Informationally complete measurements and universal detectors

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  1. special quantum states

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2. **Programmable quantum detectors**

- **Goal:** establish the *minimal set of resources* in terms of:

  1. special quantum states
  2. special measurements
  3. special unitary transformations

Informationally complete measurements

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For an informationally complete POVM $\{\Xi_i\}$ one must have

$$\mathrm{Tr}[\rho O] = \sum_i f_i(O)\mathrm{Tr}[\rho \Xi_i],$$

- $f_i(O)$ **data-processing** for O.

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for a suitable ***data-processing*** $f_i(\nu, O)$ of the outcome i.

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- Relation with **informationally complete POVM**

$$\mathrm{Tr}[\rho O] = \sum_i f_i(\nu, O)\mathrm{Tr}[\rho \Xi_i[\nu]], \qquad \Xi_i[\nu] \doteq \mathrm{Tr}_2[(I \otimes \nu)\Pi_i].$$

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Notation for entangled states

- Hilbert-Schmidt isomorphism: $|\Psi\rangle\rangle \in \mathsf{H} \otimes \mathsf{K} \iff \Psi$ operator from K to H

$$|\Psi\rangle\rangle = \sum_{nm} \Psi_{nm} |n\rangle \otimes |m\rangle \qquad \iff \qquad \Psi = \sum_{nm} \Psi_{nm} |n\rangle\langle m|.$$

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$$\begin{aligned}
(A \otimes B)|C\rangle\!\rangle &= |AC\,B^\intercal\rangle\!\rangle, \\
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(U \otimes U^*)|I\rangle\!\rangle &= |I\rangle\!\rangle, \qquad U^* \doteq (U^\dagger)^\intercal.
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- Partial trace rules

$$\mathrm{Tr}_{\mathsf{K}}[|A\rangle\rangle\langle\langle B|] = AB^\dagger\,, \qquad \mathrm{Tr}_{\mathsf{H}}[|A\rangle\rangle\langle\langle B|] = (B^\dagger A)^\tau\,,$$

Frames of operators

- A sequence of operators $\{\Xi_i\}$ is a frame for a Banach space of operators if there are constants $0 < a \leq b < +\infty$ s.t. for all operators A one has

$$a\|A\|^2 \leq \underbrace{\sum_i |\langle A, \Xi_i \rangle|^2}_{\text{Bessel series}} \leq b\|A\|^2.$$

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Hilbert-Schmidt: $\langle \Theta_i, A\rangle \doteq \mathrm{Tr}[\Theta_i^\dagger A]$.

- The sequence of operators $\{\Xi_i\}$ is a frame iff the following operator on H $\otimes$ K is bounded and invertible

$$F = \sum_i |\Xi_i\rangle\!\rangle \langle\!\langle \Xi_i| \ . \qquad \text{(frame operator)}$$

Frames of operators

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$$|\Theta_i\rangle\rangle = F^{-1}|\Xi_i\rangle\rangle + |Y_i\rangle\rangle - \sum_j \langle\langle \Xi_j|F^{-1}|\Xi_i\rangle\rangle|Y_j\rangle\rangle \ ,$$

Y_i arbitrary Bessel, and $F^{-1}|\Xi_i\rangle\rangle$ canonical dual frame.

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- For *exact* frames there is only the canonical dual frame.
- Alternate duals are useful for optimization.

Universal quantum detectors

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namely $\{\Xi_i[\nu]\}$ is a **positive frame**, and the dual frame provides the data-processing rule

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[D'Ariano, Perinotti and Sacchi, Europhys. Lett. **65** *165 (2004)]*

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- The POVM $\{\Xi_i[\nu]\}$ is necessarily not orthogonal.

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Upon diagonalizing the POVM $\{\Pi_i\}$ on $\mathsf{H} \otimes \mathsf{K}$

$$\Pi_i = \sum_{j=1}^{r_i} |\Psi_j^{(i)}\rangle\!\rangle\langle\!\langle\Psi_j^{(i)}| \,,$$

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- It follows that $\{\Pi_i\}$ is universal iff both $\{\Psi_j^{(i)}\}$ and $\{\Xi_i[\nu]\}$ are operator frames.

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Universal POVM's: the Bell abelian case

POVM on H $\otimes$ H : $\quad \Pi_i = \frac{\alpha_i}{d} |U_i\rangle\rangle \langle\langle U_i|, \quad d = \dim(\mathsf{H}),\ \alpha_i > 0,\ U_i$ unitary.

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- One can prove that the Bell POVM is necessarily orthogonal, and is universal for ancilla state ν such that $\mathrm{Tr}[U_\alpha^\dagger \nu^\tau]\neq 0$ for all α.
- Dual set (unique) for data-processing:

$$\Theta_\alpha[\nu] = \frac{1}{d}\sum_{\beta=1}^{d^2}\frac{U_\beta e^{-ic(\beta,\alpha)}}{\mathrm{Tr}\left[U_\beta\nu^*\right]}\ .$$

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$$P \doteq \tfrac{1}{d}|I\rangle\!\rangle\langle\!\langle I|, \qquad a = \frac{d^2-1}{d\,\mathrm{Tr}[(\nu^\tau)^2]-1},$$

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$$\Theta_\alpha^0[\nu] = a\,U_\alpha\,\nu^\tau U_\alpha^\dagger + b\,I\,, \qquad b = \frac{\mathrm{Tr}[(\nu^\tau)^2]-d}{d\mathrm{Tr}[(\nu^\tau)^2]-1}.$$

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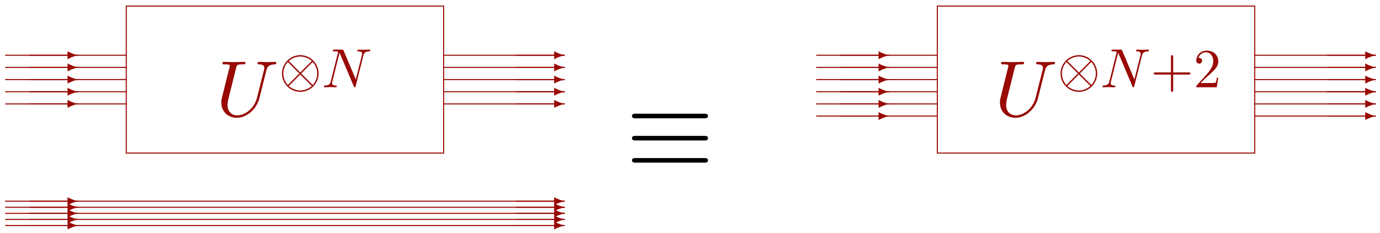
- **Other examples:** $SU(2)$ UIR's on H with $\dim(\mathrm{H}) > 2$, ...

Estimation of unitaries with multiple copies



- There is no need of entanglement assistance, since one can use entanglement bewteen copies in the input state.

- Entanglement is internal between the irrep. space and the multiplicity space.

- Fidelity can be improved from $F \sim N^{-1}$ to $F \sim N^{-2}$.

[Chiribella, D'Ariano, Perinotti and Sacchi, (in preparation)]

Universal POVM's: the separable case

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[G. M. D'Ariano, Phys. Lett. A **268** *151 (2000)]*

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$$\Rightarrow \quad \text{tomography + ancillary } \textit{quantum roulette}.$$

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- Data-processing function:

$$f_{k,l}(\nu, O) = \frac{\mathrm{Tr}[C^\dagger(l) O]}{\langle l|\nu|l\rangle} c_k(l), \qquad \langle l|\nu|l\rangle \neq 0 \ \forall l.$$

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Universal POVM's: open problems

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2. Methods for generating positive operator frames from complex operator frames.

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8. *Weakly universal* POVM's: the ancilla state ν depends on the operator O to be estimated.

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Bell measurement from local measurements

- Bell measurement corresponding to the projective UIR of the Abelian group in d dimensions: $\mathbf{G} = \mathbf{Z}_d \times \mathbf{Z}_d$

$$U(m,n) = Z^m W^n, \quad Z = \sum_j \omega^j |j\rangle\langle j|, \quad W = \sum_k |k\rangle\langle k \oplus 1|, \quad \omega = e^{\frac{2\pi i}{d}}.$$

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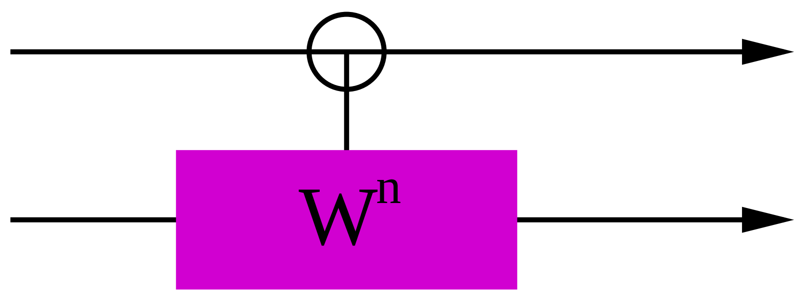

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$$X_n = U^\dagger |n\rangle\langle n| U \simeq Z_n^{(\nu)} \doteq \mathrm{Tr}_1[V^\dagger (I \otimes |n\rangle\langle n|) V (\nu \otimes I)]$$

where the observables are *close* in term of the physical distance

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Subject Index