

A Quantum-Mechanical Study of Optical Regenerators Based on Nonlinear-Loop Mirrors

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Abstract— We present a quantum-mechanical analysis of a nonlinear interferometer that achieves optical switching via cross-phase modulation (XPM) resulting from the Kerr effect. We show how it performs as a very precise optical regenerator, highly improving the transmitted bit-error rate (BER) in the presence of loss.

Index Terms— Nonlinear optics, optical fiber communications, optical Kerr effect, optical repeaters, optical switches, quantum theory, time-division multiplexing.

THE INCREASINGLY LARGE demand for information capacity in telecommunication networks urges conversion of hybrid electrooptic signal processing to all-optical processing, exploiting directly the large bandwidth available in the optical domain. For long-distance communication, losses along the line represent the crucial limitation to the maximum transmission span, and loss compensation through linear optical amplification unavoidably introduces noise of quantum-mechanical origin [1], [2]. However, when the data are digitally encoded, instead of linear amplifiers one can effectively employ *digital optical switches* or *optical regenerators*. Very promising as optical switches are the nonlinear loop mirrors, which are nonlinear Sagnac interferometers made with cross-phase modulating (XPM) Kerr medium. Experimental studies at high-power levels have shown that such devices can achieve very effective switching, and optical regeneration [3] up to 40 Gb/s has been practically demonstrated [4]. The quantum characteristics of nonlinear Mach-Zehnder (or equivalently, Sagnac) interferometers (MZI) have been previously investigated [5]. In this letter, we address the performance limits for optical switching with such devices, and present numerical results for the bit-error rate (BER) achievable in a lossy line with optical regenerators distributed along the line.

A fiber-optic nonlinear Sagnac interferometer is depicted in Fig. 1 along with its equivalent MZI. The input mode \hat{a} , assumed to be in a coherent state $|\alpha\rangle$, is split into the two arms of the interferometer by a 50/50 beam splitter. In one arm the pertaining field mode \hat{a}'' undergoes a Kerr

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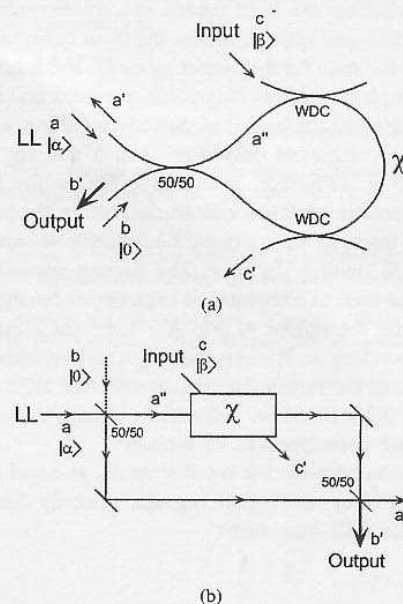


Fig. 1. (a) Schematic of a nonlinear fiber-optic Sagnac interferometer and (b) its Mach-Zehnder equivalent. WDC: Wavelength dependent coupler.

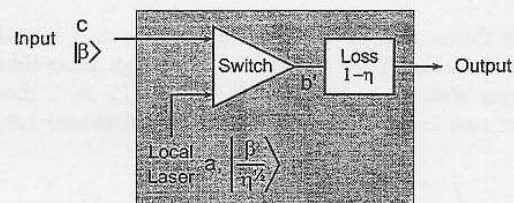


Fig. 2. Schematic representation of the nonlinear Sagnac interferometer used as a regenerator along a transmission line. Considering only the relevant ports, the regenerator is a three-port device. The optimal working point for the local laser is shown, which depends on the loss following the regenerator.

nonlinear phase shift $\hat{U} = \exp[i\chi\hat{a}''\hat{a}''\hat{c}^\dagger\hat{c}]$ that depends on the state of the other input mode \hat{c} of a different frequency and/or polarization. Here, by the hooded letters $\hat{a}, \hat{b}, \hat{c}$ we are denoting the annihilation operators of the respective field modes, by their daggered letters we mean the respective creation operators, and χ denotes the overall Kerr coupling, i.e., the third-order susceptibility of the medium integrated over its length along the direction of propagation. For \hat{c} in

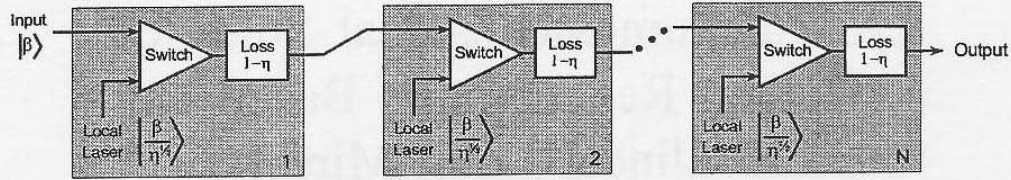


Fig. 3. Schematic of a lossy line with distributed optical regenerators.

the vacuum state there is no nonlinear phase shift for \hat{a}'' , and for an appropriate choice of phases in the two arms of the interferometer the field modes exactly recombine at the second 50/50 beam splitter, so that the input coherent state $|\alpha\rangle$ emerges as the state for the output mode \hat{a}' . If \hat{c} is nonvacuum, then a Kerr phase shift of magnitude π would make the state $|\alpha\rangle$ switch toward the output \hat{b}' . Strictly speaking, a perfect π phase shift is achieved only with \hat{c} in a number state $|n\rangle$, with $n\chi = \pi$. However, as we will see in the following, the interferometer switches effectively even in the presence of Poissonian intensity fluctuations, i.e., when \hat{c} is in a coherent state, with $|\beta\rangle$, with $|\beta|^2\chi = \pi$. The Sagnac interferometer in Fig. 1, when used as a repeater, is regarded as having the input at port c and the output at port b' . When the coherent state $|\beta\rangle$ corresponding to bit *one* enters port c the interferometer approximately retransmits the strong coherent state $|\alpha\rangle$ of the local-laser (LL) mode \hat{a} ; otherwise, it just retransmits the vacuum state corresponding to bit *zero*.

The relation between the input state $\hat{\rho}_{in}$ at \hat{c} and the output state $\hat{\rho}_{out}$ at \hat{b}' of the Sagnac repeater is easily derived; it is given by the following map:

$$\begin{aligned} \hat{\rho}_{out} &= \Gamma_{\chi,\alpha}(\hat{\rho}_{in}) \\ &\equiv \sum_{n=0}^{\infty} |\alpha e^{\frac{i}{2}n\chi} \sin(n\chi/2)\rangle \langle n|\hat{\rho}_{in}|n\rangle \langle \alpha e^{\frac{i}{2}n\chi} \sin(n\chi/2)| \end{aligned} \quad (1)$$

which shows that the resulting output state is a mixture of coherent states. When the input is in a high mean-intensity coherent state $\hat{\rho}_{in} = |\beta\rangle\langle\beta|$ with $|\beta|^2 = \pi/\chi \gg 1$, then the output state in (1) is approximated by the dephased coherent state

$$\hat{\rho}_{out} \simeq \int_{-\infty}^{+\infty} \frac{d\varphi}{\sqrt{2\pi\Delta^2}} e^{-\frac{\varphi^2}{2\Delta^2}} |i\alpha e^{i\varphi} \cos\varphi\rangle \langle i\alpha e^{i\varphi} \cos\varphi| \quad (2)$$

having a small variance $\Delta^2 = \pi^2/(4|\beta|^2) = \pi\chi/4 \ll 1$. The state in (2) has a banana-shaped Wigner function, corresponding to a quasi-Poissonian photon-number distribution with $\langle \hat{b}^\dagger \hat{b} \rangle \simeq |\alpha|^2 \exp(-\Delta^2)$ and a Gaussian phase distribution with $\langle \Delta\phi^2 \rangle = |\alpha|^{-2} + \Delta^2$.

We now consider an on-off communication scheme implemented on a lossy line with transmitted average power $P = |\gamma|^2/2$, and the *zero* and the *one* bit represented by the vacuum state $|0\rangle$ and the coherent state $|\gamma\rangle$, respectively. After a loss $1-\eta$ we insert a repeater with its Kerr coupling tuned to the switching value $\chi = \pi/(\eta|\gamma|^2)$ and the LL amplitude set to $\alpha = -i\gamma$, such that the original amplitude γ is approximately reestablished at the output [for the $-i$ phase factor see (2)]. After a further loss $1-\eta$ another repeater is inserted, and

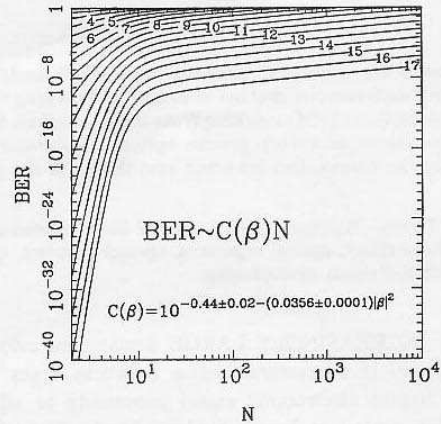


Fig. 4. BER B for a lossy line with N optical repeaters versus the number N of repeaters (see text). Different curves refer to different input field amplitudes β , ranging in unit steps from the bottom curve that corresponds to $\beta = 17$. The slowly increasing regime for large N is linear and its form is reported in the figure inset.

so forth, for an overall N number of steps (see Figs. 2 and 3). Since the only effect of loss on coherent states is just an amplitude rescaling by the factor $\sqrt{\eta}$, and because the output state from the Sagnac interferometer is a mixture of coherent states, it turns out that the overall state transformation for a loss $1-\eta$ preceded by a repeater depends only on the signal level β at the repeater input, independently of η , as long as the LL is set at the loss-compensating amplitude value $-i\eta^{-1/2}\beta \equiv -i\gamma$ and the Kerr coupling is tuned to the switching value $\chi = \pi/|\beta|^2$. Hence, the overall input-output map for the repeater-loss sequence is given by $\Gamma_{\pi/|\beta|^2, -i\beta}$. The fact that this map depends only on the input signal level β does not mean that one can recover a given signal after any arbitrarily high loss. If the input signal level is reduced too much, i.e., if $\beta \equiv \eta^{1/2}\gamma \rightarrow 0$ for a fixed peak amplitude γ , then the dephasing effect at the repeater would increasingly degrade the carrier coherence, leading to enhanced fluctuations $\Delta^2 = \pi^2/(4\eta|\gamma|^2)$ for $\eta \rightarrow 0$.

We now evaluate the transmitted BER for on-off keying and direct photodetection at the end of N optical regenerators distributed along the lossy line. The input is in the coherent state $|\beta\rangle$ and all repeaters are set at their optimal working point with the Kerr coupling $\chi = \pi/|\beta|^2$, and with the n th repeater having a LL intensity given by $|\alpha_n|^2 = |\beta|^2/\eta_n$, where $1-\eta_n$ is the loss between the n th and the $(n+1)$ th repeater. In this scheme, since there is no spontaneous emission from the repeater, the detection threshold should be set at one photon, and the BER is just the vacuum component of the output

state after iterating the map $\Gamma_{\pi/|\beta|^2, -i\beta}$ in (1) N times on the input state $|\beta\rangle$. A numerical calculation shows that after a sudden dephasing at the first step, the state remains quite stable with a slow dephasing in the following steps. A log-log plot of the BER B versus N is reported in Fig. 4 for different magnitudes of the input amplitude β . The BER increases fast for the first 10–20 steps. Then, there is a transition to a very stable linear regime $B = C(\beta)N$, with $C(\beta) = 10^{-0.44 \pm 0.02 - (0.0356 \pm 0.0001)|\beta|^2}$. So, for example, after $N = 10^2$ optical repeaters and for input amplitude $\beta = 20$ the BER is $B = 2 \times 10^{-13}$. This result should be compared with the BER achieved on a lossy line with ideal photon-number amplifiers [6], where for a gain $g = \eta^{-1} = 2$ the BER is $B = 3.4 \times 10^{-7}$. The performance is much worse when using linear phase-insensitive amplifiers due to the spontaneous-emission noise.

In conclusion, we analyzed a nonlinear Sagnac interferometer with XPM, which is used as a repeater for on-off modulation and direct detection in a lossy line with N distributed repeaters. We showed that with all repeaters set at their optimal working point, the BER increases linearly with N for large N , and the proportionality constant exponentially decreases with the input signal intensity, resulting in almost error-free communication even at very low-power levels. Finally, our monochromatic analysis remains valid even for short pulses

as long as dispersion effects are negligible and the cross-Kerr susceptibility can be considered approximately constant over the pulses' frequency bandwidth.

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