Contents lists available at ScienceDirect

Studies in History and Philosophy of Modern Physics

journal homepage: www.elsevier.com/locate/shpsb

Emergence of space–time from topologically homogeneous causal networks

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ABSTRACT

In this paper we study the emergence of Minkowski space–time from a discrete causal network representing a classical information flow. Differently from previous approaches, we require the network to be topologically homogeneous, so that the metric is derived from pure event-counting. Emergence from events has an operational motivation in requiring that every physical quantity—including space–time—be defined through precise measurement procedures. Topological homogeneity is a requirement for having space–time metric emergent from the pure topology of causal connections, whereas physically homogeneity corresponds to the universality of the physical law. We analyze in detail the case of 1+1 dimensions. If we consider the causal connections as an exchange of classical information, we can establish coordinate systems via an Einsteinian protocol, and this leads to a digital version of the Lorentz transformations. In a computational analogy, the foliation construction can be regarded as the synchronization with a global clock of the calls to independent subroutines (corresponding to the causally independent events) in a parallel distributed computation. Thus the Lorentz time-dilation emerges as an increased density of leaves within a single *tic-tac* of a clock, whereas space-contraction results from the corresponding decrease of density of events per leaf. The operational procedure of building up the coordinate system introduces an in-principle indistinguishability between neighboring events, resulting in a network that is coarse-grained, the thickness of the event being a function of the observer's clock. The illustrated simple classical construction can be extended to space dimension greater than one, with the price of anisotropy of the maximal speed, due to the Weyl-tiling problem. This issue is cured if the causal network is quantum, as e.g. in a quantum cellular automaton, and isotropy is recovered by quantum coherence via superposition of causal paths. We thus argue that in a causal network description of space–time, the quantum nature of the network is crucial.

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When citing this paper, please use the full journal title *Studies in History and Philosophy of Modern Physics*

1. Introduction

Our everyday way of looking at space–time as a stage for physical events conflicts with the requirement of defining all physical quantities—including space and time—through precise measurement protocols. This means that we should more properly regard space–time as emerging from events, instead of pre-existing them. The operational definition of space–time is defined by the protocol that sets up the coordinate system. For example, in the Einstein protocol light pulses are sent back and forth between different locations: at the place where the signal has been

originated, from the arrival time of the reflected signal one infers both the distance and the time of the remote event of signal-reflection. The protocol shows how space–time is indeed a coherent organization of inferences based on a causal structure for events. The clock itself is just a sequence of events—a light pulse bouncing between two mirrors. The closest are the mirrors, the more precise is the clock, and the more refined is the coordinate system.

The above reasoning shows that ultimately space and time are defined through pure event-counting, precisely counting *tic-tacs* of the observer's clock, and we are thus lead to regard space and time as emergent from the topology of the causal network of events. The events of the network do not need to be regarded as actual, but can be just potential, and the fabric of space–time is precisely the network of causal links between them.

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The idea of deriving space–time from purely causal structures is not new. Raphael Sorkin started an independent research line in quantum gravity based on this idea more than two decades ago (Bombelli, Lee, Meyer, & Sorkin, 1987). This was motivated by the potentialities of the approach residing in the natural discreteness of the causal network, which also provides a history-space for a path integral formulation (Henson, 2006; Markopoulou, 2002). The possibility of recovering the main features of the space–time manifold—topology, differentiable structure, and conformal metric—has been shown, starting from discrete sets of points endowed with a causal partial ordering (Surya, 2008). Along these lines, in an operational context, Hardy (2007) has also formulated a *causaloid* approach, which considers the possibility of a dynamical treatment of the causal links.

In the causal-set approach of Sorkin events are randomly scattered in order to avoid occurrence of their sparseness in the boosted frames that would lead to violation of Lorentz invariance. Clearly the randomness of Sorkin's events should not be regarded in terms of their location on a background, otherwise we contradict the same idea of space–time emergence. We then need to consider randomness at the pure topological level, and this means having random causal connections. However, regarding the causal connections as an irreducible description of the physical law, a random topology would then corresponds to having a random physical law at the most microscopic level (the Planck scale), and one may argue that a "random law" would contradict the same notion of law. Instead, the randomness should result from the law itself, e.g. in a quantum cellular automaton, where randomness comes from the quantum nature of the network. The universality of the physical law thus leads us to take the causal network as topologically homogeneous. Topological homogeneity has then the added bonus that metric simply emerges from the pure topology by just counting events along the network.

With the above motivations, in this paper we analyze the mechanism of emergence of space–time from the pure homogeneous topology in 1+1 dimensions. We present a digital version of the Lorentz transformations, along with the corresponding digital-analog conversion rule. Upon considering the causal connections as exchanges of classical information, we can establish coordinate systems via an Einsteinian protocol, leading to a digital version of the Lorentz transformations. In a computational analogy first noticed by Lamport (1978), the foliation construction can be regarded as the synchronization protocol with a global clock of the calls to independent subroutines (the causally independent events) in a parallel distributed computation. The boosts are determined by the relative lengths of the *tic* and *tac* of the clock, and the Lorentz time-dilation corresponds to an increased number of leaves within a clock *tic-tac*, whereas space-contraction results from the corresponding decreased density of events per leaf, as first noticed in D'Ariano (2010).

We will see that the operational procedure of building up the coordinate system introduces an in-principle indistinguishability between neighboring events, resulting in a network that is coarse-grained, the thickness of the event being a function of the observer's clock. The digital version of the Lorentz transformation is an integer relation which differs from the usual analog transformation by a multiplicative real constant corresponding exactly to the event thickness. The composition rule for velocities is independent on such constant, and is the same in both the analog and the digital versions. Preliminary results of the present work were already presented in D'Ariano and Tosini (2010).

It is obvious that the discreteness of the causal network will lead to violation of Lorenz covariance (and the other space symmetries) at the Planck scale level. However, one must have a theory where covariance is restored in the large scale limit—the Fermi scale—corresponding to counting huge numbers of events.

This limit will not be considered in the details in the present paper whose focus is on the exact digital description of the space–time emergent from the network. An example of dynamical model based on a discrete causal network at the Planck scale with the usual covariance restored in the particle physics regime is shown in D'Ariano (2011) and Bisio, D'Ariano, and Tosini (2012). In these works a one-dimensional quantum cellular automaton at the Planck scale is considered which in the large scale limit, corresponding to small momenta, recovers the usual Dirac equation. The choice of a quantum cellular automaton is also crucial in the emergence of space–time from a causal network in dimension greater than one. Indeed the illustrated simple classical construction can be extended to higher dimensions, but at the price of anisotropy of the maximal speed, due to the Weyl-tiling problem (Weyl). This issue is cured if the causal network is quantum, as in a quantum cellular automaton, and isotropy is recovered with quantum coherence, corresponding to superposition of causal paths. We will thus argue that in a causal network description of space–time, the quantum nature of the network is crucial.

2. Setting up the digital coordinate system

The first problem to address is which specific lattice should be adopted for the causal network. In our convention the causal arrow is directed from the bottom to the top of the network. The dimension of the emerging space–time corresponds to the graph-dimension of the network, which is the dimension of the embedding manifold such that all links can be taken as segments of straight line with the same length. We will require the lattice to be pure topology (namely with all events equivalent), corresponding to a locally homogeneous space–time, and with no redundant links. It is then easy to see that in 1+1 dimensions there are only three possible lattices: the square, the triangular, and the honeycomb one (Fig. 1). The honeycomb-lattice has two inequivalent types of events (having one input and two output links and viceversa), and the corresponding "undressed" topology—where each couple of connected inequivalent events is merged into a single event—reduces to the square lattice. The triangular-lattice, on the other hand, has redundant causal links (the middle vertical ones). We are thus left with the square-lattice.

We always assume the network links as oriented according to the causal arrow. In the square-lattice network there are thus two types of link: toward the right and toward the left—shortly *r*-link and *l*-link. Two events are in the same position (for some boosted reference frame) if they are connected by a path made with a sequence of *r*-links followed by a sequence of *l*-links. When the two sequences contain the same number of links the reference is at rest.

A clock is a sequence of causally connected events periodically oscillating between two positions. For an Einstein clock the oscillation (*tic-tac*) is exactly the same couple of sequences of *l*- and *r*-links identifying events in the same position. The precision of the clock, namely the minimum amount of time that it can measure, is the number of links of a complete *tic-tac*. The *tic-tac* is
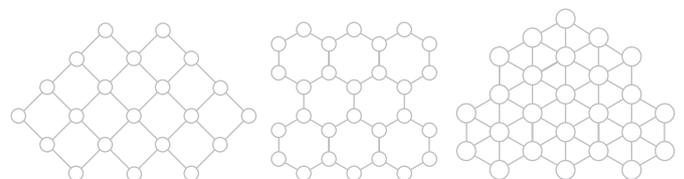

Fig. 1. The three possible homogeneous topologies for causal networks in 1+1 dimensions.

indivisible, namely the sole *tic* (or *tac*) is not a complete measured time interval, since it involves two different positions.

In the following we will call *light signals* those sequences of events that are connected only by *r*-links or by *l*-links, namely making segments at 45° with the horizontal in the network. Their "speed" is equal to "one event-per-step", and it is the maximum speed allowed by the causality of the network, since connecting events along a line making an angle smaller than 45° with the horizontal would require following some causal connections in the backward direction from the output to the input. In this way a homogeneous causal network suffices to guarantee a bound for the speed of information flow.

In the following we will take the clock *tic-tac* made with α *r*-links followed by β *l*-links (see Fig. 2). Any clock allows the introduction of a reference frame $\Re$ which is just a foliation of the network built up using the Einstein protocol. From the start of the clock *tic-tac* a light signal is sent to an event in a different position and then received back at the clock. The intermediate time between the sending and the receiving event is taken as synchronous with the event at the turning point, and the number of *tic-tacs* divided by two is taken as the distance from the turning point and the clock conventionally located at the beginning of the *tic-tac*. In this way we build the foliation corresponding to a given clock. A set of synchronous events identifies a leaf of the foliation.
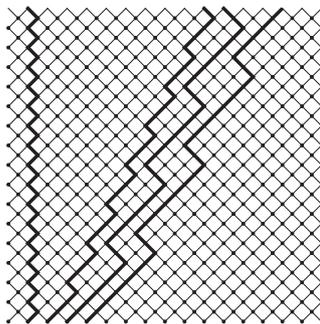

Fig. 2. Different clocks on the causal network. The *tic-tac* of the clock is represented by the two numbers α and β (see text). From the left to the right we have the rest-frame clock, clock corresponding to $\alpha = \beta = 1$, and boosted-frames for $\alpha = 3$, $\beta = 1$, $\alpha = 6$, $\beta = 2$, and $\alpha = 7$, $\beta = 2$, respectively, corresponding to digital speed $v = 1/2$, $v = 1/2$, and $v = 5/9$, respectively. The case $\alpha = 6$, $\beta = 2$ has doubled imprecision, compared to the case $\alpha = 3$, $\beta = 1$.

In Fig. 3 the Einstein protocol is illustrated in two particular reference frames. The figure on the left corresponds to the rest-frame, with the blue lines depicting the coordinate system established using the clock with $\alpha = \beta = 1$ (see Fig. 2). The green lines represent light signals bouncing between the clock and four particular events in the network. These events are synchronous, since the intermediate time between the sending and the receiving event on the clock is the same for all of them. They lie on the same leaf of the foliation, but at different position, 0, 1, 4, 7, respectively: the spatial coordinate is obtained by counting the *tic-tacs* between the sending and the receiving event divided by two. The right figure represents a boosted frame for $\alpha = 3$, $\beta = 1$, built up using the same protocol as in the left figure.

Due to indivisibility of the *tic-tac*, we see that there are indiscernible events, for which the synchronization occurs in the middle of the *tic-tac*. We are thus led to identify events, and merge them into thicker coarse-grained events. This is done as follows. We identify the events along the *tic* and those along the *tac* so that the *tic-tac* is always regarded as the bouncing between two next-neighbour events. Then we merge events into minimal sets so that the topology is left invariant (see Figs. 4 and 5). We can distinguish between two different kinds of coarse-graining: one due to the boosting (in yellow in the figures), and one due to intrinsic imprecision of the clock (in gray). The difference between the two is clarified in Fig. 5. In the top figure events along the *tic* and events along the *tac* are identified in the boosted frame. Then events are merged into minimal sets (in yellow) so that the topology is left invariant (the merged events are again events of a square-lattice network). In the central figure the coarse graining associated to the intrinsic imprecision is added in gray, and finally, in the bottom figure the circuit is stretched so to have all synchronous events on horizontal lines, and events located in the same position on vertical lines. Notice that in the special case of the rest-frame, see Fig. 4, the coarse-graining is just due to the intrinsic imprecision of the clock.

3. The digital Lorentz transformations

The velocity of the boosted frame can be easily written in terms of the α and β of the *tic-tac* of the clock, by simply evaluating the ratio of the distances in space and time between the two ending
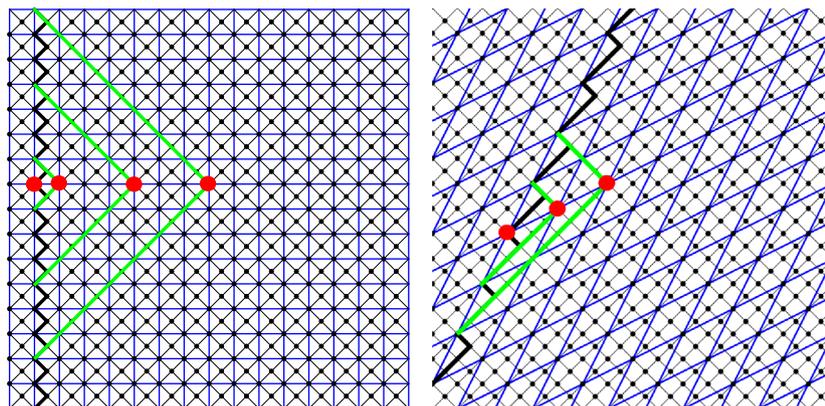

Fig. 3. Illustration of the Einstein protocol for establishing a reference frame. *Left figure*: rest-frame. The blue lines represent the reference frame established using the clock with $\alpha = 1$, $\beta = 1$ (see Fig. 2). The green lines represent light signals bouncing between the clock and four particular events in the network. These events are synchronous, since the intermediate time between the sending and the receiving event on the clock is the same for all of them. They lie on the same leaf of the foliation, but at different position, 0, 1, 4, 7, respectively: the spatial coordinate is obtained by counting the *tic-tacs* between the sending and the receiving event (0, 2, 8, 14, respectively) divided by two. *Right figure*: boosted reference frame (blue lines) for $\alpha = 3$, $\beta = 1$, built up using the same protocol as in the felt figure. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this article.)
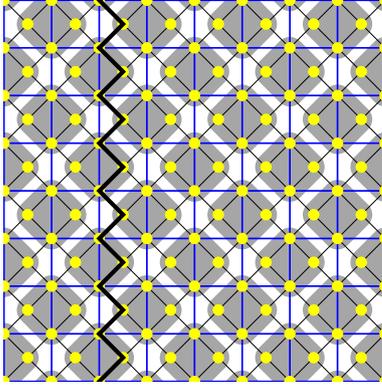
Fig. 4. Coarse-graining in the rest-frame. The coarse-graining due to boosts is trivial (depicted in yellow, containing just one event), whereas the coarse-graining associated to the intrinsic imprecision of the clock is not trivial (depicted in gray, containing four events). (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this article.)

points of the *tic-tac*, namely

$$v = \frac{\alpha - \beta}{\alpha + \beta}. \tag{1}$$

Thanks to invariance of topology under the boost coarse-graining, the above identity holds also for the motion relative to any boosted frame, whence, upon defining with $\alpha_{12} = \alpha_2/\alpha_1 \in \mathbb{Q}$ ($\mathbb{Q}$ denoting rational numbers) and $\beta_{12} = \beta_2/\beta_1 \in \mathbb{Q}$ for frames $\mathfrak{R}_1$ and $\mathfrak{R}_2$, and by $v_{12} \in \mathbb{Q}$ the relative velocity of frame $\mathfrak{R}_2$ with respect to frame $\mathfrak{R}_1$, one has

$$v_{12} = \frac{\alpha_{12} - \beta_{12}}{\alpha_{12} + \beta_{12}}. \tag{2}$$

Now, by using the trivial identities $\alpha_{13} = \alpha_{12}\alpha_{23}$ and $\beta_{13} = \beta_{12}\beta_{23}$ one has

$$v_{13} = \frac{\alpha_{12}\alpha_{23} - \beta_{12}\beta_{23}}{\alpha_{12}\alpha_{23} + \beta_{12}\beta_{23}}, \tag{3}$$

which by simple algebraic manipulations immediately gives

$$v_{13} = \frac{\left(\frac{\alpha_{12}-\beta_{12}}{\alpha_{12}+\beta_{12}}\right) + \left(\frac{\alpha_{23}-\beta_{23}}{\alpha_{23}+\beta_{23}}\right)}{1 + \left(\frac{\alpha_{12}-\beta_{12}}{\alpha_{12}+\beta_{12}}\right)\left(\frac{\alpha_{23}-\beta_{23}}{\alpha_{23}+\beta_{23}}\right)} = \frac{v_{12} + v_{23}}{1 + v_{12}v_{23}}. \tag{4}$$

The last identity is the composition rule of parallel velocities (the only possibility in 1+1 dimension) in special relativity.

Now we use the Einstein protocol to construct the boosted coordinated system with respect to the rest-frame along with the relative coordinate systems between any couple of boosted frames. We will now see that the coordinates of an event transform from the frame $\mathfrak{R}_2$ to the frame $\mathfrak{R}_1$ as follows:

$$s_1 = \tfrac{1}{2}(\alpha_{12} + \beta_{12})s_2 + \tfrac{1}{2}(\alpha_{12} - \beta_{12})t_2, \tag{5}$$

$$t_1 = \tfrac{1}{2}(\alpha_{12} + \beta_{12})t_2 + \tfrac{1}{2}(\alpha_{12} - \beta_{12})s_2. \tag{6}$$

In fact, from a simple inspection of Fig. 6 one can check Eqs. (5) and (6) with the frame $\mathfrak{R}_1$ as the rest frame. In the left figure the reference frame $\mathfrak{R}_1$ with $\alpha_1 = \beta_1 = 1$ is represented by the tiny network in black, whereas the coarser network in blue represents the boosted reference frame $\mathfrak{R}_2$ with $\alpha_2 = 4, \beta_2 = 2$. According to Eq. (2) the relative velocity of $\mathfrak{R}_2$ with respect to $\mathfrak{R}_1$ is $v_{12} = 1/3$. In order to connect the coordinate systems in the two frames we have chosen the same origin $(0,0)$ on both $\mathfrak{R}_2$ and $\mathfrak{R}_1$. The generic event on $\mathfrak{R}_2 \cap \mathfrak{R}_1$ has coordinates $(s_2, t_2) = (3, 2)$ and $(s_1, t_1) = (11, 9)$ in the two frames, respectively. In the figure on the right one can see that a spatial step in $\mathfrak{R}_2$ corresponds to $(\alpha_{12} + \beta_{12})/2$ space and $(\alpha_{12} - \beta_{12})/2$ time steps in $\mathfrak{R}_1$. In the same way a time step in $\mathfrak{R}_2$ corresponds to $(\alpha_{12} - \beta_{12})/2$ space and $(\alpha_{12} + \beta_{12})/2$ time steps in $\mathfrak{R}_1$. This correspondence allows
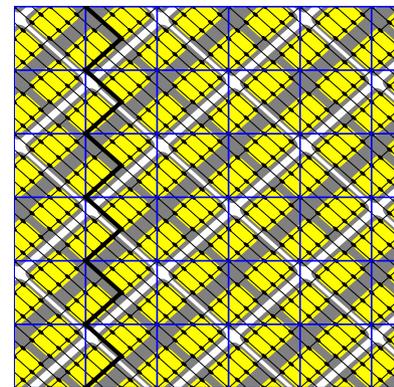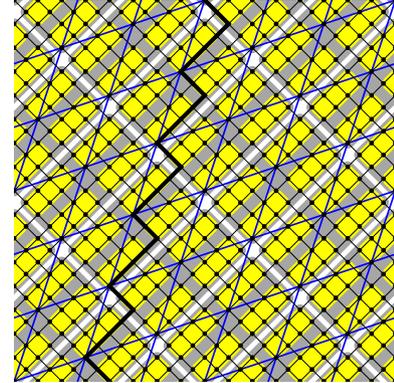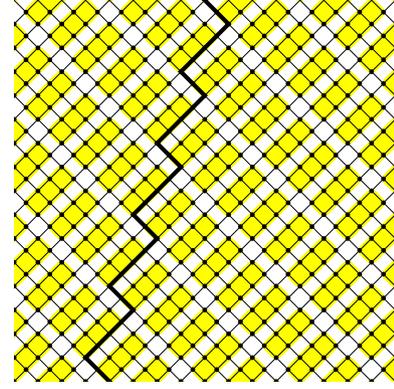

Fig. 5. Illustration of the coarse-graining procedure due to boost (in yellow) and to intrinsic imprecision of the clock (in gray). *Top figure*: events along the *tic* and events along the *tac* are identified in the boosted frame. Then events are merged into minimal sets (in yellow) so that the topology is left invariant (the merged events are again events of a square-lattice network). *Center figure*: the coarse graining associated to the intrinsic imprecision is added in gray. *Bottom figure*: the circuit is stretched so to have all synchronous events on horizontal lines, and events located in the same position on vertical lines. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this article.)

the determination of the coordinates (s_1, t_1) of a given event in the frame $\mathfrak{R}_1$ in terms of its coordinates (s_2, t_2) in the frame $\mathfrak{R}_2$. The resulting transformations are in Eqs. (5) and (6). Invariance of topology with boost, guarantees that they also hold between any couple of boosted frames. By elementary manipulation Eqs. (5) and (6) can be written in the more customary way

$$s_1 = \tfrac{1}{2}(\alpha_{12} + \beta_{12})(s_2 + v_{12}t_2), \tag{7}$$

$$t_1 = \tfrac{1}{2}(\alpha_{12} + \beta_{12})(t_2 + v_{12}s_2). \tag{8}$$

Upon defining the following constant depending on the clocks of the two frames:

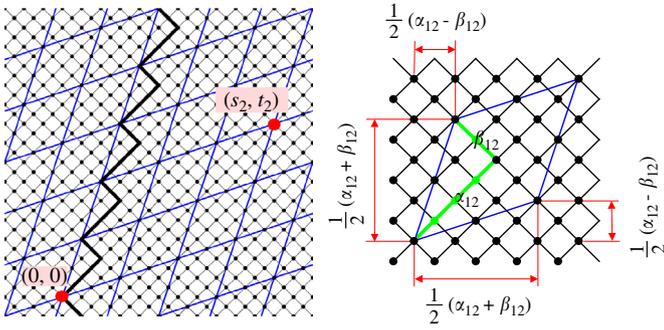$$\chi_{12} := \sqrt{\alpha_{12}\beta_{12}}, \tag{9}$$

Fig. 6. Illustration of the derivation of Eqs. (5) and (6), leading to the digital version of the Lorentz transformations (7) and (8). *Left figure*: the reference frame $\Re_1$ with $\alpha_1 = \beta_1 = 1$ is represented by the tiny network in black, whereas the coarser network in blue represents the boosted reference frame $\Re_2$ with $\alpha_2 = 4$, $\beta_2 = 2$. According to Eq. (2) the relative velocity of $\Re_2$ with respect to $\Re_1$ is $v_{12} = 1/3$. In order to connect the coordinate systems in the two frames we have chosen the same origin $(0,0)$ on both $\Re_2$ and $\Re_1$. The generic event on $\Re_2$ has coordinates $(s_2, t_2) = (3,2)$ and $(s_1, t_1) = (11,9)$ in the two frames, respectively. *Right figure*: a spatial step in $\Re_2$ corresponds to $(\alpha_{12} + \beta_{12})/2$ space and $(\alpha_{12} - \beta_{12})/2$ time steps in $\Re_1$. In the same way a time step in $\Re_2$ corresponds to $(\alpha_{12} - \beta_{12})/2$ space and $(\alpha_{12} + \beta_{12})/2$ time steps in $\Re_1$. This correspondence allows the determination of the coordinates (s_1, t_1) of a given event in the frame $\Re_1$ in terms of its coordinates (s_2, t_2) in the frame $\Re_2$. The resulting transformations are in Eqs. (5) and (6). (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this article.)

and using the identity

$$\frac{1}{2}(\alpha_{12} + \beta_{12}) = \frac{\chi_{12}}{\sqrt{1 - v_{12}^2}}, \tag{10}$$

we obtain the *digital* Lorentz transformations

$$s_1 = \chi_{12} \frac{s_2 + v_{12} t_2}{\sqrt{1 - v_{12}^2}}, \quad t_1 = \chi_{12} \frac{t_2 + v_{12} s_2}{\sqrt{1 - v_{12}^2}}. \tag{11}$$

Eq. (11) differ from the usual *analog* Lorentz transformations by the multiplicative factor χ_{12}, which is logically required to make the transformations rational, compensating the irrationality of the boost factor $\sqrt{1 - v_{12}^2}$. The digital-analog conversion is thus just a rescaling of both space and time coordinates by the factor $(\alpha\beta)^{1/2}$ depending on the boost, which is exactly the square-root of the volume of the coarse-grained event measured as the number of rest-frame events that it contains. Such event volume also affects the Lorentz space-contraction and time-dilation factor, which in the digital case is given by $\frac{1}{2}(\alpha_{12} + \beta_{12})$, whereas in the analog case is rescaled by the ratio of event volumes, leading to $(1/2\sqrt{\alpha_{12}\beta_{12}})(\alpha_{12} + \beta_{12})$. Thus, for example, for $\alpha_{12} = 1$ and $\beta_{12} = 3$ corresponding to $v_{12} = 1/2$ the digital factor is 2 whereas the analog one is $2/\sqrt{3}$. The digital factor agrees with that of the Lorentz time-dilation and space-contraction mechanism of D'Ariano (2010), given in terms of increased density of leaves and corresponding decreased density of events per leaf, as illustrated in Fig. 7.

4. Conclusions and discussion

We have analyzed the mechanism of emergence of space–time from homogeneous topology in 1+1 dimensions, deriving the digital version of the Lorentz transformations along with the corresponding digital-analog conversion rule. The homogeneity of topology physically represents the universality of the physical law (it is worth mentioning that such law is stripped of the conventionality of space and time homogeneity: see e.g. Malament, 1977). We have built the digital coordinate system using the Einstein's protocol, with signals sent back-and-forth to
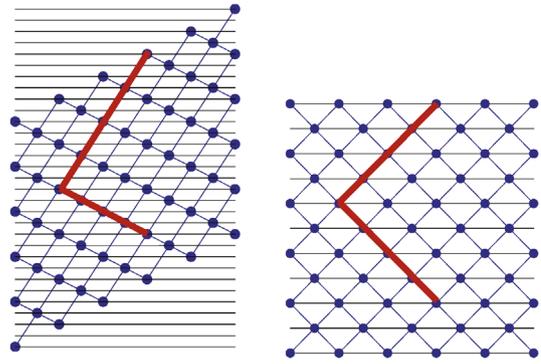

Fig. 7. The mechanism for the digital Lorentz time-dilation and space-contraction given in D'Ariano (2010), here for a boost with $v = 1/2$, corresponding to a digital time-dilation by a factor 2 (analog factor $2/\sqrt{3}$) and space-contraction by a factor $1/2$ (compare with the same factors in Eq. (11)).
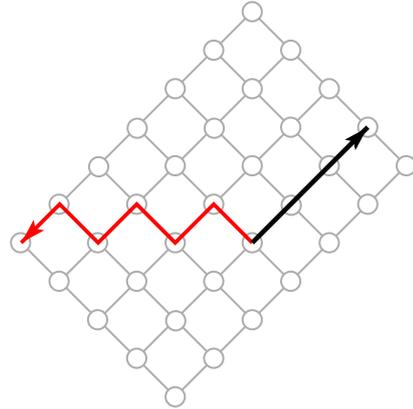

Fig. 8. Square (2+1)-dimensional computational network: view of a leaf in the rest frame. Information must zig-zag to flow at the maximal speed in diagonal direction. This leads to a slow-down of a $\sqrt{2}$ factor of the analog speed compared to cubic axis direction.

events from an observer's clock. We found that the procedure introduces an in-principle indistinguishability between neighbouring events, due to the limited precision of the clock, resulting in a network that is coarse-grained, with the event thickness also depending on the boost. The digital version of the Lorentz transformation is an integer relation which differs from the usual analog transformation by a multiplicative real constant corresponding to the event thickness.

The present purely classical kinematical construction does not straightforwardly extend from one dimension to larger dimensions, due to the Weyl-tiling issue, namely that continuum geometry cannot simply emerge from counting sites on a discrete lattice, since e.g. in a square tiling one counts the same number of tiles along a side and along the diagonal of a square (Weyl). Thus, for example, as shown in Fig. 8, in a causal network shaped as a square-lattice the fastest speed would be along the cubic axes, whereas along diagonals information should zig-zag, resulting in a slowdown by a factor $\sqrt{2}$ (or even $\sqrt{3}$ in three dimensions). Indeed, a general theorem of Fritz (2011) shows that the polytope of points that can be reached in no more than N links in a periodic graph does not approach a circle for large N. Since the polytope has necessarily distinguished directions, this means that there is no periodic graph for which this velocity set is isotropic. This result represents a no-go theorem for the emergence of an isotropic space from a discrete homogeneous causal network representing a *classical* information flow.

The situation, however, is completely different if one considers the possibility that information can flow in a superposition of
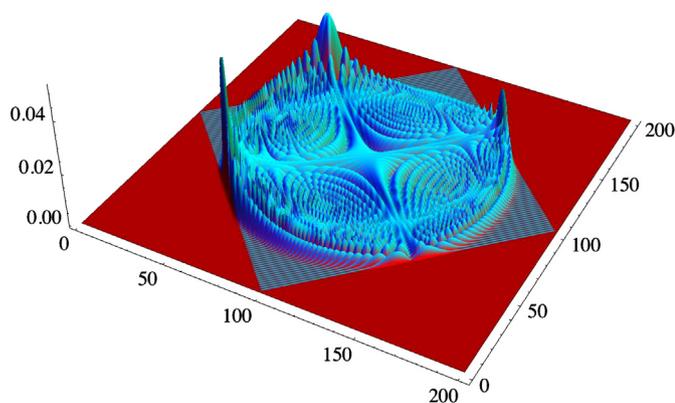
Fig. 9. Evolution of a localized state for 60 steps of the probability distribution of finding a particle or antiparticle in a two-dimensional quantum Weyl automaton of the kind of Bialynicki-Birula (1994) on a square-lattice. One can see how the propagation speed is isotropic after few steps.

paths, along the network, as in a quantum cellular automaton, corresponding to a homogeneous quantum computational network. In Fig. 9 a concrete example of evolution is given for a two-dimensional quantum Weyl automaton of the kind of (Bialynicki-Birula, 1994) on a square-lattice. One can see that the maximum propagation speed is isotropic after just few steps. In a similar way full Lorentz covariance is expected to be restored in the same limit of infinitely many events–a kind of large-scale limit corresponding to observing the automaton at the Fermi scale. Describing the automaton in the momentum space the large-scale limit is rigorously defined by considering small momenta (Bisio et al., 2012), indeed mathematically a cut-off on the momenta is dually related to a coarse graining in the position space.

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