

Superbroadcasting of conjugate quantum variables

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Abstract. – We consider the problem of broadcasting arbitrary states of radiation modes from N to $M > N$ copies by a map that preserves the average value of the field and optimally reduces the total noise in conjugate variables. For $N \geq 2$ the broadcasting can be achieved perfectly, and for sufficiently noisy input states one can even purify the state while broadcasting—the so-called *superbroadcasting*. For purification (*i.e.* $M \leq N$), the reduction of noise is independent of M . Similar results are proved for broadcasting with phase-conjugation. All the optimal maps can be implemented by linear optics and linear amplification.

The impossibility of exact quantum cloning, namely copying the unknown state of a quantum system to a larger number of copies [1], has stimulated the search for quantum devices that can emulate cloning with the highest possible fidelity. After the simplest case of qubits [2–4] many optimal cloners have been found, for general finite-dimensional systems [5], restricted sets of input states [6, 7], and infinite-dimensional systems such as harmonic oscillators—the so-called continuous variables cloners [8]. However, when considering mixed states, a less stringent type of cloning transformation can be used—so-called *broadcasting*—in which the output copies are in a globally correlated state whose local reduced states are identical to the input states. This issue has been considered in ref. [9], where it has been shown that broadcasting a single copy from a noncommuting set of density matrices is always impossible. Later, this result has been considered in the literature as the generalization of the no-cloning theorem to mixed states. However, more recently, for qubits an effect called *superbroadcasting* [10] has been discovered, which consists in the possibility of broadcasting the state while even increasing the purity of the local state, for at least $N \geq 4$ input copies, and for sufficiently short input Bloch vector (and even for $N = 3$ input copies for phase-covariant instead of universal covariant broadcasting [11]).

In the present letter, we study the broadcasting of conjugate quantum variables $x_a = \frac{a+a^\dagger}{2}$ and $y_a = \frac{a-a^\dagger}{2i}$, where a and a^\dagger are the customary annihilation and creation operators for the harmonic oscillator (or single-mode radiation field), *i.e.* $[a, a^\dagger] = 1$. We look for the map that from N uncorrelated states with the same complex amplitude provides $M > N$ states, while preserving the amplitude and optimally reducing the total noise in conjugate quadratures. We derive a bound from the quantum limits on noise in *linear* amplifiers that can be easily

achieved experimentally. We will show indeed that such a bound cannot be overcome even for general nonlinear transformations (*i.e.* allowing for arbitrary quantum operations). Explicit examples will be given for displaced thermal states, which are equivalent to coherent states that have suffered Gaussian noise.

As we will see, superbroadcasting is possible for continuous variables for $N \geq 2$, namely one can produce a larger number M of purified copies at the output, locally on each use, and with the same amplitude of the input copies N . For displaced thermal states, *e.g.*, N to M superbroadcasting can be achieved for input thermal photon number $\bar{n}_{in} > \frac{M-N}{M(N-1)}$. For purification (*i.e.* $M \leq N$), quite surprisingly the purification rate is $\bar{n}_{out}/\bar{n}_{in} = N^{-1}$, independently of M . We mention that the particular case of $2 \rightarrow 1$ purification for noisy coherent states has been reported in ref. [12]. We will also consider the optimal broadcasting with phase-conjugate output, showing analogous effects.

From N uncorrelated modes a_0, a_1, \dots, a_{N-1} with

$$\langle a_i \rangle = \alpha, \quad \Delta x_{a_i}^2 + \Delta y_{a_i}^2 = \gamma_i, \quad (1)$$

a broadcasting transformation provides $M > N$ (generally correlated) modes b_0, b_1, \dots, b_{M-1} , with the same complex amplitude and noise Γ , *i.e.*

$$\langle b_i \rangle = \alpha, \quad \Delta x_{b_i}^2 + \Delta y_{b_i}^2 = \Gamma, \quad (2)$$

and we are looking for the minimal Γ . This can be obtained by applying a fundamental theorem for linear amplifiers: the sum of the uncertainties of conjugate quadratures of an amplified mode with (power) gain G is bounded as follows [14]

$$\Delta X_B^2 + \Delta Y_B^2 \geq G(\Delta X_A^2 + \Delta Y_A^2) + \frac{|G \mp 1|}{2}, \quad (3)$$

where the upper (lower) sign holds for phase-preserving (phase-conjugating) amplifiers, and A and B denote the input and the amplified mode, respectively. In fact, our transformation can be seen as a phase-preserving amplification from mode $A = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} a_i$ to mode $B = \frac{1}{\sqrt{M}} \sum_{i=0}^{M-1} b_i$ with gain $G = \frac{M}{N}$, and hence eq. (3) should hold. Notice that generally for any mode c one has

$$\Delta x_c^2 + \Delta y_c^2 = \frac{1}{2} + \langle c^\dagger c \rangle - |\langle c \rangle|^2. \quad (4)$$

In the present case, since modes a_i are uncorrelated, from eqs. (1) and (4) we have $\langle A^\dagger A \rangle = \gamma + N|\alpha|^2 - \frac{1}{2}$, with $\gamma = \frac{1}{N} \sum_{i=0}^{N-1} \gamma_i$, whereas from eqs. (2) and (4)

$$\langle B^\dagger B \rangle = \frac{1}{M} \sum_{i,j=0}^{M-1} \langle b_i^\dagger b_j \rangle \leq \frac{1}{M} \sum_{i,j=0}^{M-1} \sqrt{\langle b_i^\dagger b_i \rangle \langle b_j^\dagger b_j \rangle} = M \left(\Gamma + |\alpha|^2 - \frac{1}{2} \right). \quad (5)$$

From eqs. (3), (4) we obtain the following bound for the noise Γ :

$$\Gamma - \frac{1}{2} \geq \frac{1}{N} \left(\gamma - \frac{1}{2} \right) + \frac{1}{N} - \frac{1}{M}. \quad (6)$$

Notice that $\gamma, \Gamma \geq \frac{1}{2}$, due to the Heisenberg uncertainty relations. A similar derivation gives a bound for purification, where $N > M$. In such a case $G < 1$, and one obtains

$$\Gamma - \frac{1}{2} \geq \frac{1}{N} \left(\gamma - \frac{1}{2} \right). \quad (7)$$

We will see that the M output purified copies are, however, correlated.

From the bound on phase-conjugating amplifiers (3), similarly it follows

$$\Gamma - \frac{1}{2} \geq \frac{1}{N} \left(\gamma + \frac{1}{2} \right), \quad (8)$$

for phase-conjugating broadcasting (and purification). The bound (8) is independent of the number of output copies, and corresponds to (6) for broadcasting in the limit $M \rightarrow \infty$.

The bound (6) for broadcasting can be achieved by the following experimental setup. By means of a N -splitter the signal is concentrated in one mode, whereas the other $N - 1$ modes are discarded. The mode is then amplified by a phase-insensitive amplifier with power gain $G = \frac{M}{N}$. Finally, the amplified mode is mixed in a M -splitter with $M - 1$ vacuum modes. In the concentration stage the N modes with amplitude $\langle a \rangle = \alpha$ and noise $\Delta x^2 + \Delta y^2 = \gamma_i$ are reduced to a single mode with amplitude $\sqrt{N}\alpha$ and noise γ . The amplification stage gives a mode with amplitude $\sqrt{M}\alpha$ and noise $\gamma' = \gamma \frac{M}{N} + \frac{M}{2N} - \frac{1}{2}$. Finally, the distribution stage gives M modes, with amplitude α and noise $\Gamma = \frac{1}{M} \left(\gamma' + \frac{M-1}{2} \right)$ each. We notice that the linear amplifier can be replaced by a beam-splitter, heterodyne detection and feed-forward, as in ref. [15].

The condition for superbroadcasting is given by $\Gamma < \gamma$, namely $\gamma - \frac{1}{2} \geq \frac{M-N}{M(N-1)}$, which can be true for any $N > 1$, and up to $M \leq \infty$. Consider, for example, the case of N displaced thermal states $D(\alpha)\rho_{\bar{n}}D^\dagger(\alpha)$, where

$$\rho_{\bar{n}} = \frac{1}{\bar{n} + 1} \left(\frac{\bar{n}}{\bar{n} + 1} \right)^{a^\dagger a}, \quad (9)$$

and \bar{n} denotes the thermal photon number. The output state is given by $D(\alpha)^{\otimes M} \lambda D^\dagger(\alpha)^{\otimes M}$, with

$$\lambda = \int \frac{M d^2 \gamma}{\pi \bar{n}'} |\gamma\rangle \langle \gamma|^{\otimes M} e^{-\frac{M|\gamma|^2}{\bar{n}'}} , \quad (10)$$

where $\bar{n}' = \frac{M(\bar{n}+1)}{N} - 1$. Such a state is permutation-invariant and separable, with displaced thermal state at each use, with thermal photon number

$$\bar{n}'' = \frac{\bar{n}'}{M} = \frac{\bar{n}}{N} + \frac{M-N}{MN}. \quad (11)$$

The superbroadcasting condition (output purity higher than the input one), is equivalent to require smaller thermal photon number at the output than at the input, namely $\bar{n} \geq \frac{M-N}{M(N-1)}$. In fact, $\gamma = \bar{n} + \frac{1}{2}$ and $\Gamma = \bar{n}'' + \frac{1}{2}$. Notice that for $\bar{n} = 0$ one has N coherent states at the input, and $\bar{n}'' = \frac{M-N}{MN}$, namely one finds the optimal cloning for coherent states of ref. [13]. In fig. 1 we sketch the scheme for optimal 2 to 3 broadcasting. The superbroadcasting effect arises for $\bar{n} > \frac{1}{3}$.

For achieving the optimal purification for any $M \leq N$, one simply uses an N -splitter which concentrates the signal in one mode and discards the other $N - 1$ modes. Then by N -splitting with $N - 1$ vacuum modes, one obtains N purified signals (although correlated), with equality in eq. (7).

The bound for phase-conjugating broadcasting and purification (8) can be obtained by N -splitting, heterodyne measurement on one of the modes, and preparation of M coherent states with amplitude $\alpha = \frac{\alpha_o^*}{\sqrt{N}}$, where α_o denotes the outcome of the measurement.

We would like to stress that the bounds (6), (7), and (8) hold for any state and relies on the theorem of the added noise in *linear* amplifiers, namely only linear transformations of

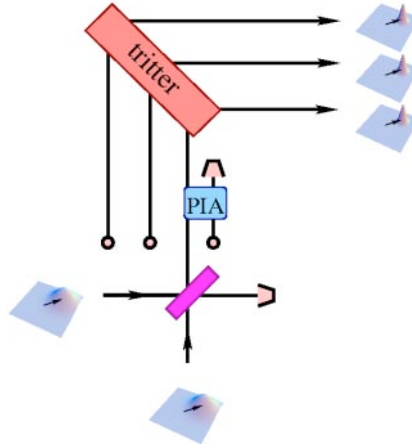


Fig. 1 – Experimental scheme for optimal superbroadcasting from 2 to 3 copies. The scheme makes use of a beam splitter, a phase-insensitive amplifier and a tritter (*i.e.* two suitably balanced beam splitters). The output copies carry the same signal as the input, and are locally less noisy, the noise being confined into the correlations between them.

modes are considered. Hence, in principle, these bounds might be violated when considering a restricted set of states and allowing for more exotic and nonlinear transformations.

In the following we give a rigorous proof that these bounds indeed cannot be overcome by any quantum transformation. Let us consider a generic state Ξ_α of N uncorrelated modes with noise γ_i , and $\langle a_i \rangle = \alpha$ for all modes. Then, Ξ_α can be written as $D(\alpha)^{\otimes N} \Xi_0 D^\dagger(\alpha)^{\otimes N}$, where $D(\alpha) = \exp[\alpha a^\dagger - \alpha^* a]$ denotes the displacement operator and $\Xi_0 = \otimes_{i=0}^{N-1} \xi_i$ is the tensor product of N states, each with zero amplitude (*i.e.*, for a single-mode radiation field, zero average value of the field) and noise γ_i . We look for a broadcasting map \mathcal{B} that preserves the unknown amplitude on each copy

$$\text{Tr}[b_i \mathcal{B}(D(\alpha)^{\otimes N} \Xi_0 D^\dagger(\alpha)^{\otimes N})] = \alpha, \quad (12)$$

for all $i \in [0, M-1]$ and complex α , such that each copy has minimal noise Γ , where, using eq. (4),

$$\Gamma = \frac{1}{2} + \text{Tr}[b_i^\dagger b_i \mathcal{B}(D(\alpha)^{\otimes N} \Xi_0 D^\dagger(\alpha)^{\otimes N})] - |\alpha|^2. \quad (13)$$

The optimal broadcasting map can be searched among *covariant* maps \mathcal{B} that satisfy for all σ and α ⁽¹⁾

$$\mathcal{B}(D(\alpha)^{\otimes N} \sigma D^\dagger(\alpha)^{\otimes N}) = D(\alpha)^{\otimes M} \mathcal{B}(\sigma) D^\dagger(\alpha)^{\otimes M}.$$

It is useful to consider the Choi-Jamiołkowski bijective correspondence of completely positive (CP) maps \mathcal{B} from \mathcal{H}_{in} to \mathcal{H}_{out} and positive operators $R_{\mathcal{B}}$ acting on $\mathcal{H}_{\text{out}} \otimes \mathcal{H}_{\text{in}}$, which is

⁽¹⁾In fact, for any map \mathcal{B} , one can construct a covariant one $\tilde{\mathcal{B}}$ by averaging over the group, and still satisfying eqs. (12) and (13). Actually, being the group noncompact, a limit procedure has to be taken, *e.g.*

$$\tilde{\mathcal{B}}(\sigma) = \lim_{\Delta \rightarrow \infty} \int \frac{d^2\alpha}{\pi\Delta^2} e^{-\frac{|\alpha|^2}{\Delta^2}} D^\dagger(\alpha)^{\otimes M} \mathcal{B}[D(\alpha)^{\otimes N} \sigma D^\dagger(\alpha)^{\otimes N}] D(\alpha)^{\otimes M}.$$

given by the following relations

$$\begin{aligned} R_{\mathcal{B}} &= \mathcal{B} \otimes \mathcal{S}(|\Omega\rangle\langle\Omega|), \\ \mathcal{B}(\rho) &= \text{Tr}_{\text{in}}[(I_{\text{out}} \otimes \rho^\tau)R_{\mathcal{B}}], \end{aligned} \quad (14)$$

where $|\Omega\rangle = \sum_{n=0}^{\infty} |\psi_n\rangle|\psi_n\rangle$ is a maximally entangled vector of $\mathcal{H}_{\text{in}}^{\otimes 2}$, and X^τ denotes transposition of X in the basis $|\psi_n\rangle$. In terms of the operator $R_{\mathcal{B}}$ the covariance property can be written as

$$[R_{\mathcal{B}}, D(\alpha)^{\otimes M} \otimes D(\alpha^*)^{\otimes N}] = 0, \quad \forall \alpha \in \mathbb{C}, \quad (15)$$

and conditions (12) and (13) are equivalent to

$$\text{Tr}[b_i \otimes \Xi_0^\tau R_{\mathcal{B}}] = 0, \quad (16)$$

$$\Gamma = \frac{1}{2} + \text{Tr}[b_i^\dagger b_i \otimes \Xi_0^\tau R_{\mathcal{B}}]. \quad (17)$$

In order to deal with the covariance constraint we introduce the multisplitter operators U_a and U_b , that satisfy

$$U_a a_k U_a^\dagger = \frac{1}{\sqrt{N}} \sum_{l=0}^{N-1} e^{\frac{2\pi i k l}{N}} a_l, \quad U_b b_k U_b^\dagger = \frac{1}{\sqrt{M}} \sum_{l=0}^{M-1} e^{\frac{2\pi i k l}{M}} b_l, \quad (18)$$

and the squeezing transformation $S_{a_0 b_0}$ defined by

$$S_{a_0 b_0} a_0^\dagger S_{a_0 b_0}^\dagger = \mu a_0^\dagger - \nu b_0, \quad S_{a_0 b_0} b_0 S_{a_0 b_0}^\dagger = \mu b_0 - \nu a_0^\dagger, \quad (19)$$

with $\mu = \sqrt{\frac{M}{M-N}}$ and $\nu = \sqrt{\frac{N}{M-N}}$. Condition (15) then becomes

$$[S_{a_0 b_0}^\dagger (U_b^\dagger \otimes U_a^\dagger) R_{\mathcal{B}} (U_b \otimes U_a) S_{a_0 b_0}, D_{b_0}(\alpha)] = 0. \quad (20)$$

Hence, upon introducing an operator B of modes $b_1, \dots, b_{M-1}, a_0, \dots, a_{N-1}$, the operator $R_{\mathcal{B}}$ can be written in the form

$$R_{\mathcal{B}} = (U_b \otimes U_a) S_{a_0 b_0} (I_{b_0} \otimes B) S_{a_0 b_0}^\dagger (U_b^\dagger \otimes U_a^\dagger). \quad (21)$$

Notice that $R_{\mathcal{B}} \geq 0$ is equivalent to $B \geq 0$. The further condition that \mathcal{B} is trace-preserving in terms of $R_{\mathcal{B}}$ becomes $\text{Tr}_b[R_{\mathcal{B}}] = I_a$, where b and a denotes collectively all output and input modes. This condition is verified iff

$$\text{Tr}_{b \setminus b_0, a_0}[B] = \nu^2 I_{a \setminus a_0}, \quad (22)$$

where $a \setminus a_i$ denote all the input modes except a_i , and similarly for $b \setminus b_i$.

Consider now the expectation value of the total number of photons of the M clones $W = \text{Tr}[\sum_{l=0}^{M-1} b_l^\dagger b_l \mathcal{B}(\Xi_0)]$. Since the multisplitter preserves the total number of photons we have

$$\begin{aligned} W &= \text{Tr} \left[\left(\sum_{l=0}^{M-1} b_l^\dagger b_l \otimes U_a^\dagger \Xi_0^\tau U_a \right) S_{a_0 b_0} (I_{b_0} \otimes B) S_{a_0 b_0}^\dagger \right] \\ &\geq \text{Tr} \left[\left(b_0^\dagger b_0 \otimes U_a^\dagger \Xi_0^\tau U_a \right) S_{a_0 b_0} (I_{b_0} \otimes B) S_{a_0 b_0}^\dagger \right]. \end{aligned} \quad (23)$$

Using the relation which holds for any state σ

$$\text{Tr}_{b_0}[S_{a_0 b_0}^\dagger(b_0^\dagger b_0 \otimes \sigma)S_{a_0 b_0}] = \frac{1}{\nu^4}(a_0^\dagger a_0 \otimes \text{Tr}_{a_0}[\sigma] + \mu^2 I_{a_0} \otimes \text{Tr}_{a_0}[a_0^\dagger a_0 \sigma] + I_{a_0} \otimes \text{Tr}_{a_0}[\sigma]), \quad (24)$$

along with condition (22), continuing from eq. (23), we obtain

$$\begin{aligned} W &= \frac{\mu^2 \text{Tr}[a_0^\dagger a_0 U_a^\dagger \Xi_0^\dagger U_a] + 1}{\nu^2} = \frac{\mu^2 \frac{1}{N} \text{Tr}[\sum_{i,j=0}^{N-1} a_i^\dagger a_j \Xi_0^\dagger] + 1}{\nu^2} \\ &= \frac{M}{N} \left(\gamma - \frac{1}{2} \right) + \frac{M-N}{N}, \end{aligned} \quad (25)$$

which, from eq. (17), allows one to recover the bound (6). With the choice $B = \nu^2 |0\rangle\langle 0|_{b \setminus b_0} \otimes |0\rangle\langle 0|_{a_0} \otimes I_{a \setminus a_0}$ one can check that both the bound in eq. (25) is achieved and eq. (16) is satisfied. In fact, such a choice of B gives a map that produces M identical clones $D(\alpha)\rho D^\dagger(\alpha)$ with

$$\rho = \int \frac{d^2\alpha}{\pi} e^{-\frac{|\alpha|^2}{2}(\frac{1}{N} - \frac{2}{M} + 1)} \{ \text{Tr}[\Xi_0 D^\dagger(\alpha/N)^{\otimes N}] D(\alpha) \}.$$

The proof of the bound (7) for purification (for $M < N^{(2)}$) can be obtained by analogous derivation, where now $\mu = \sqrt{\frac{N}{N-M}}$ and $\nu = \sqrt{\frac{M}{N-M}}$, while the trace-preserving condition becomes $\text{Tr}_b[B] = \mu^2 I_{a \setminus a_0}$. The map corresponding to $B = \mu^2 |0\rangle\langle 0|_b \otimes I_{a \setminus a_0}$ achieves the bound (7), and produces M purified copies $D(\alpha)\rho D^\dagger(\alpha)$ with

$$\rho = \int \frac{d^2\alpha}{\pi} e^{-\frac{|\alpha|^2}{2}(1 - \frac{1}{N})} \{ \text{Tr}[\Xi_0 D^\dagger(\alpha/N)] \}^N D(\alpha).$$

A covariant phase-conjugating broadcasting map \mathcal{C} satisfies for all σ and α

$$\mathcal{C}(D(\alpha)^{\otimes N} \sigma D^\dagger(\alpha)^{\otimes N}) = D^*(\alpha)^{\otimes M} \mathcal{C}(\sigma) D^\tau(\alpha)^{\otimes M},$$

which, in terms of $R_{\mathcal{C}}$, corresponds to $[D(\alpha)^{* \otimes (M+N)}, R_{\mathcal{C}}] = 0$. Introducing the beam-splitter transformation

$$U_{a_0 b_0} b_0 U_{a_0 b_0}^\dagger = \eta b_0 + \theta a_0, \quad U_{a_0 b_0} a_0 U_{a_0 b_0}^\dagger = -\theta b_0 + \eta a_0, \quad (26)$$

with $\eta = \sqrt{\frac{M}{M+N}}$ and $\theta = \sqrt{\frac{N}{M+N}}$, the covariance relation gives $R_{\mathcal{C}}$ of the form

$$R_{\mathcal{C}} = U_b \otimes U_a U_{a_0 b_0} (I_{b_0} \otimes C) U_{a_0 b_0}^\dagger U_b^\dagger \otimes U_a^\dagger, \quad (27)$$

where C is an operator of modes $b_1, \dots, b_{M-1}, a_0, \dots, a_{N-1}$, with the trace-preserving condition $\text{Tr}_{b \setminus b_0, a_0}[C] = \theta^2 I_{a \setminus a_0}$. The proof of the bound (8) is analogous to the case for the broadcasting map, where one just replaces $U_{a_0 b_0}$ with $S_{a_0 b_0}$. The bound can be achieved by

(²)The proof for the case $M = N$ is technically slightly different, since the squeezing operator $S_{a_0 b_0}$ is ill-defined in this case. By introducing the EPR states $|D(\beta)\rangle\rangle = \sum_{n=0}^{\infty} (D(\beta) \otimes I)|n\rangle|n\rangle$, the covariance condition implies

$$R_{\mathcal{B}} = (U_b \otimes U_a) \int \frac{d^2\gamma}{\pi} |D(\gamma)\rangle\rangle \langle\langle D(\gamma)|_{a_0 b_0} \otimes \Delta_{a \setminus a_0, b \setminus b_0}(\gamma)(U_b^\dagger \otimes U_a^\dagger),$$

while the trace-preserving constraint is given by $\int d^2\gamma \text{Tr}_{b \setminus b_0}[\Delta_{a \setminus a_0, b \setminus b_0}(\gamma)] = \pi I_{a \setminus a_0}$. Then, the bound is proved as before, and is achieved for $\Delta_{a \setminus a_0, b \setminus b_0}(\gamma) = \pi \delta^2(\gamma) I_{a \setminus a_0} \otimes |0\rangle\langle 0|_{b \setminus b_0}$.

$C = \theta^2 |0\rangle\langle 0|_{b\backslash b_0} \otimes |0\rangle\langle 0|_{a_0} \otimes I_{a\backslash a_0}$, and the state of the clones are given by $D^*(\alpha)\rho D^\tau(\alpha)$, with

$$\rho = \int \frac{d^2\alpha}{\pi} e^{-\frac{|\alpha|^2}{2}(1+\frac{1}{N})} \{\text{Tr}[\Xi_0 D^\dagger(\alpha/N)]\}^N D(\alpha).$$

In conclusion, we showed the optimal N to M phase-preserving/phase conjugating broadcasting and purification maps for continuous variables. For $N \geq 2$, the superbroadcasting can be achieved, namely $M > N$ copies can be obtained along with a reduction of noise in conjugate variables. Since the noise cannot be removed without violating the quantum data processing theorem, the price to pay for having higher purity at the output is that the copies are correlated. Essentially noise is moved from local states to their correlations, and the superbroadcasting channel that we presented does this optimally. All the optimal maps can be easily implemented by linear optics and linear amplification (or beam-splitting and feed-forward). Superbroadcasting is relevant for foundations, opening new perspectives in the understanding of correlations and their interplay with noise, and may be also promising from a practical point of view, for communication tasks in the presence of noise.

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