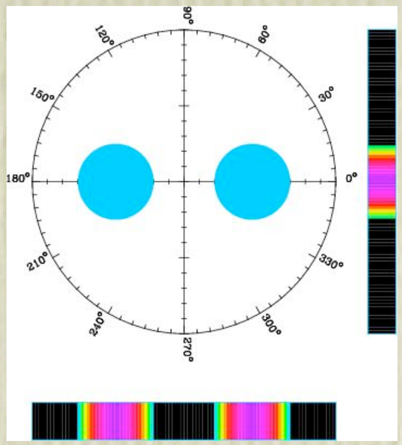
OPERATIONS $\mathscr{M}$

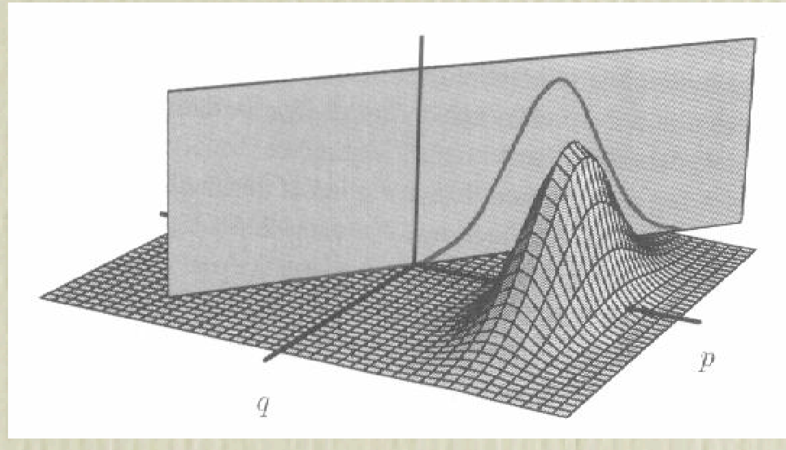


$$R \Longleftrightarrow \mathscr{M}$$

$$R = \mathscr{M} \otimes \mathscr{I}(F)$$

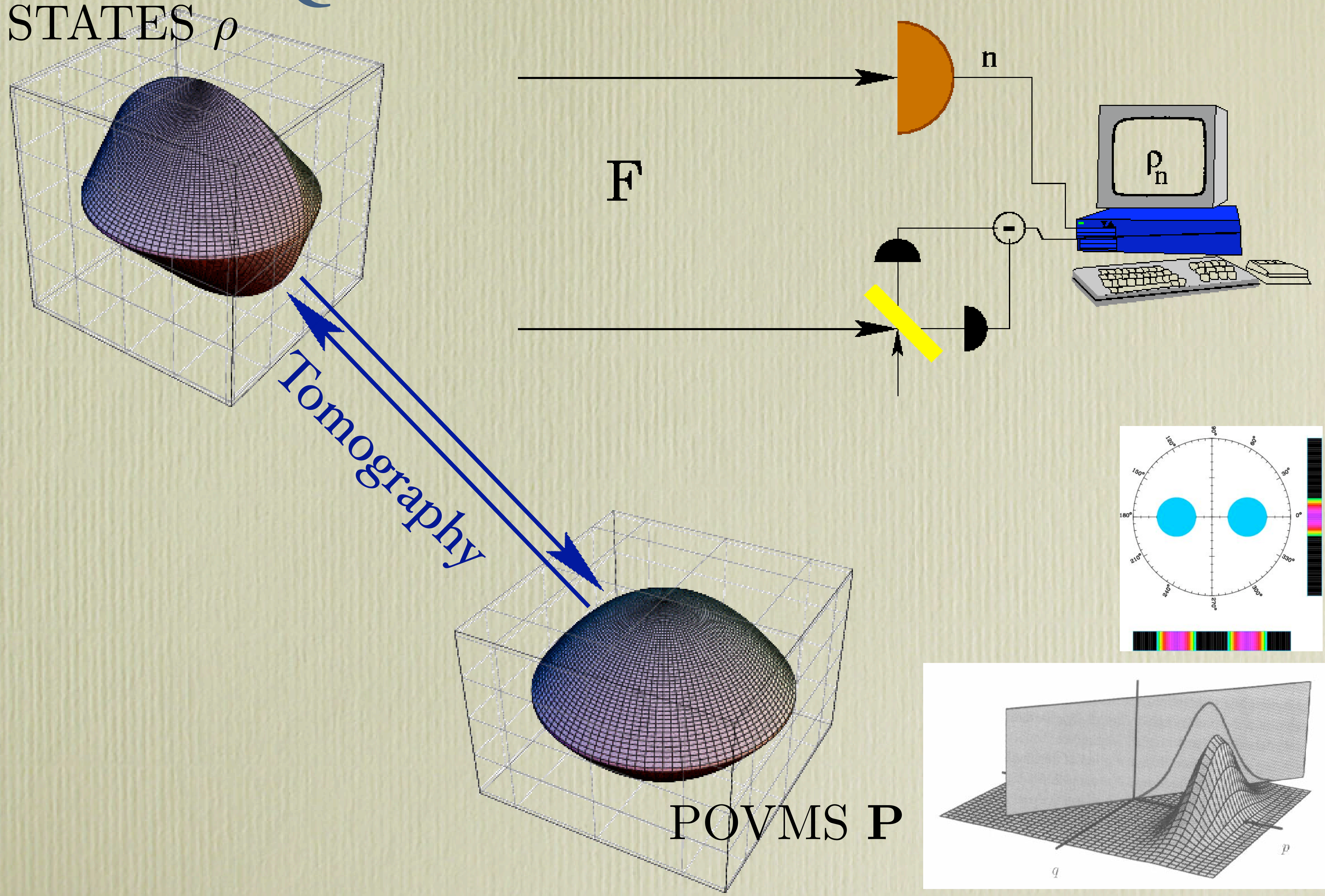




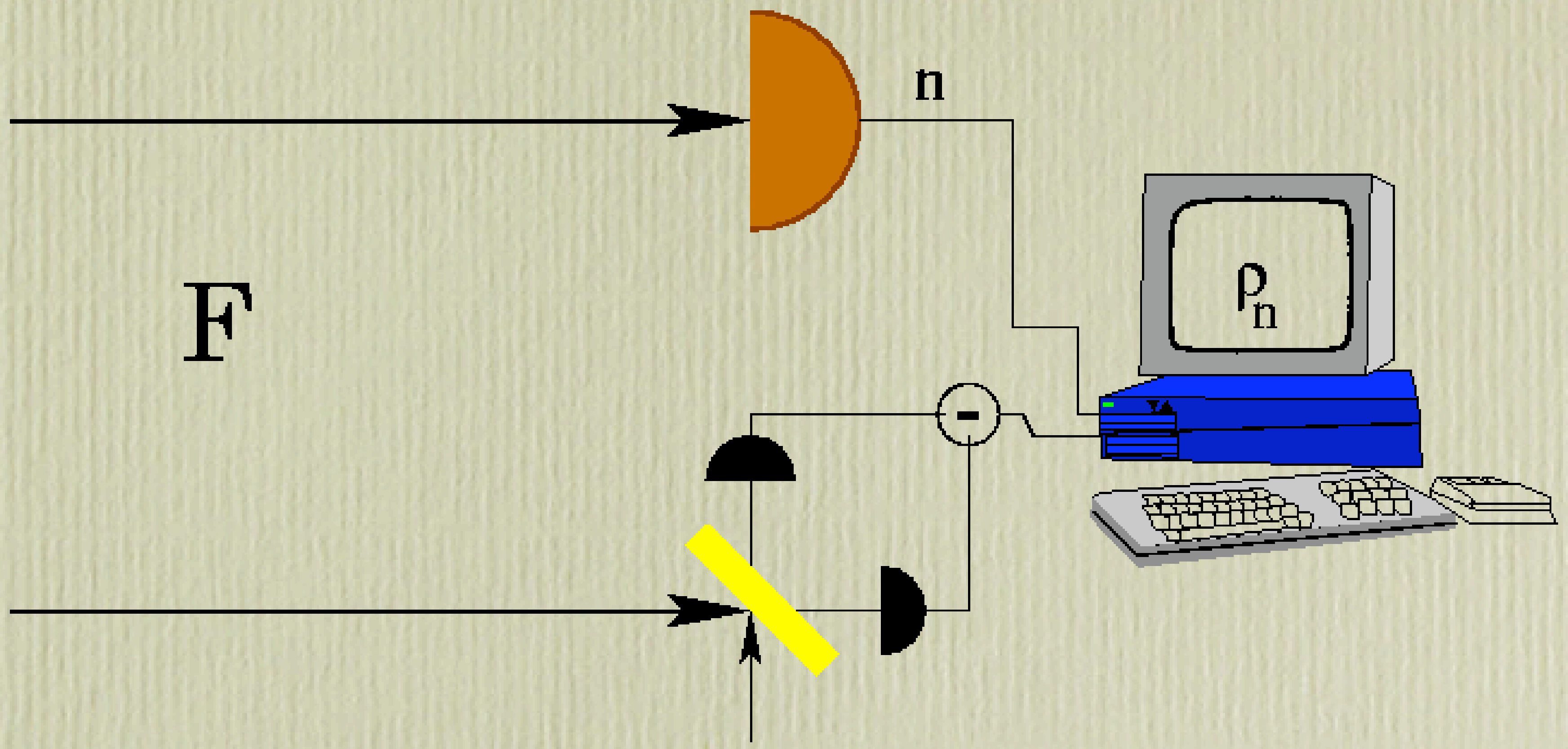


G. M. D'Ariano and P. Lo Presti, Phys. Rev. Lett. **91** 047902-(2003)

Quantum Calibration



Quantum Calibration



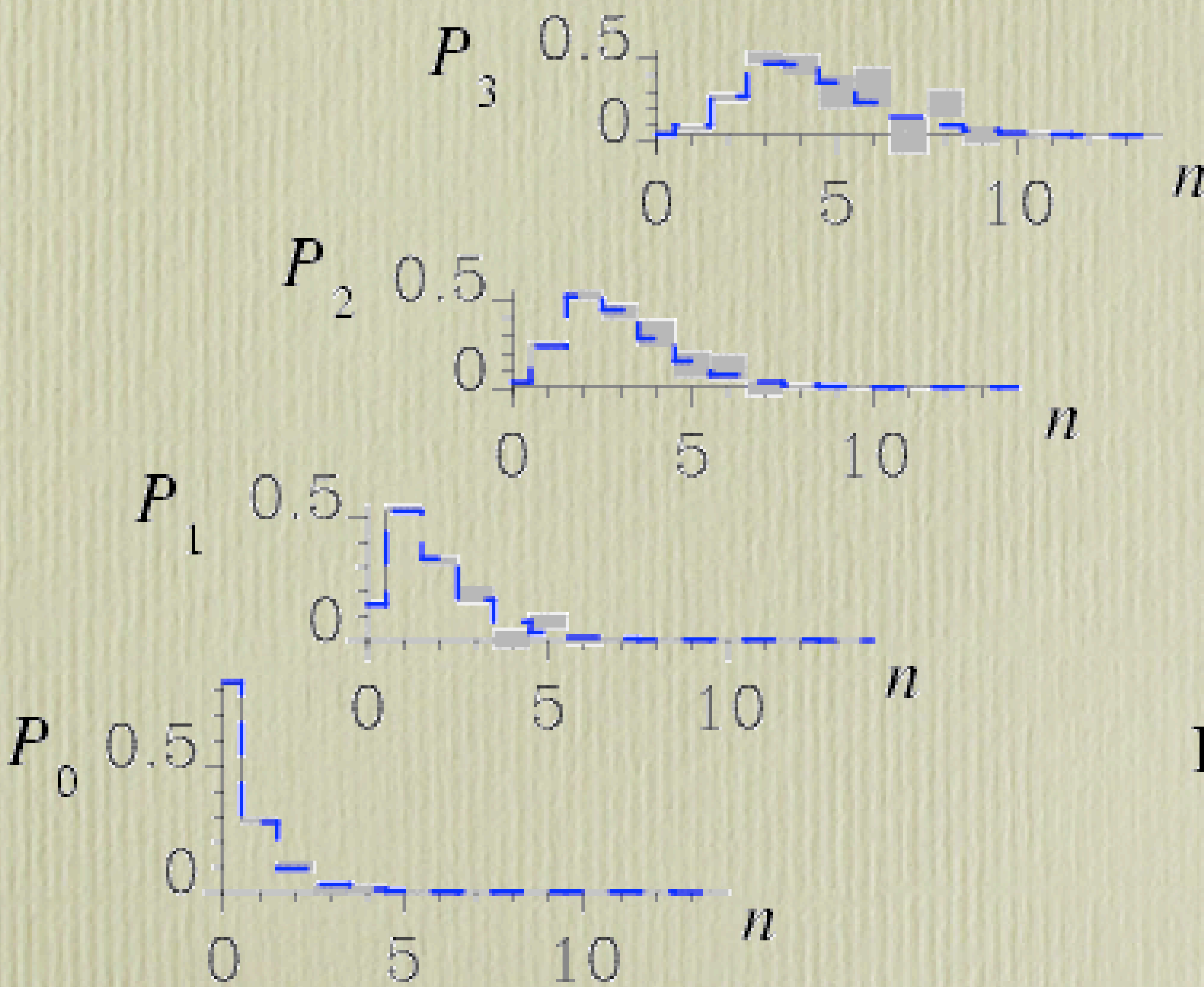
$$p_n \rho_n = \mathscr{F}(P_n), \quad P_n = \mathscr{F}^{-1}(p_n \rho_n),$$

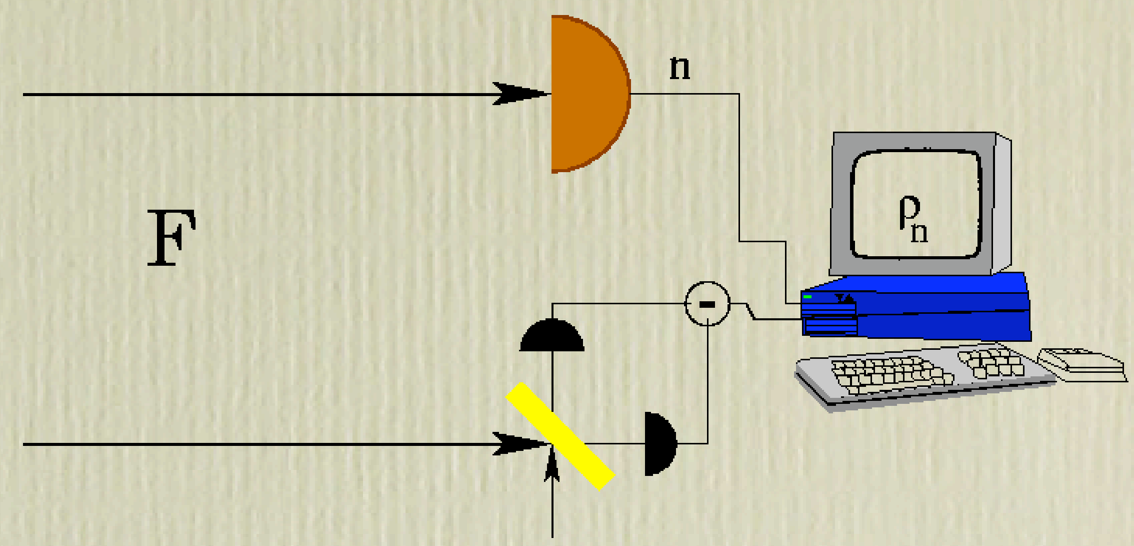
$$\mathscr{F}(X) = \mathrm{Tr}_2[(I \otimes X) F]$$

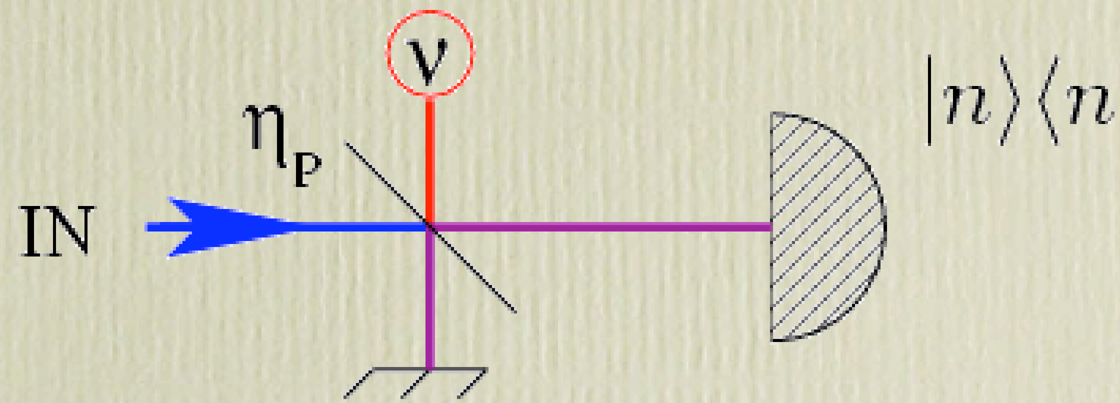
- p_n probability of the outcome n,
- ρ_n conditioned state, to be determined by quantum tomography,
- $\mathscr{F}$ associated map of the faithful state F.

G. M. D'Ariano, P. Lo Presti, and L. Maccone,
Phys. Rev. Lett. (in press) (quant-ph0408116)

Quantum Calibration

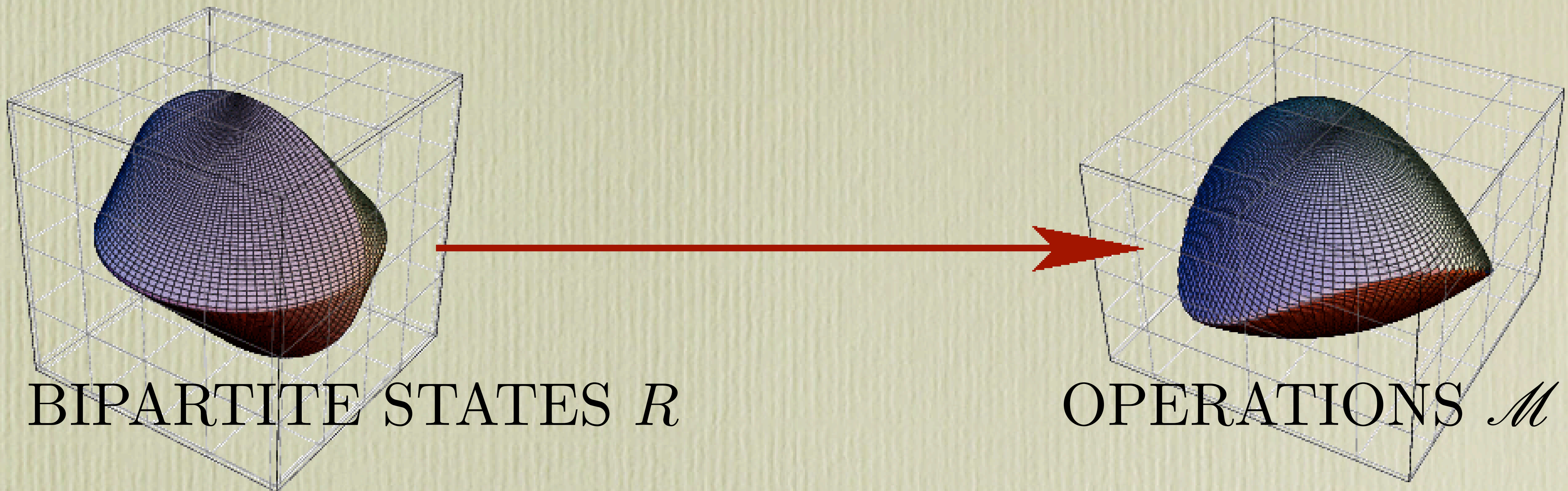






G. M. D'Ariano, P. Lo Presti, and L. Maccone, Phys. Rev. Lett. (in press) (quant-ph0408116)

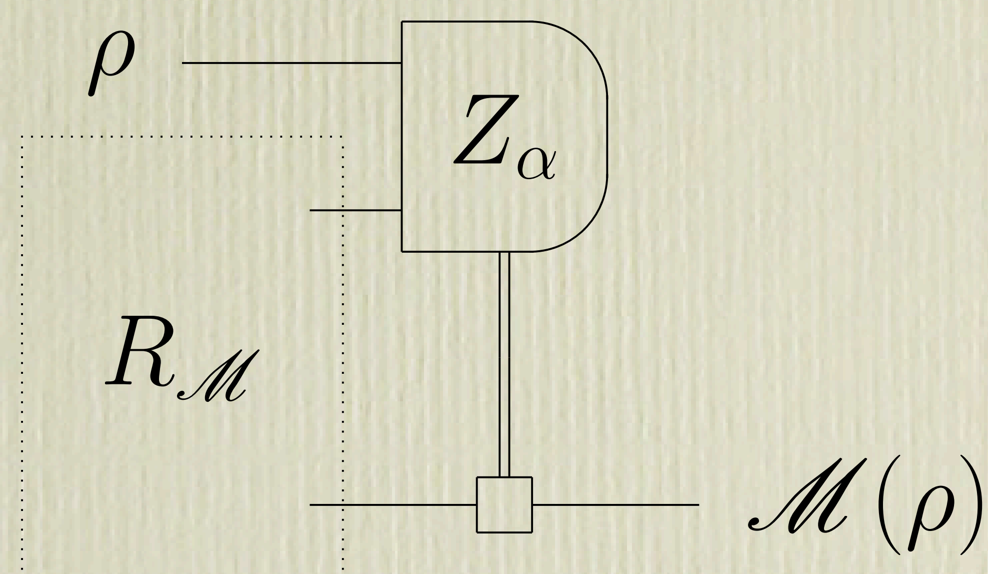
Programmability of operations



Probababilistic

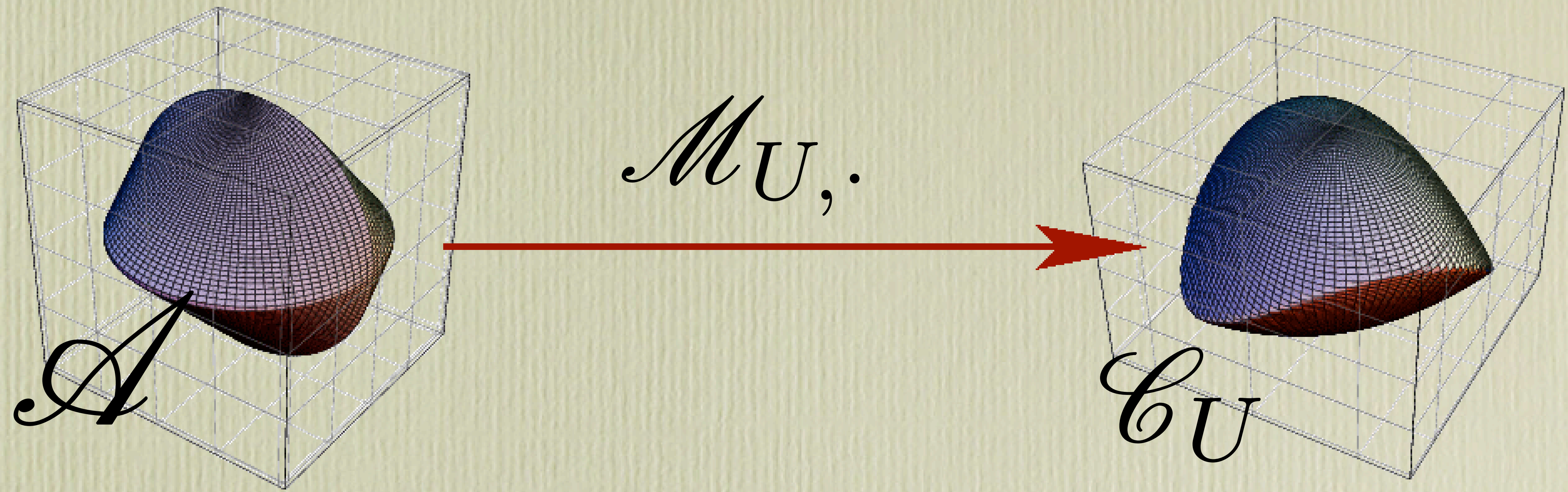
$$p\mathscr{M}(\rho) = \mathrm{Tr}_2[(I \otimes \rho^{\intercal})R_{\mathscr{M}}]$$
$$= \mathrm{Tr}_{23}[I \otimes |\Omega\rangle\!\rangle\langle\!\langle\Omega|)(R_{\mathscr{M}} \otimes \rho)]$$

$$R_{\mathscr{M}} = \mathscr{M} \otimes \mathscr{I}(I \otimes |\Omega\rangle\!\rangle\langle\!\langle\Omega|)$$



$$\Omega = \frac{1}{\sqrt{d}}I, \quad Z_0 = |\Omega\rangle\!\rangle\langle\!\langle\Omega|, \quad p = \frac{1}{d^2}$$

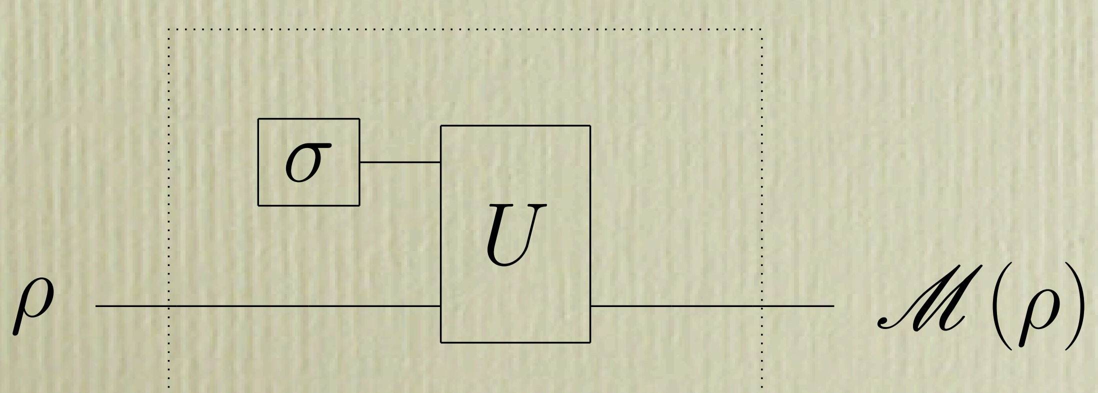
Programmability of operations



Deterministic

$$\mathscr{M}_{U,\sigma}(\rho) \doteq \mathrm{Tr}_2[U(\rho \otimes \sigma)U^\dagger]$$

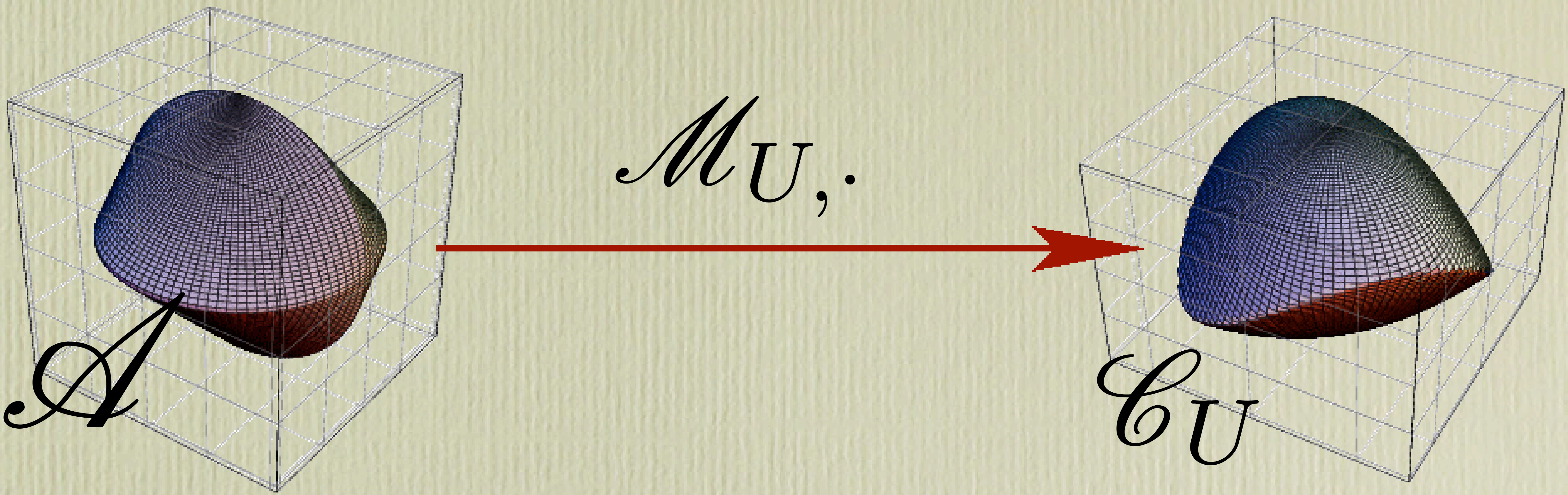
$$\mathscr{C}_U \doteq \mathscr{M}_{U,\mathscr{A}}$$



No go theorem (Nielsen-Chuang)

It is impossible to program all unitary channels with a single U and a finite-dimensional ancilla

Programmability of operations



Deterministic

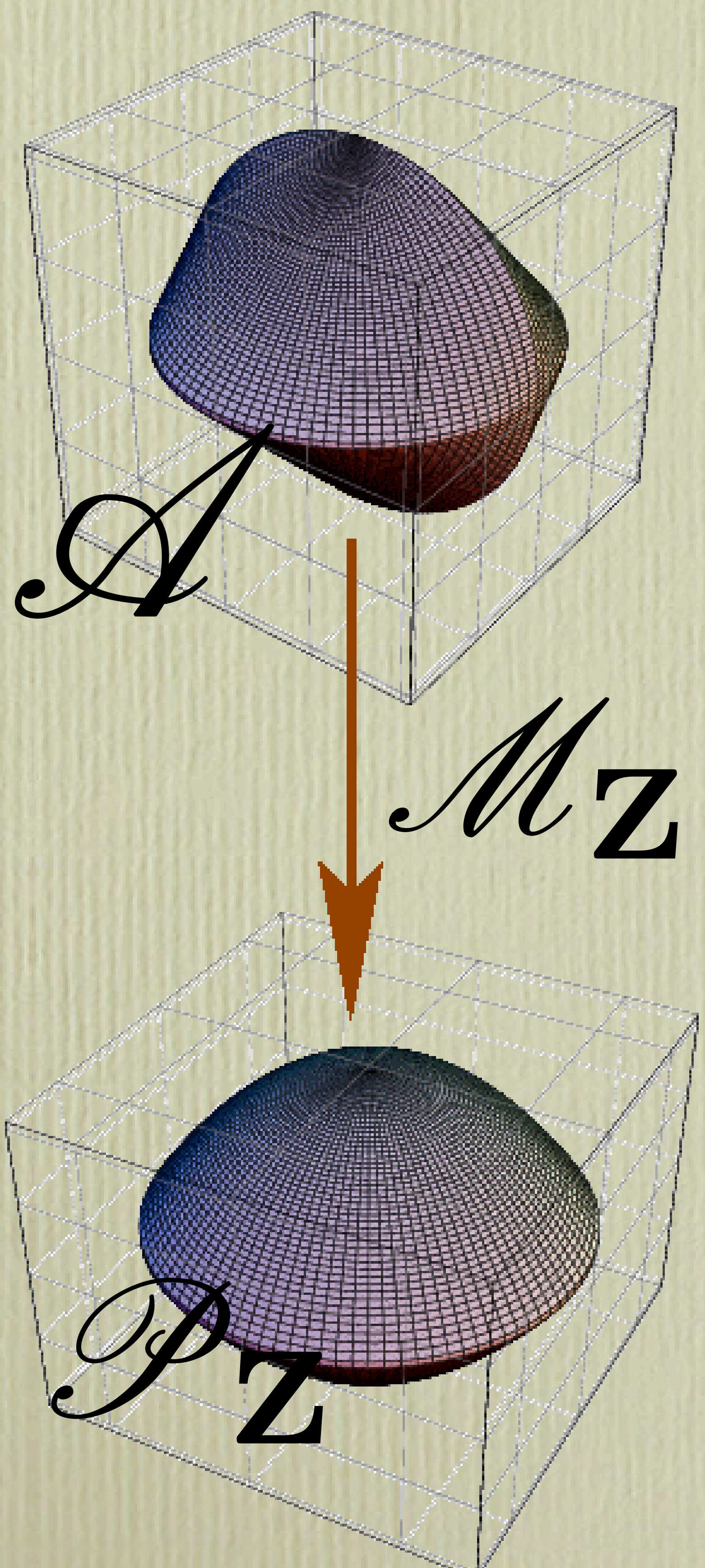
$$\mathscr{M}_{U,\sigma}(\rho) \doteq \mathrm{Tr}_2[U(\rho \otimes \sigma)U^\dagger]$$

$$\mathscr{C}_U \doteq \mathscr{M}_{U,\mathscr{A}}$$

Problem: *The "big U"*

For given $d = \dim(\mathcal{A})$ find the unitary operator U that maximizes the "size" of the convex set $\mathscr{C}_U$.

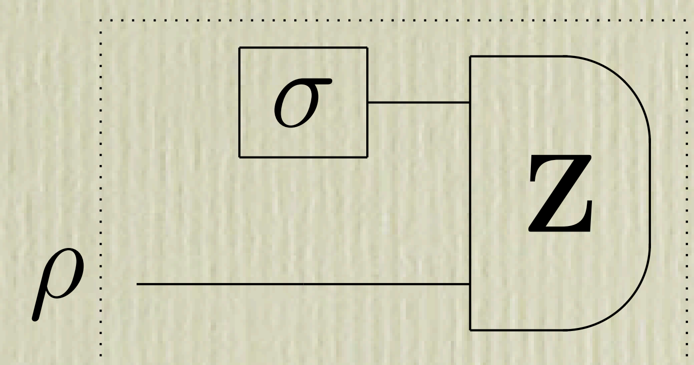
Programmability of POVMs



Deterministic

$$\mathscr{M}_{\mathbf{Z},\sigma} \doteq \mathrm{Tr}_2[(I \otimes \sigma)\mathbf{Z}] = \mathbf{P}$$

$$\mathscr{P}_{\mathbf{Z}} \doteq \mathscr{M}_{\mathbf{Z},\mathscr{A}}$$



No go theorem

It is impossible to program all observables with a single ***Z*** and a finite-dimensional ancilla

Programmability of POVMs

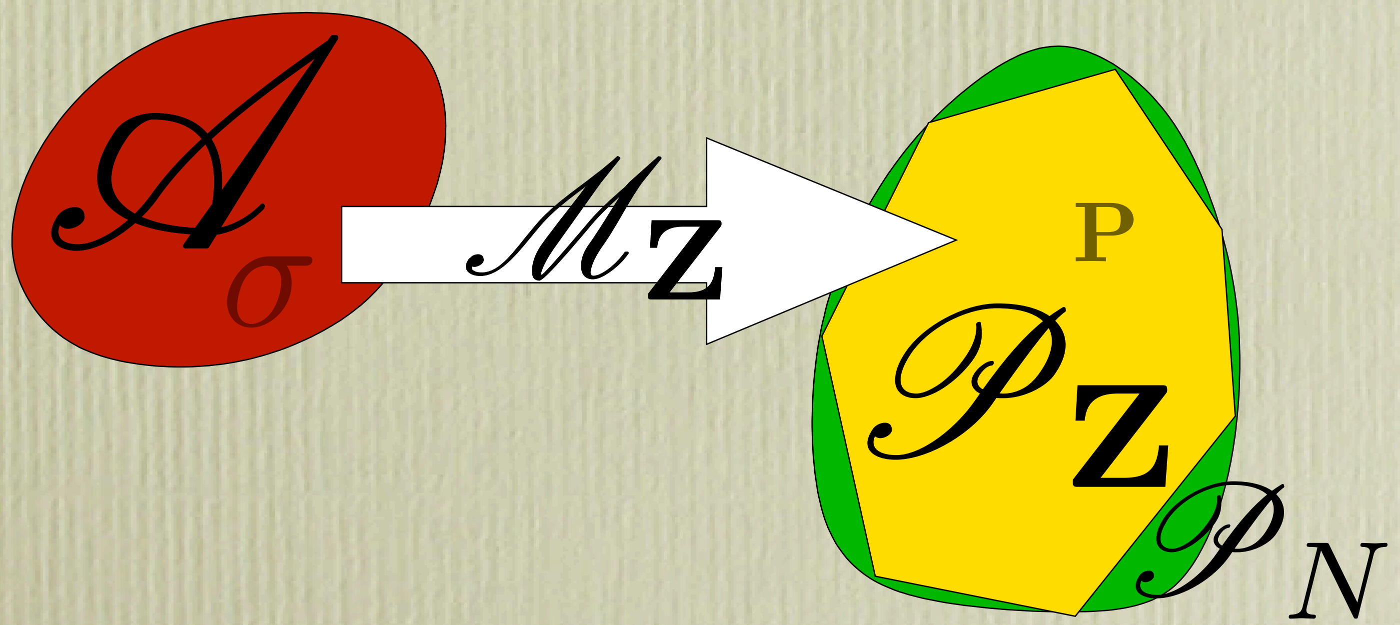
Deterministic

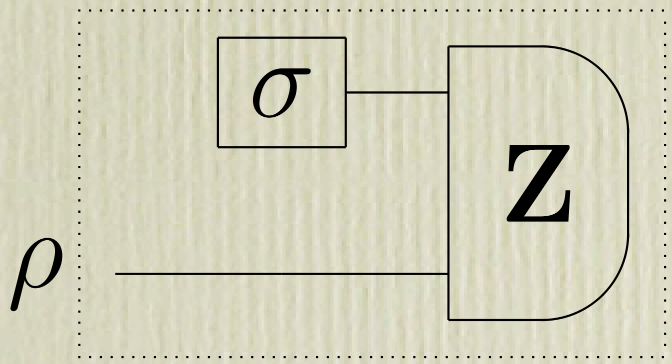
$$\mathscr{M}_{\mathbf{Z},\sigma} \doteq \mathrm{Tr}_2[(I \otimes \sigma)\mathbf{Z}] = \mathbf{P}$$

$$\mathscr{P}_{\mathbf{Z}} \doteq \mathscr{M}_{\mathbf{Z},\mathscr{A}}$$

No go theorem

It is impossible to program all observables with a single ***Z*** and a finite-dimensional ancilla





Programmability of POVMs

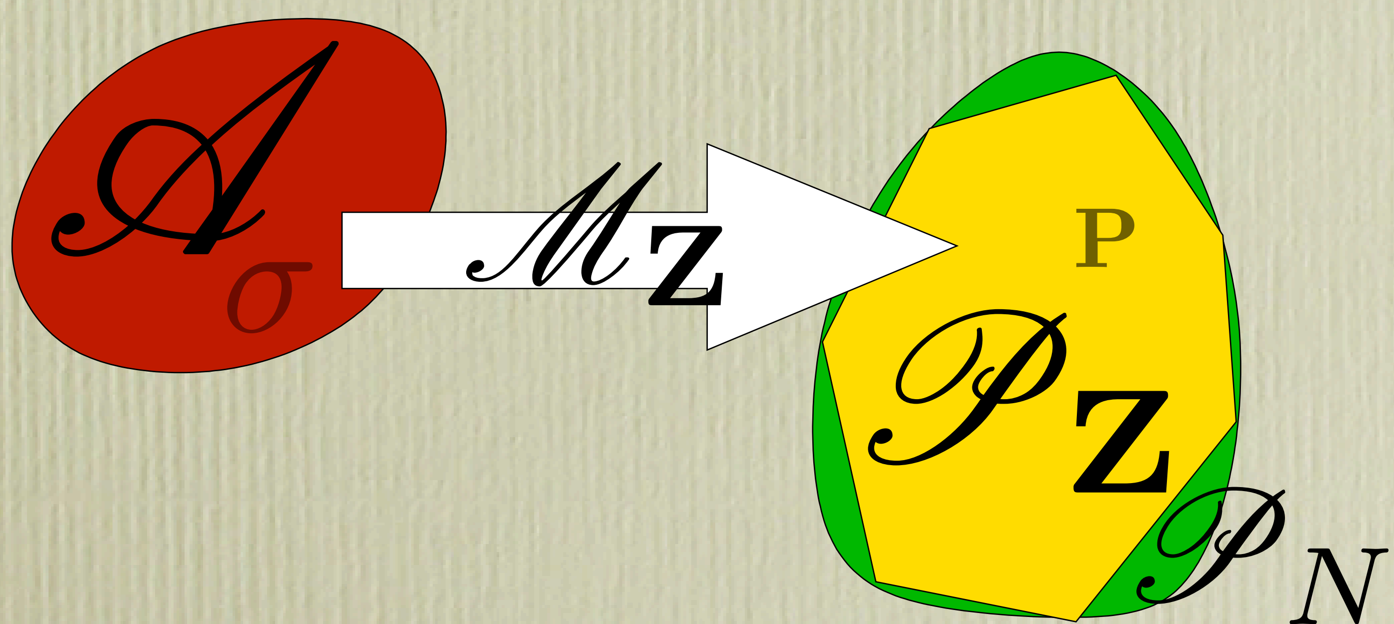
Deterministic

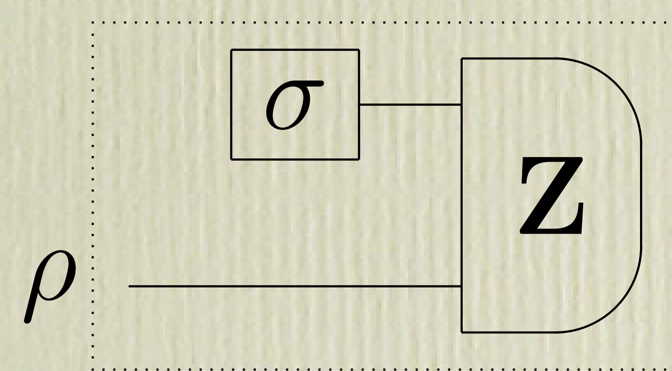
$$\mathscr{M}_{\mathbf{Z},\sigma} \doteq \mathrm{Tr}_2[(I \otimes \sigma)\mathbf{Z}] = \mathbf{P}$$

$$\mathscr{P}_{\mathbf{Z}} \doteq \mathscr{M}_{\mathbf{Z},\mathscr{A}}$$

Problem: *The "big* $\mathbf{Z}$*"*

For given $d = \dim(\mathcal{A})$ and $N = |\mathbf{Z}| = |\mathbf{P}|$, find the observable $\mathbf{Z}$ that maximizes the "size" of the convex set $\mathscr{P}_{\mathbf{Z}}$.





Programmability of POVMs

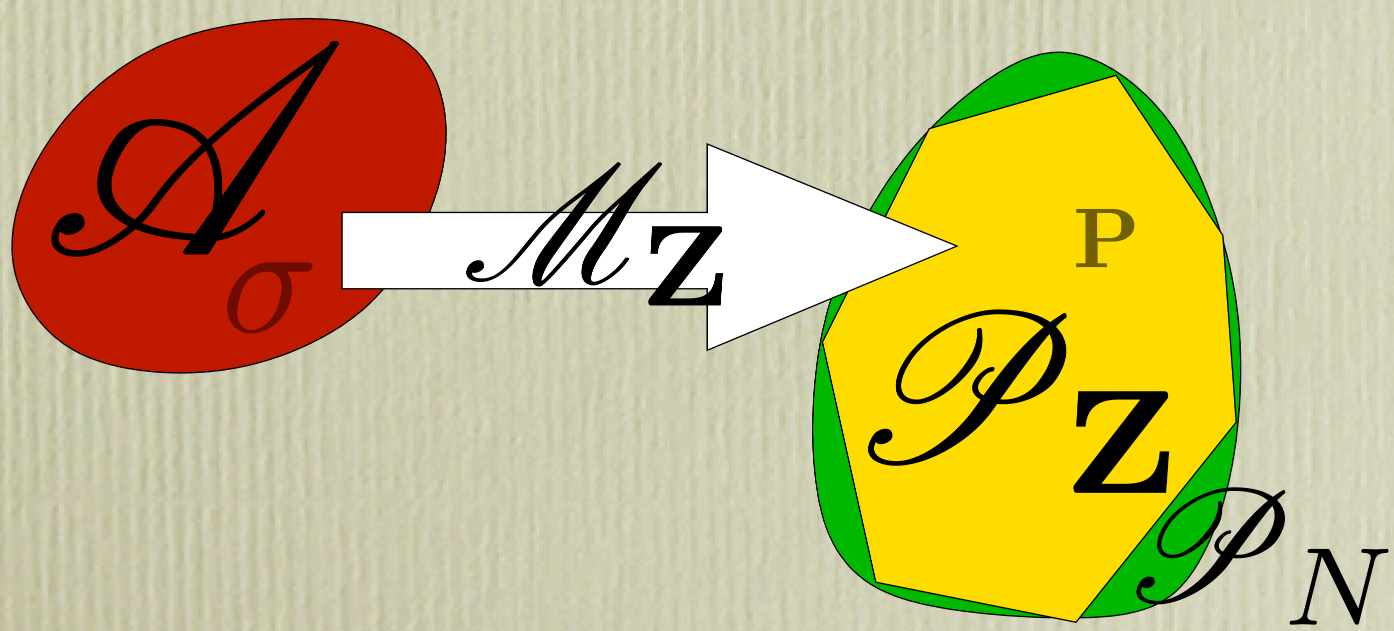
Deterministic

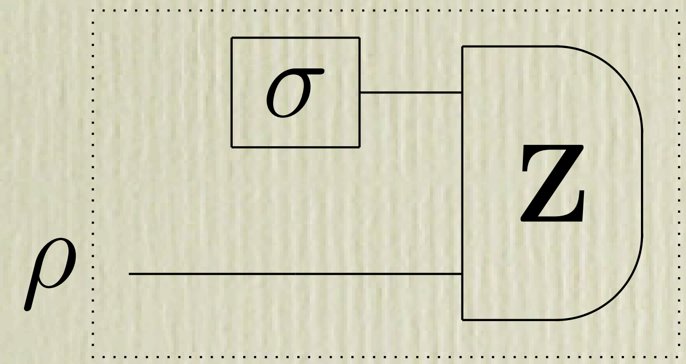
$$\mathscr{M}_{\mathbf{Z},\sigma} \doteq \mathrm{Tr}_2[(I \otimes \sigma)\mathbf{Z}] = \mathbf{P}$$

$$\mathscr{P}_{\mathbf{Z}} \doteq \mathscr{M}_{\mathbf{Z},\mathscr{A}}$$

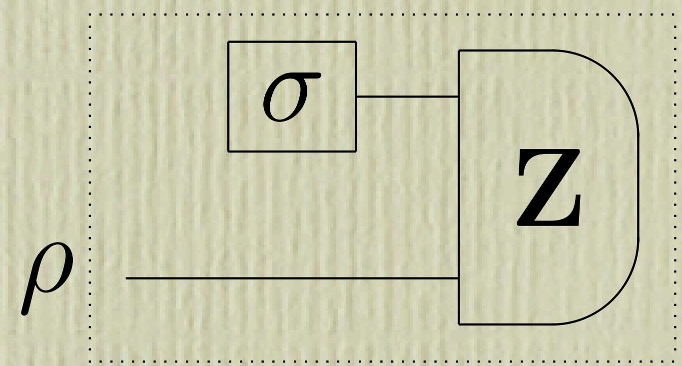
A "measure" of the **green** region can be given in terms of the **accuracy** ε^{-1} of the programmability

$$\varepsilon \doteq \max_{\mathbf{P}\in\mathscr{P}_N} \min_{\mathbf{Q}\in\mathscr{P}_{\mathbf{Z}}} \delta(\mathbf{P},\mathbf{Q})$$





Approximate programmability



programmability with **accuracy** ε^{-1}:

$$\varepsilon \doteq \max_{\mathbf{P}\in\mathscr{P}_N} \min_{\mathbf{Q}\in\mathscr{P}_{\mathbf{Z}}} \delta(\mathbf{P},\mathbf{Q})$$

$$\delta(\mathbf{P},\mathbf{Q}) = \max_{\rho} \sum_i |\operatorname{Tr}[\rho(P_i - Q_i)]|$$

Using a joint observable $\mathbf{Z}$ of the form

Controlled-U

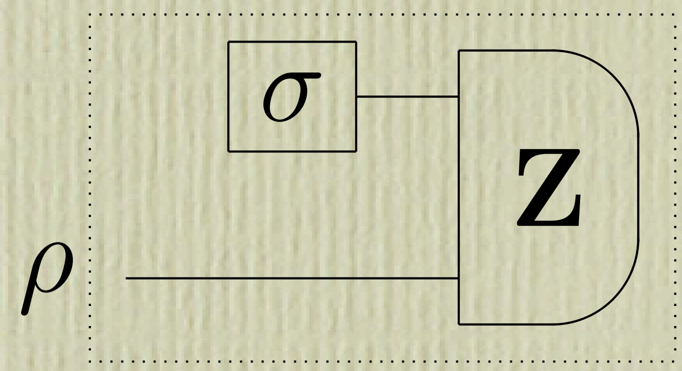
$$Z_i = U^\dagger(|\psi_i\rangle\langle\psi_i| \otimes I_A)U, \qquad U = \sum_{k=1}^{\dim(\mathcal{A})} W_k \otimes |\phi_k\rangle\langle\phi_k|$$

with $\{\psi_i\}$ and $\{\phi_k\}$ orthonormal sets and W_k unitary, we can program observables with accuracy ε^{-1} using an ancilla with **polynomial** growth

$$\dim(\mathcal{A}) \leqslant \kappa(N) \left(\frac{1}{\varepsilon}\right)^{N(N-1)}$$

G. M. D'Ariano, **P. Perinotti**, submitted to Phys. Rev. Lett. (quant-ph0410169)

Approximate programmability



programmability with **accuracy** ε^{-1}:

$$\varepsilon \doteq \max_{\mathbf{P}\in\mathscr{P}_N} \min_{\mathbf{Q}\in\mathscr{P}_\mathbf{Z}} \delta(\mathbf{P},\mathbf{Q})$$

$$\delta(\mathbf{P},\mathbf{Q}) = \max_{\rho} \sum_i |\operatorname{Tr}[\rho(P_i - Q_i)]|$$

polynomial growth

$$\dim(\mathcal{A}) \leqslant \kappa(N) \left(\frac{1}{\varepsilon}\right)^{N(N-1)}$$

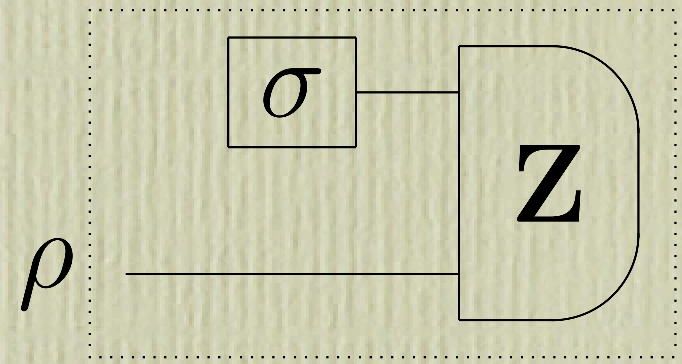
to be compared with the best formerly known **exponential** growth (Fiurasek)

$$\dim(\mathcal{A}) = \frac{1}{2} 4^{\varepsilon^{-1}}$$

G. M. D'Ariano, **P. Perinotti**, submitted to Phys. Rev. Lett. (quant-ph0410169)

Approximate programmability

For qubits: *linear* growth!



Program for the observable $\mathbf{P} = \{U_g|\pm\frac{1}{2}\rangle\langle\pm\frac{1}{2}|U_g^\dagger\}$

$$\sigma = V_g|jj\rangle\langle jj|V_g^\dagger$$

in dimension $\dim(\mathcal{A}) = 2j+1$, with joint observable

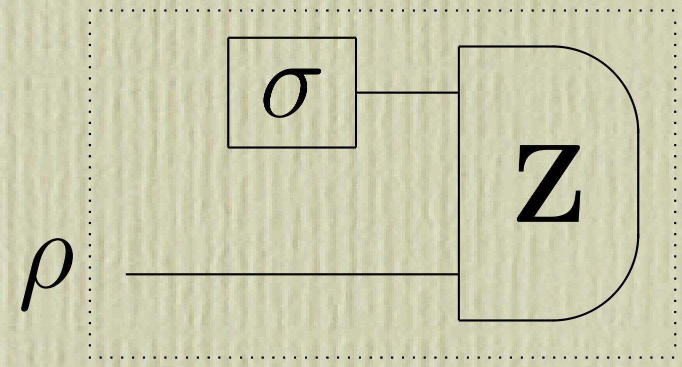
$$\mathbf{Z} = \{\Pi_{j\pm\frac{1}{2}}\}$$

gives the programmability accuracy

$$\varepsilon = \delta(\mathbf{P},\mathbf{Q}) = \frac{2}{2j+1} \quad \Rightarrow \quad \dim(\mathcal{A}) = 2\varepsilon^{-1}$$

G. M. D'Ariano, **P. Perinotti**, submitted to Phys. Rev. Lett. (quant-ph0410169)

Exact programmability



Covariant measurements are exactly programmable

G-covariant POVM densities (Holevo theorem)

$$P_g \, \mathrm{d}\, g = U_g \xi U_g^\dagger \, \mathrm{d}\, g, \qquad g \in \mathbf{G}$$

programmable as

$$P_g = \mathrm{Tr}_2[(I \otimes \sigma) F_g], \qquad \xi = V \sigma^\intercal V^\dagger$$

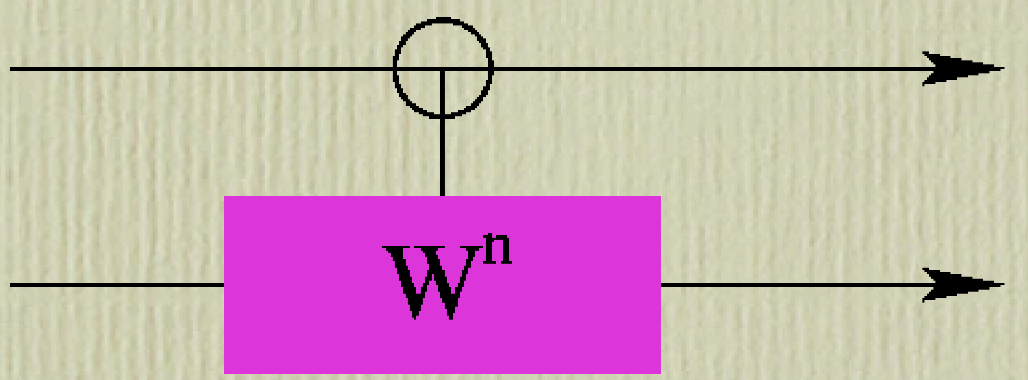
with covariant Bell POVM density

$$F_g = (U_g \otimes I) |V\rangle\!\rangle \langle\!\langle V| (U_g^\dagger \otimes I)$$

G. M. D'Ariano, **P. Perinotti**,
Phys. Lett. A **329** 188 (2004)

Bell from local observables

Unitary operator U connecting the Bell observable with local observables



$$U(|m\rangle \otimes |n\rangle) = \frac{1}{\sqrt{d}} |U_{m,n}\rangle\rangle$$

of the controlled-U form

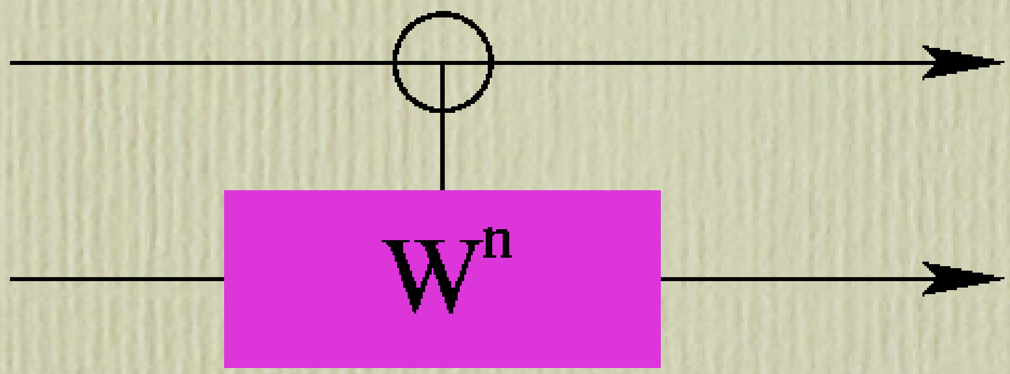
Controlled-U

$$U = \sum_n |n\rangle\langle n| \otimes W^n$$

e. g. for projective d-dimensional UIR of the Abelian group $\mathbf{G} = \mathbf{Z}_d \times \mathbf{Z}_d$

$$U_{m,n} = Z^m W^n, \quad Z = \sum_j \omega^j |j\rangle\langle j|, \quad W = \sum_k |k\rangle\langle k \oplus 1|, \quad \omega = e^{\frac{2\pi i}{d}}.$$

Bell from local observables



Unitary operator U connecting the Bell observable with local observables

$$U(|m\rangle \otimes |n\rangle) = \frac{1}{\sqrt{d}}|U_{m,n}\rangle\rangle$$

Problem: *The "Bell-izing U's"*

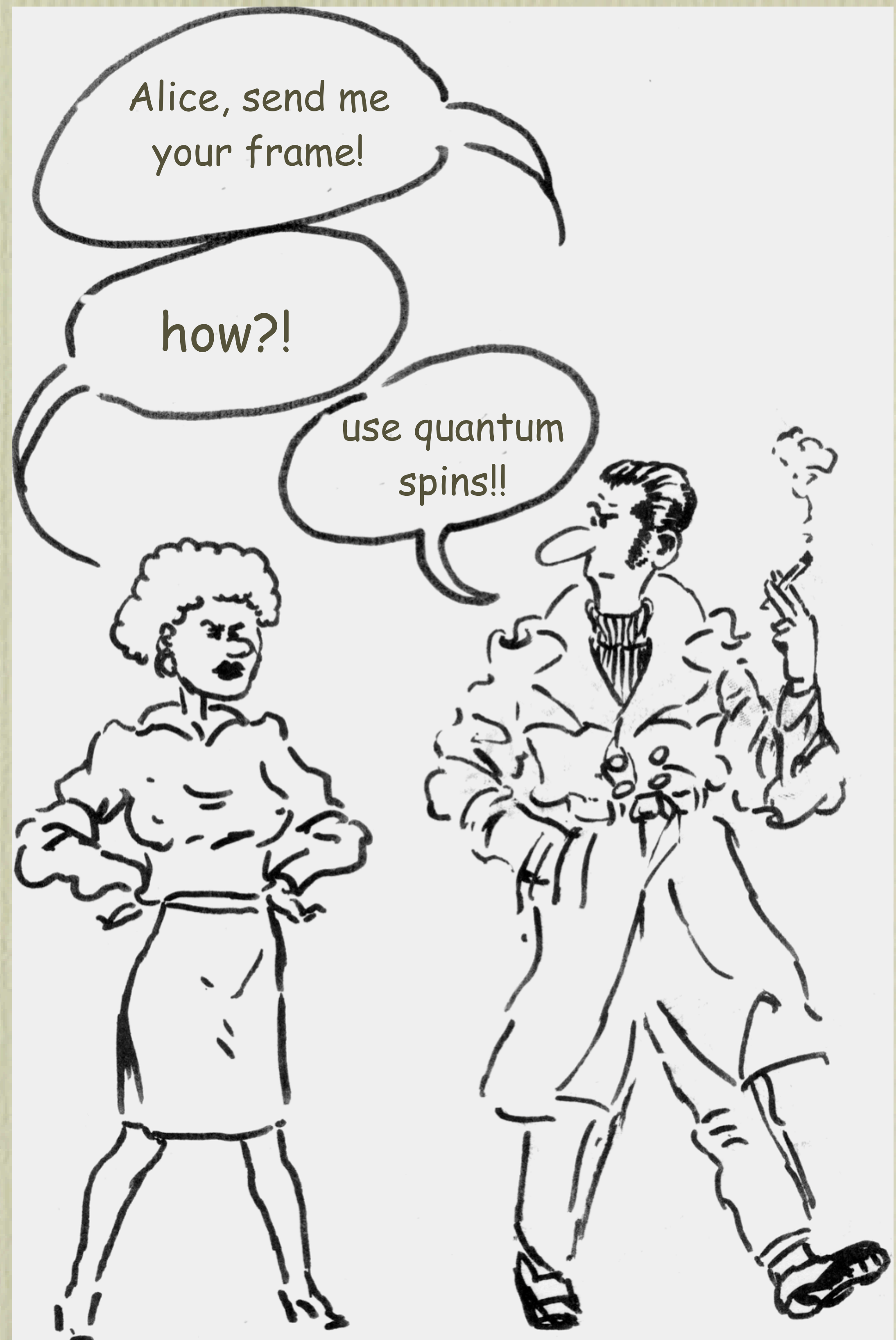
Find the unitary operators U that connect a fixed separable orthonormal basis to any Bell orthonormal basis

Problem: *The "Bell basis classification"*

Classify all Bell orthonormal basis.

Equivalently: classify all orthonormal basis of unitary operators.

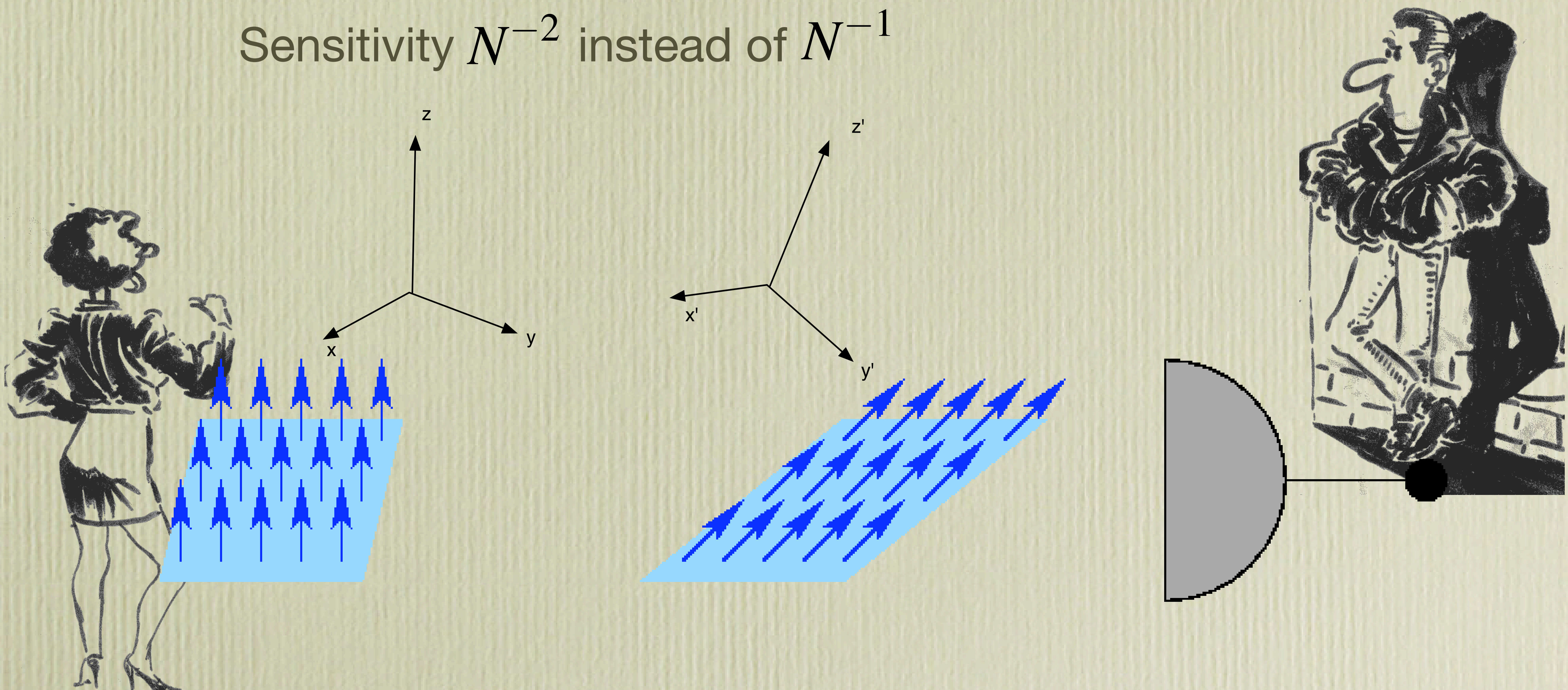
Transmission of frames



G. Chiribella, G. M. D'Ariano, P. Perinotti, and M. Sacchi, Phys. Rev. Lett. **93** 180503 (2004)

Transmission of frames

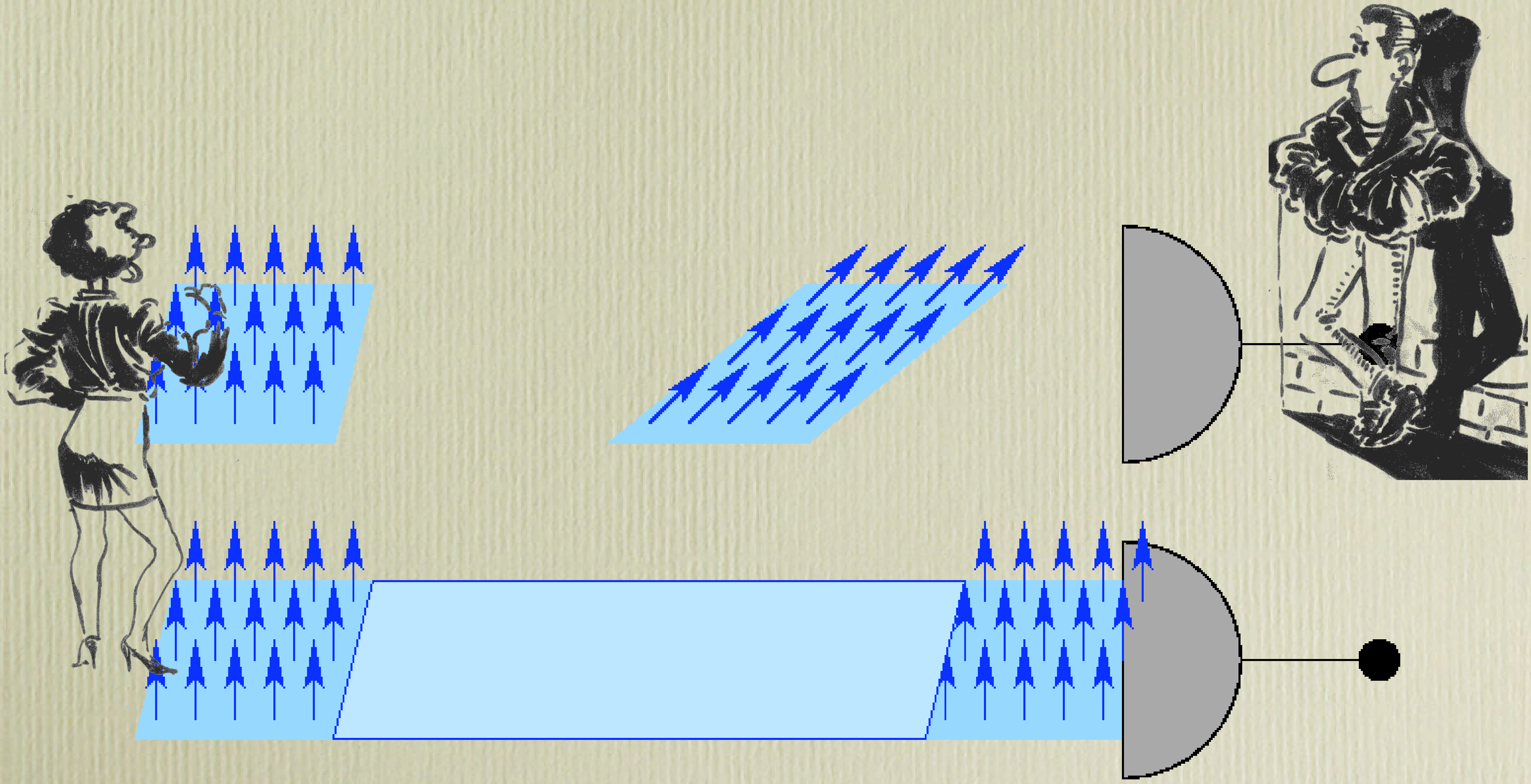
Sensitivity N^{-2} instead of N^{-1}



$$H^{\otimes N} = \oplus_\nu (H_\nu \otimes \mathbb{C}^{m_\nu})$$

Transmission of frames

No need of shared entanglement!



Transmission of frames

- Use N spins that can carry information about the rotation g_* that connects the two frames
- Alice prepares N spins in $|A\rangle$
- She sends the spins to Bob who receives

$$|A_{g^*}\rangle = U_{g*}^{\otimes N}|A\rangle$$

- Bob performs a measurement to infer g_* and rotates his frame by the estimated rotation g

G. Chiribella, G. M. D'Ariano, P. Perinotti, and M. Sacchi, Phys. Rev. Lett. **93** 180503 (2004)

Transmission of frames

The deviation between estimated and true axes is

$$e(g, g_*) = \sum_{\alpha=x,y,z} |g n_\alpha^B - g_* n_\alpha^B|^2$$

The state and the measurement are chosen in order to minimize the average transmission error

$$\langle e \rangle = \int dg_* \int dg \, p(g|g_*) \, e(g, g_*)$$

The previous literature claimed as optimal an asymptotic sensitivity $\propto 1/N$

BUT... the use of equivalent irreducible representations dramatically improves the sensitivity up to $\propto 1/N^2$!

Optimal solution: write the Clebsch-Gordan decomposition

$$H^{\otimes N} \equiv \oplus_{j=0}^{J} H_j \otimes M_j \qquad J = N/2$$

G. Chiribella, G. M. D'Ariano, P. Perinotti, and M. Sacchi, Phys. Rev. Lett. **93** 180503 (2004)

Transmission of frames

Choose a state of the form

$$|A\rangle = A_J|JJ\rangle + \sum_{j=0}^{J-1} A_j|I_j\rangle\rangle$$

with $$|I_j\rangle\rangle = \frac{1}{\sqrt{2j+1}} \sum_{m=-j}^{j} |jm\rangle \otimes |m\rangle$$

Entanglement between representation and multiplicity space, but no shared entanglement between Alice and Bob

Bob's measurement that maximizes the likelihood

$$U_g^{\otimes N}|B\rangle\langle B|U_g^{\dagger\otimes N}$$

with $$|B\rangle = \sqrt{2J+1}|JJ\rangle + \sum_{j=0}^{J-1}(2j+1)|I_j\rangle\rangle$$

G. Chiribella, G. M. D'Ariano, P. Perinotti, and M. Sacchi, Phys. Rev. Lett. **93** 180503 (2004)

Transmission of frames

Comparison of the protocol exploiting equivalent representation with the optimal one without equivalent representations

Number of spins	$\langle e \rangle_{with}$	$\langle e \rangle_{without}$
N= 3	1.6114	1.8138
N= 5	0.9136	1.3292

Asymptotic behavior for large N:

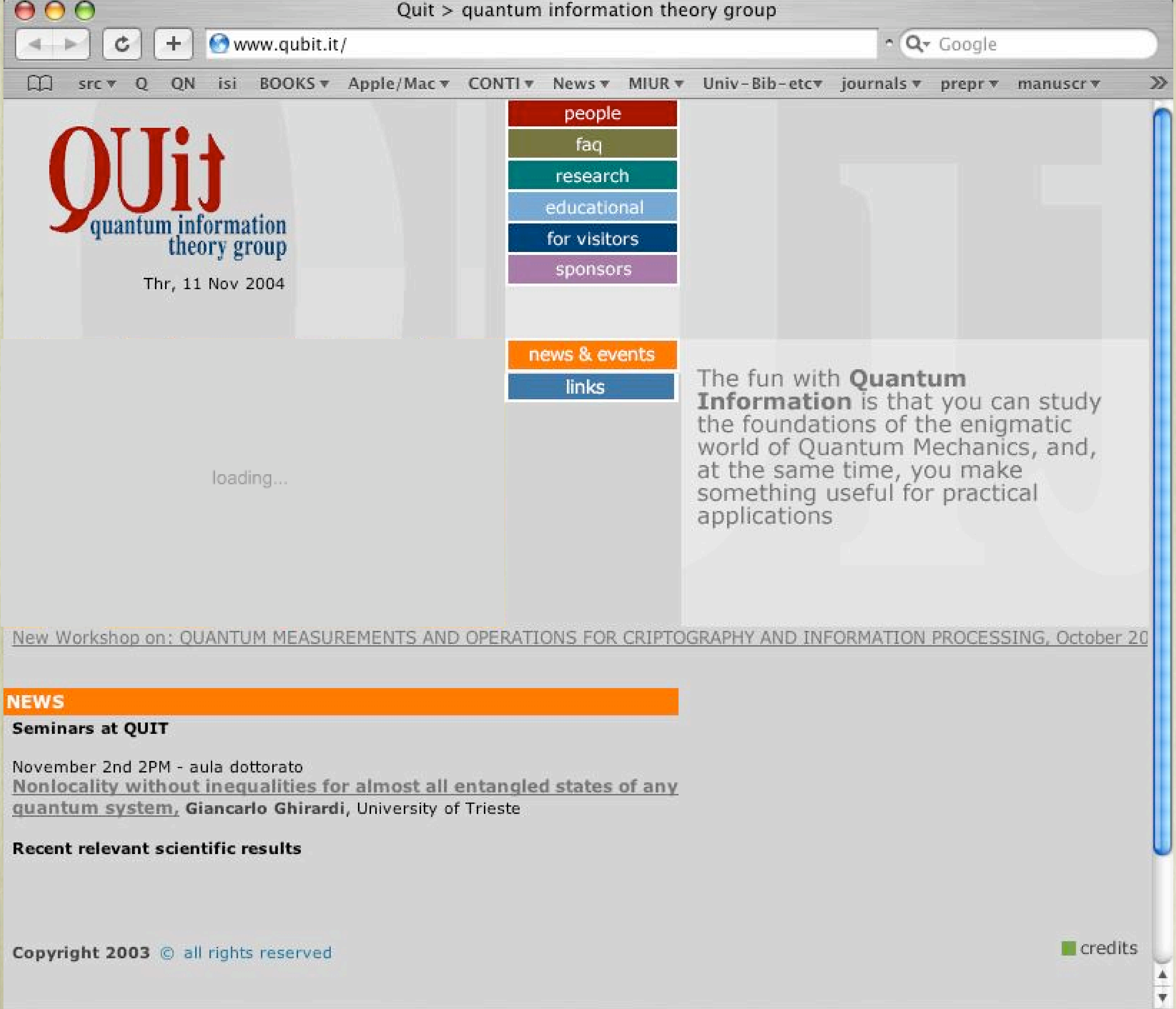
$$\langle e \rangle_{with} \sim \frac{8\pi^2}{N^2} \qquad \langle e \rangle_{without} \sim \frac{8}{N}$$

Note remarkable increase of transmission efficiency due to equivalent representations.

G. Chiribella, G. M. D'Ariano, P. Perinotti, and M. Sacchi, Phys. Rev. Lett. **93** 180503 (2004)

Conclusions

- Convex structures of POVM's and channels
- Quantum calibration of channels and detectors
- Programmable channels and detectors
  - Open problems:
    - *Big U and Big* ***Z***
    - *Bell-izing U's*
- Transmission of reference frames with high sensitivity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