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COMMUNICATION WITH NONCLASSICAL STATES IN THE PRESENCE OF LOSS

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Abstract

Communication using direct detection of Fock states achieves the maximum mutual information (the ideal channel capacity). Also squeezed-state based communication with homodyne detection leads to great improvements of the mutual information with respect to the classical channel based on heterodyne detection of coherent states. However, any nonclassical state is made "classical" by loss, and even for not too high losses, the conventional coherent-state channel gives the best mutual information anyway. Using the Blahut's algorithm we optimize the *a priori* probability for the Fock-state based channel in the presence of loss and at constrained fixed power. For high losses, the optimization improvement of the mutual information achieves 60%. Moreover, we show that there is a threshold (around loss $\eta = .6$) where the optimized *a priori* probability departs from the monotonic thermal-like probability, and develops "holes" of zero probability at intermediate numbers of photons. However, we find that even such an optimized channel is worst than the classical one for sufficiently high power.

1 Introduction

The results about the mutual information of ideal narrow-band quantum-optical channels that employ Fock states, coherent states and squeezed states are well-known [1]. The ultimate capacity is achieved by the number-state channel with thermal *a priori* probability and direct detection at the output. Also the communication channel based on homodyne detection of quadrature-squeezed states with optimized squeezing is more efficient (1 bit is gained for high power) than the classical channel based on heterodyne detection of coherent states. These results hold true for optimal channels, and are more interesting from the theoretical point of view rather than in practice. For applications, it is necessary to determine the relative robustness of the above communication schemes with respect to losses along the line. Here we analyze the detrimental effect of loss on the efficiency of the communication channels. The influence of loss is parametrized by a value η between 0 and 1, corresponding to the energy attenuation factor. Due to its peculiar form, the master equation that describes the loss keeps coherent states as coherent, so that, at sufficiently high average power, heterodyne channel turns out to be more effective than the number-state and squeezed-state channels. Nevertheless, one can try to optimize the *a priori* probability as a function of the attenuation factor η, and consider unconventional *a priori* probabilities—for example with gaps for the number-state channel—in order to make the states more distinguishable at the receiver. Indeed, as we will show in the following, the optimal number *a priori* probability departs from the thermal-like monotonic behaviour and develops "holes" of zero probability for sufficiently high losses (for $\eta < .6$). Optimization is obtained numerically by means of the recursive Blahut's algorithm [2] and leads to a sizeable enhancement (over 60% for high loss) of the mutual information for the number-state channel.

2 Coherent-state and squeezed-state channels

The effect of loss on a single-mode communication channel can be modelled by the following master equation

$$\partial_t \hat{\varrho} = \mathcal{L}_\Gamma \hat{\varrho} \doteq \Gamma(n_a + 1)L[a] + \Gamma n_a L[a^\dagger] \,. \tag{1}$$

In Eq. (1), the superoperator $\mathcal{L}_\Gamma$ gives the time derivative of the density matrix $\hat{\varrho}$ of the radiation state in the interaction picture, through the action on $\hat{\varrho}$ of the Lindblad superoperators $L[a]\hat{\varrho} = a\hat{\varrho}a^\dagger - \frac{1}{2}(a^\dagger a\hat{\varrho} + \hat{\varrho}a^\dagger a)$ [3]. The coefficient Γ represents the damping rate, whereas n_a denotes the mean number of thermal photons at the frequency of mode a, and can be neglected at optical frequencies. The loss $\eta \equiv e^{-\Gamma t}$, which represents the energy attenuation factor, is introduced from the following relation

$$\langle a^\dagger a(t)\rangle \equiv \mathrm{Tr}[a^\dagger a\, \hat{\varrho}(t)] = \mathrm{Tr}[a^\dagger a\, e^{\mathcal{L}_\Gamma t} \hat{\varrho}(0)] = \eta \langle a^\dagger a(0)\rangle \,. \tag{2}$$

2.1 Coherent-state channel

The communication channel based on coherent states employs a Gaussian *a priori* probability density and heterodyne detection at the output. The heterodyne channel noise is additive (3 dB) and Gaussian, and the Gaussian form for the *a priori* probability achieving the capacity is requested by Shannon's theorem [4] for Gaussian channels with quadratic constraint (corresponding to fixed average power). The channel capacity—i. e. the maximum quantity of information that may be transmitted without error over the channel—for average photon number N is given by

$$C = \ln(1 + \eta N) \,, \tag{3}$$

and the corresponding optimal *a priori* probability density writes

$$p(\alpha) = \frac{1}{\pi N} \exp\left(-\frac{|\alpha|^2}{N}\right) \,. \tag{4}$$

Notice that Eq. (4) is independent on η. This means that the optimal *a priori* probability for the ideal channel still remains the optimal one in the presence of loss. As we will show, this does not hold true for the squeezed-state and number-state channels.

2.2 Squeezed-state channel

The squeezed-state channel encodes a real variable x on the quadrature-squeezed state

$$|x\rangle_r = D(x)S(r)|0\rangle \,. \tag{5}$$

The state in Eq. (5) is generated from vacuum $|0\rangle$ by the action of displacement and squeezing operators, respectively

$$D(x) = \exp\left[x\left(a^\dagger - a\right)\right] \ , \qquad S(r) = \exp\left[\frac{r}{2}\left(a^{\dagger 2} - a^2\right)\right] \ . \tag{6}$$

Decoding is performed through homodyne detection of the quadrature $\hat{X} \equiv (a + a^\dagger)/2$. The conditional probability density of getting the value x' when the transmitted state is $|x\rangle_r$ writes

$$Q_\eta(x'|x) = \sqrt{\frac{1}{2\pi\Delta_\eta^2}} \exp\left[-\frac{(x' - \eta^{1/2}x)^2}{2\Delta_\eta^2}\right] \ , \qquad \Delta_\eta^2 = \frac{1}{4}\left[1 - \eta(1 - e^{-2r})\right] \ . \tag{7}$$

The mutual information for Gaussian *a priori* probability and fixed average number of photons N is given by

$$I = \frac{1}{2}\ln\left[1 + \frac{4\eta(N - \sinh^2 r)}{1 - \eta(1 - e^{-2r})}\right] \ . \tag{8}$$

Upon maximizing Eq. (8) with respect to $\xi \equiv e^{-2r}$ one obtains

$$I = \frac{1}{2}\ln\left[1 + \frac{4\xi N - (1 - \xi)^2}{\xi^2 + \frac{1-\eta}{\eta}\xi}\right] \tag{9}$$

with

$$\xi = \frac{\eta + \sqrt{1 + 4\eta(1 - \eta)N}}{(4N + 1)\eta + 1} \ . \tag{10}$$

The well-known result $I = \ln(1 + 2N)$ for the ideal squeezed-state channel is easily recovered from Eq. (9) with $\eta = 1$. The optimal number of squeezing photons is given by $(\xi + \xi^{-1} - 2)/4$: for increasing loss, the demand of squeezing photons that optimize the mutual information rapidly decreases.

Following Hall [5], one can prove the following upper bound for communication channels based on homodyne detection and degraded by loss η

$$I \leq \ln\left(1 + 2\eta N\right) \ . \tag{11}$$

Is the upper bound (11) achievable? While for ideal transmission ($\eta = 1$) the bound is achieved by a Gaussian ensemble of squeezed states, in the presence of loss the bound is not reached, even by optimizing the squeezing parameter r versus η. Notice that Eq. (8) has been derived for Gaussian *a priori* probability, namely taking r as constant, and we cannot rule out the possibility of further improving the mutual information by optimizing r versus the signal x in Eq. (5). However, such an optimization is difficult to achieve, since the corresponding conditional probability density is no longer Gaussian, and even numerically, we have no method available to evaluate the ultimate capacity of the lossy homodyne channel and to determine the relative *a priori* probability density.

3 Number-state channel

The ideal communication channel that encodes Fock-states with thermal *a priori* probability distribution and direct detection at the output achieves the ultimate capacity (Holevo's bound [6, 7]) for single-mode channels subjected to a fixed power constraint. For ideal transmission the conditional probability density is given by the Kronecker delta $\delta_{m,n}$, which is replaced by a binomial distribution in the presence of loss.

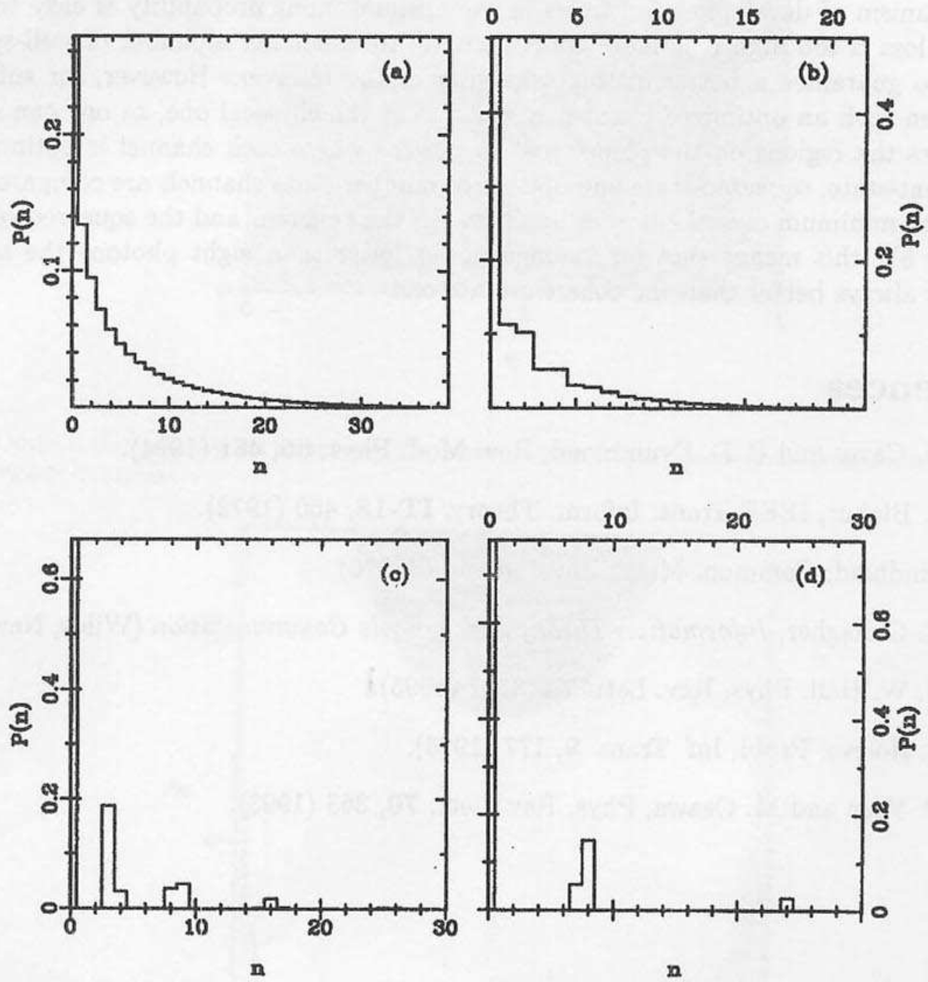

Figure 1: Optimized *a priori* probability $p(n)$ versus n for different values of the attenuation factor η and average power N: a) $\eta = .8$, $N = 5.79$, $I = 2.41$ bit. b) $\eta = .6$, $N = 2.41$, $I = 1.34$ bit. c) $\eta = .4$, $N = 1.89$, $I = 0.89$ bit. d) $\eta = .2$, $N = 2.24$, $I = 0.63$ bit.

We have used the Blahut's recursive algorithm [2] to evaluate the Fock-state channel capacity in the presence of loss and to determine the corresponding optimal *a priori* probability. Fig. 1 shows the optimal number probability for different powers and different values of η. The Hilbert space has been truncated at 200, but truncation at 100 gives indistinguishable results. For $\eta = .8$ the corresponding distribution p_n is thermal-like (with enhanced vacuum-probability) and the improvement of mutual information with respect to the non-optimized channel is around 4%.

For higher loss ($\eta = .6$) the optimal *a priori* probability departs from the monotonic thermal-like distribution and develops "holes" of zero probability (as for $\eta = .4$ and, more dramatically, for $\eta = .2$). Correspondingly, the improvement of mutual information due to the optimization procedure is much better: 10%, 24% and 60% for $\eta = .6$, .4 and .2, respectively. In these cases (low power), this improvement makes the number channel more efficient than the coherent channel. The mechanism of development of holes in the optimal input probability is easy to understand: when the loss is too high it is more convenient to use a smaller alphabet of well-spaced letters, in order to guarantee a better distinguishability at the receiver. However, for sufficiently high power, even such an optimized channel is worst than the classical one, as one can see in Fig. 2, which gives the regions on the plane "loss *vs* power" where each channel is optimal. In Fig. 3 the coherent-state, squeezed-state and optimized number-state channels are compared. Notice the value of the minimum over the border line between the coherent and the squeezed regions ($\eta = .5$ and $N = 8$): this means that for average power lower than eight photons the squeezed-state channel is always better than the coherent-state one.

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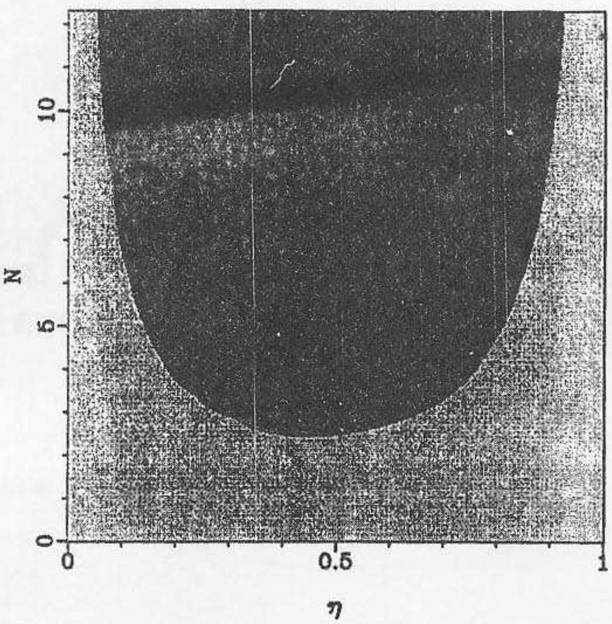

Figure 2: Optimal channel diagram comparing the coherent-state (black region) and the optimized number-state (dark grey region) channels.

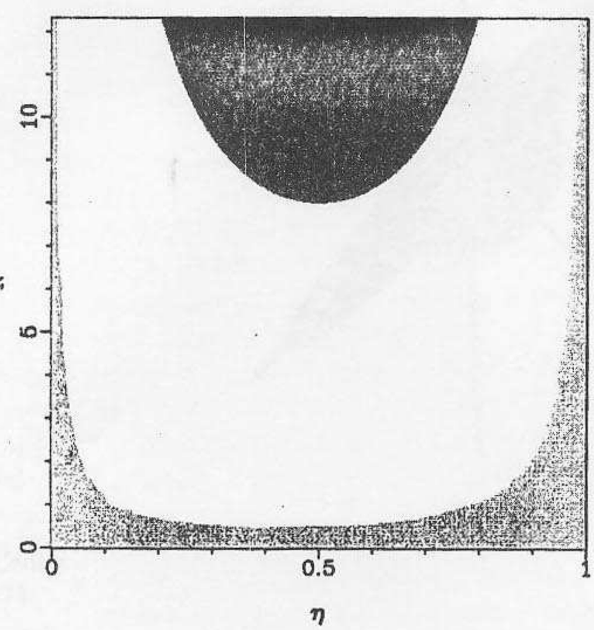

Figure 3: Optimal channel diagram comparing the coherent-state (black region), the squeezed-state (light grey region) and the optimized number-state (dark grey region) channels.