

Homodyne detection of the Phase through Independent Measurements

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Abstract. A feedback assisted phase detection scheme based on independent measurements of conjugated quadratures is presented. It is shown that the feedback works very effectively, achieving the best sensitivity in a few measurement steps. The influence of nonunit quantum efficiency at detectors is taken into account.

1 Introduction

Homodyne detection is the most commonly adopted scheme for measuring phase shifts. This is due to the fact that it combines experimental simplicity with optimal sensitivity [1]. However, the optimum powerlaw $\delta\phi \sim \langle \hat{n} \rangle^{-1}$ is degraded by non-unit quantum efficiency at detectors. A way to overcome this difficulty is to consider novel phase detection schemes that are based on couples of independent homodyne measurements of conjugated quadratures of the field. This means that every measurement is performed on the field prepared in the same input state before every detection step, not on the state after reduction from the preceding measurement. This situation is very common in actual experiments: in practice the whole sequence of measurements is performed on a stable radiation source, well within the stability time of the source. This is what happens, for example, in quantum tomography experiments [2], where up to $10^4 \div 10^5$ homodyne measurements at different reference phases with respect to the local oscillator (LO) are performed within the stability time of the source.

In this paper we will analyze this two-quadrature scheme, showing numerically that it allows to achieve high sensitivities for a large range of the involved parameters.

The paper is organized as follows. In the first part we describe the detection scheme and we explain how a sequence of conjugated quadratures measurements is requested in order to extract informations about the phase. Then we illustrate our results on the basis of a Monte Carlo simulation of real experiments. The last part is devoted to some concluding remarks.

2 Detection Scheme

The homodyne detector is schematically depicted in Fig. 1. The input field a

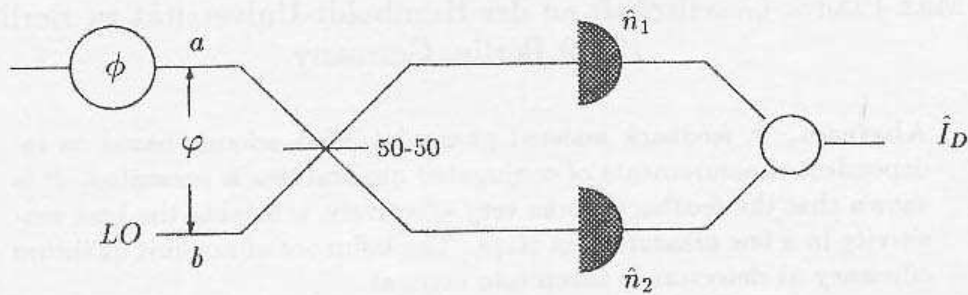


Figure 1: Scheme of a homodyne detector.

undergoes an unknown phase shift ϕ , then it impinges into a 50-50 beam splitter together with a local oscillator (LO) b . The LO has the same frequency, is synchronous and has a stable phase difference with respect to a . The two output beams follow two different paths and are finally recombined to detect the difference photocurrent $\hat{I}_D = \hat{n}_2 - \hat{n}_1$ at zero frequency. If the LO is prepared in a highly excited coherent state $|z\rangle$ with $|z| \gg 1$, the reduced photocurrent $\hat{I}_D/2|z|$ approaches a quadrature $\hat{x}_\varphi = \frac{1}{2}(ae^{-i\varphi} + a^\dagger e^{i\varphi})$ of the field. The phase φ with respect to the local oscillator can be adjusted at will by changing the path lengths. The homodyne output probability distribution when measuring the quadrature \hat{x}_φ for input state $\hat{\rho}$ will be denoted as follows

$$p(x; \varphi) = {}_\varphi \langle x | \hat{\rho} | x \rangle_\varphi, \quad (2.1)$$

where $|x\rangle_\varphi$ denotes the eigenvector of \hat{x}_φ corresponding to eigenvalue x , namely $\hat{x}_\varphi |x\rangle_\varphi = x|x\rangle_\varphi$. In the following, we will consider a fixed reference state $\hat{\rho}$ which undergoes an unknown phase shift ϕ . In such case, the quadrature probability distribution is given by

$${}_\varphi \langle x | e^{i\phi \hat{a}^\dagger} \hat{\rho} e^{-i\phi \hat{a}^\dagger} | x \rangle_\varphi \equiv {}_{\varphi-\phi} \langle x | \hat{\rho} | x \rangle_{\varphi-\phi} \doteq p(x; \varphi - \phi). \quad (2.2)$$

The scheme that we suggest consists of two independent measurements of $\hat{x}_{\phi_{LO}}$ and $\hat{x}_{\phi_{LO}-\frac{\pi}{2}}$, adjusting the phase φ relative to the LO alternatively as $\varphi = \phi_{LO}$ and $\varphi = \phi_{LO} - \pi/2$. Noticeably, this scheme does not suffer the 3 dB noise [4] added by a joint measurement, however at the expense of doubling the number of measurements. The probability distribution $P(x, y; \phi_{LO} - \phi)$ for the couple of outputs (x, y) is factorized into the two independent homodyne distributions $p(x; \phi_{LO} - \phi - \frac{\pi}{2})$ and $p(y; \phi_{LO} - \phi)$ as follows

$$P(x, y; \phi_{LO} - \phi) = p(x; \phi_{LO} - \phi - \frac{\pi}{2})p(y; \phi_{LO} - \phi), \quad (2.3)$$

It has been shown [1] that the narrowest $P(x, y; \phi_{LO} - \phi)$ is obtained for input weakly squeezed states with principal squeezing axes parallel to the two quadratures, and that phase sensitivity depends dramatically on the phase difference $(\phi_{LO} - \phi)$ [3]. Thus, for the shifted input we choose weakly squeezed states $|\alpha, \zeta\rangle$ with amplitude $\alpha = |\alpha|e^{i\phi}$ and squeezing parameter $\zeta = re^{i2\phi}$. For such states, the probabilities in Eq. (2.3) are Gaussians, with average values given by

$$\bar{x} = |\alpha| \cos(\phi_{LO} - \phi), \quad \bar{y} = |\alpha| \sin(\phi_{LO} - \phi), \quad (2.4)$$

and variances

$$\begin{aligned} \Delta_x^2 &= \frac{1}{4} [\cosh(2r) + \sinh(2r) \cos(2\phi_{LO} - 2\phi)] + \frac{1-\eta}{4\eta}, \\ \Delta_y^2 &= \frac{1}{4} [\cosh(2r) - \sinh(2r) \cos(2\phi_{LO} - 2\phi)] + \frac{1-\eta}{4\eta}, \end{aligned} \quad (2.5)$$

where the smearing effect of non unit quantum efficiency η has been taken into account. For each event (x, y) , we define an output phase as follows

$$\phi_* = \phi_{LO} - \arctan\left(\frac{y}{x}\right), \quad (2.6)$$

where \arctan is evaluated in $[-\pi, \pi]$, taking into account the individual signs of x and y . Moreover, from Eqs. (2.3) and (2.5), we see that, when the direction of the local oscillator ϕ_{LO} coincides with the input phase shift ϕ , the probability distribution in the complex plane corresponds to the generalized Wigner function $W_s(x+iy, x-iy)$ for $s = 1 - \eta^{-1}$ [5], and one has

$$P(x, y; \phi_{LO} - \phi) = \frac{1}{2\pi\Delta_x^2\Delta_y^2} \exp\left[-\frac{(x - |\alpha|)^2}{2\Delta_x^2} - \frac{y^2}{2\Delta_y^2}\right], \quad (2.7)$$

where Δ_x^2 and Δ_y^2 are evaluated in $\phi = \phi_{LO}$ [for $\eta = 1$, Eq. (2.7) becomes the usual Wigner function $W(x+iy, x-iy)$]. But ϕ is just the value that we want to measure, and thus it is necessary to adjust ϕ_{LO} after every couple of measurements in order to reach the optimum working point. The linear map describing the feedback action is given by

$$\phi_{LO} \longrightarrow (1 - \lambda)\phi_{LO} + \lambda\phi_*, \quad (2.8)$$

where $\lambda \in [0, 2]$ is the coupling parameter. This feedback does not influence the input mode, because it only acts on the path length of the LO. After a sufficiently

high number of steps, this mechanism is expected to drive ϕ_{LO} towards the optimum value $\phi_{LO} \equiv \phi$. Moreover it is expected to add some noise to the stationary probability distribution for ϕ_* , with respect to the ideal case $\phi_{LO} = \phi$, due to statistical diffusion of ϕ_* around the desired value ϕ . However, our Monte Carlo simulation has shown that this process is negligible in a large range of the coupling parameter λ , and this feedback assisted measurement scheme allows reaching sensitivities that approach the ideal power-law $\delta\phi \sim \langle \hat{n} \rangle^{-1}$.

3 Monte Carlo experiments

In our Monte Carlo simulations of a real experiment, we set the local oscillator phase ϕ_{LO} initially at random, due to our ignorance about the real value of ϕ . The feedback gradually centers ϕ_{LO} around the “true” value ϕ , thus improving the sensitivity of the apparatus. The iterative relation between the ϕ_{LO} distribution at

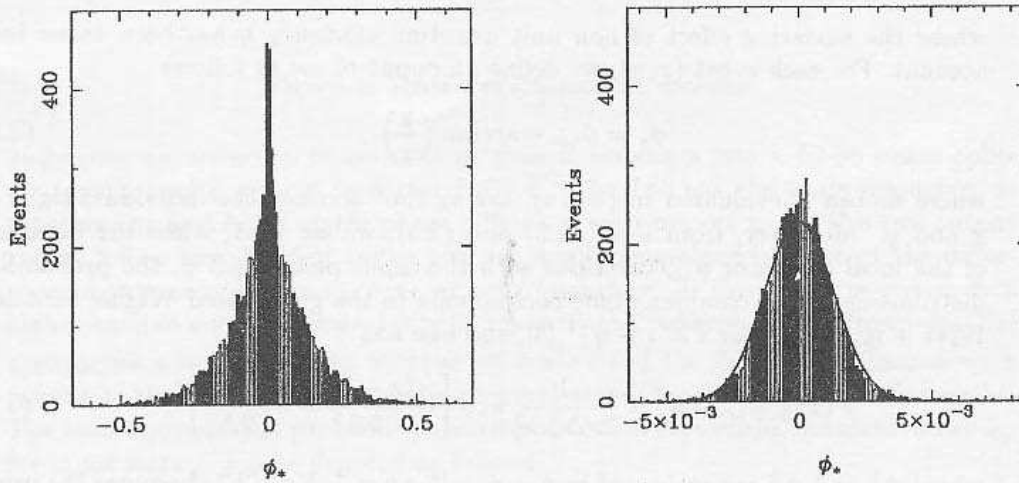


Figure 2: Marginal distribution $m(\phi_*, \phi_{LO} - \phi)$ from a Monte Carlo simulation with 10^4 collected data in each step. On the left: step $N = 2$, on the right: step $N = 64$ with a superimposed Gaussian function. In both the plots $\lambda = 0.6$, $\eta = 1$, $\phi = 0$, $\langle \hat{n} \rangle = 1000$, and the number of squeezing photons is the 5 percent of the total. Notice the non Gaussian shape of the histogram after only 2 steps and the very narrow Gaussian after 64 steps (scales on the plot are different).

the N -th step $p_{LO}^{(N)}(\phi_{LO}; \phi)$ and $p_{LO}^{(N-1)}(\phi_{LO}; \phi)$ at the preceding step can be written in terms of the marginal $m(\phi_*, \phi_{LO} - \phi)$ of the joint probability $P(x, y; \phi_{LO} - \phi)$,

$$m(\phi_*; \phi_{LO} - \phi) = \int_0^\infty d\rho \rho P(\rho \cos(\phi_{LO} - \phi_*), \rho \sin(\phi_{LO} - \phi_*); \phi_{LO} - \phi). \quad (3.9)$$

In this way, we can derive the expression for the probability distribution $p^{(N)}(\phi_*; \phi)$ of the outputs ϕ_* at the N -th step

$$p^{(N)}(\phi_*; \phi) = \int_{-\pi}^\pi d\phi_{LO} m(\phi_*; \phi_{LO} - \phi) p_{LO}^{(N)}(\phi_{LO}; \phi). \quad (3.10)$$

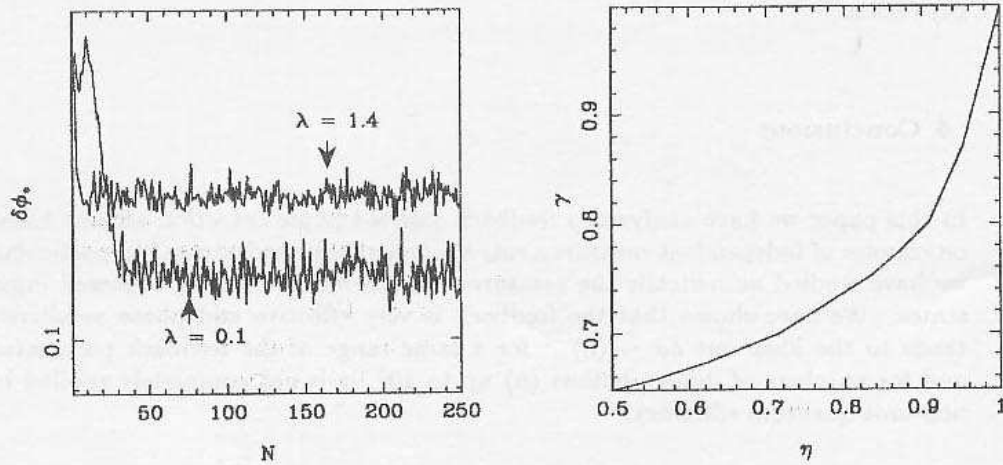


Figure 3: On the left: Semi-log plot of the r.m.s. width of $p^{(N)}(\phi_*; \phi)$ for $\eta = 1$ versus the number of feedback steps N for two different values of λ . The simulation is performed on a state with $\phi = 0$, $\langle \hat{n} \rangle = 10$ and a 3.4 percentage of squeezing photons. On the right: function γ of the phase sensitivity power-law $\delta\phi_* \sim \langle \hat{n} \rangle^{-\gamma}$ vs the quantum efficiency η .

An example of two successive steps of the evolution of $p^{(N)}(\phi_*; \phi)$ is given in Fig. 2, where the value of ϕ is set equal to zero (every step consists of a couple of measurements). The input squeezed state has a total number of photons $\langle \hat{n} \rangle = 1000$, a small fraction of which is for squeezing; the quantum efficiency of the detectors is equal to 1, and the coupling parameter is $\lambda = 0.6$. As at the beginning ϕ_{LO} is chosen at random, the distribution of ϕ_* after the first steps is very broad. Then it gradually shrinks, approaching the marginal of $W(x+iy, x-iy)$ (for $\eta = 1$), and evolving towards a stationary Gaussian of r.m.s. width $\delta\phi_* \sim \langle \hat{n} \rangle^{-1}$.

Depending on the value of the parameters, some aspects of the evolution towards the stationary value change. In particular, if the feedback parameter λ increases, the number of measurement steps which are requested to reach the stationary state decreases, but the phase sensitivity decreases. In Fig. 3 on the left, we give the evolution of $\delta\phi_*$ versus the number of steps, for $\eta = 1$, and for two different values of the feedback parameter. In the limit of λ approaching zero, the r.m.s. width of the outputs probability distribution $p^{(N)}(\phi_*, \phi)$ reaches the optimum value $\delta\phi_* = (2\langle\hat{n}\rangle)^{-1}$, but for larger and larger number of measurement steps. Regarding the effect of non unit quantum efficiency, this degrades phase sensitivity, but for numbers $\langle\hat{n}\rangle$ of input photons up to thousands, the degradation is smoother than in the usual single homodyne measurement. In particular, for input squeezed states with number of photons which is less or equal to $[\alpha_{opt}(1 - \eta)]^{-1}$, where $\alpha_{opt} = \sinh^2 r / \langle n \rangle$ is the optimum fraction of squeezing photons, the sensitivity follows the law $\delta\phi_* \sim \langle\hat{n}\rangle^{-\gamma}$, where the exponent γ is a function of η . On the other hand, for higher numbers of input photons, the sensitivity approaches the shot noise limit. In Fig. 3, we report the exponent γ as a function of η in the low $\langle\hat{n}\rangle$ region.

4 Conclusions

In this paper we have analyzed a feedback assisted phase detection scheme based on couples of independent measurements of conjugated quadratures. In particular, we have studied numerically the measurement scheme, for weakly squeezed input states. We have shown that the feedback is very effective and phase sensitivity tends to the ideal one $\delta\phi \sim \langle\hat{n}\rangle^{-1}$ for a large range of the feedback parameter, and for numbers of input photons $\langle\hat{n}\rangle$ up to 10^3 , it is not completely spoiled by non-unit quantum efficiency.

5 References

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- [3] G. M. D'Ariano, M. G. A. Paris and R. Seno, unpublished.
- [4] H. P. Yuen, Phys. Lett. A 91,101 (1982).
- [5] The generalized Wigner distribution function $W_s(\alpha, \bar{\alpha})$ in the phase space is defined as

$$W_s(\alpha, \bar{\alpha}) = \int \frac{d^2\lambda}{\pi^2} e^{\alpha\bar{\lambda} - \bar{\alpha}\lambda + \frac{1}{2}s|\lambda|^2} \text{Tr} \left(\rho e^{\lambda a^\dagger - \bar{\lambda} a} \right).$$