Why is not so easy to change Quantum Mechanics and one of the only possible changes is GRW

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Is quantum theory exact? The endeavor for the theory beyond standard quantum mechanics.

Laboratori Nazionali di Frascati, April 29 2014

A. Bisio, G. M. D'Ariano, A. Tosini, arXiv:1212.2839
Quantum Mechanics = Quantum Theory + Mechanics

1. Information-theoretic Axioms for QT
   - Operational probabilistic theory (OPT) framework

2. QT → QFT → QM

3. Which modifications destroy the epistemological value of QT, and why GRW is compatible with (1)

4. Proposal: through (2) we can make GRW Lorentz covariant and for QFT

Problems with GRW

- Lorentz covariance
- Indistinguishable particles
- QFT

A. Rimini (private comm.)
Historical background

- The experience in Quantum Information has led us to look at Quantum Theory (QT) under a completely new angle
- QT is a theory of information

Principles for Quantum Theory

P1. Causality
P2. Local discriminability
P3. Purification
P4. Atomicity of composition
P5. Perfect distinguishability
P6. Lossless Compressibility

Book from CUP (by the end of 2014)
Principles for Quantum Theory

The informational framework

Logic ⊆ Probability ⊆ OPT

joint probabilities + connectivity

\[ p(i, j, k, \ldots | \text{circuit}) \]

Marginal probability

\[ \sum_{i, k, \ldots} p(i, j, k, \ldots | \text{circuit}) = p(j | \text{circuit}) \]
Principles for Quantum Theory

The informational framework

Logic ⊂ Probability ⊂ OPT

joint probabilities + connectivity

\[ p(i, j, k, \ldots | \text{circuit}) \]

**Leaf:**
Maximal set of independent systems

\[
\rho_i \quad B \quad := \quad I \quad \mathcal{A}_i \quad B
\]

preparation

\[
A \quad \mathcal{a}_j \quad := \quad A \quad \mathcal{A}_j \quad I
\]

observation

\[
p(i, j, k, l, m, n, p, q | \text{circuit})
\]
Principles for Quantum Theory

The informational framework

Logic ⊂ Probability ⊂ OPT

joint probabilities + connectivity

\[ p(i, j, k, \ldots | \text{circuit}) \]

*Leaf:* Maximal set of independent systems
Principles for Quantum Theory

The *informational* framework

Logic ⊂ Probability ⊂ OPT

joint probabilities + connectivity

\[ p(i, j, k, \ldots | \text{circuit}) \]

*Leaf:*
Maximal set of independent systems

*Foliation*
Principles for Quantum Theory

The *informational* framework

Logic ⊆ Probability ⊆ OPT

joint probabilities + connectivity

Probabilistic equivalence classes
Principles for Quantum Theory

P1. **Causality**

P2. Local discriminability

P3. Purification

P4. Atomicity of composition

P5. Perfect distinguishability

P6. Lossless Compressibility

The probability of preparations is independent of the choice of observations

Control of experiment

no signaling without interaction

\[ p(i, j|\mathcal{X}, \mathcal{Y}) := \rho_i \]

\[ p(i|\mathcal{X}, \mathcal{Y}) = p(i|\mathcal{X}, \mathcal{Y}')} = p(i|\mathcal{X}) \]

Marginal state

\[ =: \rho_a \]
Principles for Quantum Theory

P1. Causality
P2. Local discriminability
P3. Purification
P4. Atomicity of composition
P5. Perfect distinguishability
P6. Lossless Compressibility

It is possible to discriminate any pair of states of composite systems using only local measurements.

Local discriminability of states

Local characterization of transformations

Origin of the complex tensor product

Local testability of the physical law
Principles for Quantum Theory

P1. Causality
P2. Local discriminability
P3. Purification
P4. Atomicity of composition
P5. Perfect distinguishability
P6. Lossless Compressibility

The composition of two atomic transformations is atomic

Complete information can be accessed on a step-by-step basis
Principles for Quantum Theory

P1. Causality
P2. Local discriminability
P3. Purification
P4. Atomicity of composition
P5. **Perfect distinguishability**
P6. Lossless Compressibility

Every state that is not completely mixed (i.e. on the boundary of the convex) can be perfectly distinguished from some other state.

Falsifiability of the theory
Principles for Quantum Theory

P1. Causality
P2. Local discriminability
P3. Purification
P4. Atomicity of composition
P5. Perfect distinguishability
P6. **Lossless Compressibility**

For states that are not completely mixed there exists an ideal compression scheme.

Any face of the convex set of states is the convex set of states of some other system.

Encoding only unknown information.
Principles for Quantum Theory

P1. Causality
P2. Local discriminability
P3. **Purification**
P4. Atomicity of composition
P5. Perfect distinguishability
P6. Lossless Compressibility

Every state has a purification. For fixed purifying system, every two purifications of the same state are connected by a reversible transformation on the purifying system.

Conservation of information. Reversibility.
**Principles for Quantum Theory**

P1. Causality
P2. Local discriminability
**P3. Purification**
P4. Atomicity of composition
P5. Perfect distinguishability
P6. Lossless Compressibility

Every state has a purification. For fixed purifying system, every two purifications of the same state are connected by a reversible transformation on the purifying system.

**Consequences**

1. **Existence of entangled states:** the purification of a mixed state is an entangled state; the marginal of a pure entangled state is a mixed state;

2. Every two normalized pure states of the same system are connected by a reversible transformation

\[
\begin{align*}
\psi' &\xrightarrow{B} \psi &\xrightarrow{U_B} \psi
\end{align*}
\]

3. **Steering:** Let \( \Psi \) purification of \( \rho \). The for every ensemble decomposition \( \rho = \sum_x p_x \alpha_x \) there exists a measurement \( \{ b_x \} \), such that

\[
\begin{align*}
\psi &\xrightarrow{A} \alpha_x &\xrightarrow{b_x} \psi
\end{align*}
\]

\( \forall x \in X \)

4. **Process tomography (faithful state):**

\[
\begin{align*}
\Psi &\xrightarrow{A} \mathcal{A} &\xrightarrow{A'} \Psi
\end{align*}
\]

\( \forall \rho \)

5. **No information without disturbance**
Principles for Quantum Theory

P1. Causality
P2. Local discriminability
P3. **Purification**
P4. Atomicity of composition
P5. Perfect distinguishability
P6. Lossless Compressibility

Every state has a purification. For fixed purifying system, every two purifications of the same state are connected by a reversible transformation on the purifying system.

Conservation of information. Reversibility.

### Consequences

6. **Teleportation**

\[
\Phi \begin{array}{c}
A \\
B \\
A
\end{array} \begin{array}{c}
B_x \\
A
\end{array} = p_x \begin{array}{c}
A \\
\mathcal{U}_x
\end{array} A \quad \forall x \in X
\]

7. **Reversible dilation of “channels”**

\[
C \begin{array}{c}
A \\
A
\end{array} = \begin{array}{c}
\eta \\
E \\
A
\end{array} \begin{array}{c}
\mathcal{U} \\
E \\
A
\end{array} \begin{array}{c}
e \\
A
\end{array}
\]

8. **Reversible dilation of “instruments”**

\[
A \begin{array}{c}
A \\
\mathcal{A}_x \\
A
\end{array} = \begin{array}{c}
\eta \\
E \\
B
\end{array} \begin{array}{c}
\mathcal{U} \\
E \\
B
\end{array} \begin{array}{c}
b_x \\
e \\
E
\end{array} \quad \forall x \in X
\]

9. **State-transformation cone isomorphism**

10. **Rev. transform. for a system make a Lie group**
Moving to the Mechanics

- The Weyl, Dirac, and Maxwell equations are derived from information-theoretic principles only, without assuming SR
- Only denumerable quantum systems in interaction
- QCA to be regarded as a theory unifying scales from Planck to Fermi (no continuum limit!)

- QFT is recovered in the relativistic limit \(k \ll 1\)
- In the ultra-relativistic limit (Planck scale) Lorentz covariance is an approximate symmetry, and one has the Doubly Special Relativity of Amelino-Camelia/Smolin/Magueijo

Additional principles

Min algorithmic complexity of the processing

- linearity
- unitarity
- locality
- homogeneity
- transitivity
- isotropy
- minimal-dimension

Quantum Cellular Automaton (QCA)

GOOD FEATURES

1. no SR assumed: emergence of relativistic quantum field and space-time
2. quantum ab-initio
3. no divergencies and all the problems from the continuum
4. no “violations” of causality
5. computable
6. dynamics stable (dispersive Schrödinger equation for narrow-band states valid at all scales)
7. solves the problem of localization in QFT
8. natural scenario for the holographic principle
QFT from info-principles

- minimal-dimension
- linearity
- homogeneity
- transitivity

| System $\psi(g)$, $\psi$ $s$-dimensional field operator, labeled by $g \in G$, $|G| \leq \aleph$
| --- |
| $s > 1$ ($s=1$ trivial evolution)
| Interactions described by transition matrices $A_{gg'} \in M_s(C)$ between systems $g \in G$:
  - single evolution step $\psi(g) \rightarrow \psi(g) = \sum_{g' \in S_g} A_{gg'} \psi(g')$
  - $S_g \subseteq G$ set of systems interacting with $g$

- homogeneity
- transitivity

| $\{A_{gg'}\}_{g' \in S_g}$ independent of $g$, Cayley graph $K(G,S_+)$
| $G$ group, $G=\langle h_1, h_2, \ldots, h_N, r_1, r_2, \ldots, r_M \rangle$
| $S_g = S_g$, $S:=\{h_1, h_2, \ldots, h_N\}$, $S=S_+US_-$, $S_-=S_+^{-1}$

Cayley graph $K(G, S_+)$
**QFT from info-principles**

- **minimal-dimension**
- **linearity**
- **homogeneity**
- **transitivity**
- **locality**
- **unitarity**

*Problem: find \{A_h\} ∈ M_s(C) such that A^†A = I*

- **isotropy**
  G finitely-generated group, K(G, S+) quasi-isometrically embeds in \(\mathbb{R}^3\), G contains a free Abelian subgroup \(A\) of finite index, with \(\text{rank}(A) \leq 3\) (Misha Kapovich, priv. comm.)

unitary \(s\)-dimensional (projective) \{\(L_l\)\} of \(L\) determines the statistics of \(\psi\), if Fermion, Boson, Anyon

---

**the theoretical minimum**

- \(s > 1\) \((s=1\) trivial evolution\)
- Interactions described by transition matrices \(A_{gg'} ∈ M_s(C)\) between systems \(g ∈ G\):
  - single evolution step \(\psi(g) \rightarrow \psi(g) = \sum_{g' ∈ S_g} A_{gg'} \psi(g')\)
  - \(S_g ⊆ G\) set of systems interacting with \(g\)
- \(\{A_{gg}\}_{g' ∈ S_g}\) independent of \(g\), Cayley graph \(K(G, S_+)\)
- \(G\) group, \(G = \langle h_1, h_2, ..., h_N, |r_1, r_2, ..., r_M⟩\)
- \(S_g = S_g, S = \{h_1, h_2, ..., h_N\}, S = S_+US_-, S = S_+^{-1}\)
- \(|S| < ∞ \Leftrightarrow G\) finitely generated

\[
A = \sum_{h ∈ S_+ ∪ S_+^{-1}} T_h ⊗ A_h
\]

\(T_h\) unitary repr. of \(G\) on \(l^2(G)\)

- \(A_h ≠ 0 \Leftrightarrow A_h^{-1} ≠ 0\)
- There exists a group \(L\) of permutations of \(S_+\), transitive over \(S_+\) that leaves \(K(G, S_+)\) invariant
- a nontrivial unitary \(s\)-dimensional (projective) representation \(\{L_l\}\) of \(L\) such that:

\[
A = \sum_{h ∈ S} T_h ⊗ A_h = \sum_{h ∈ S} T_{lh} ⊗ L_l A_h L_l^†
\]
The Weyl QCA

- Minimal dimension for nontrivial unitary $A$: $s=2$
- Unitarity $\Rightarrow$ the only possible $G$ is the BCC!!
- $A_h$ are proportional to rank-one projectors
- Isotropy $\Rightarrow$ Fermionic $\psi$ ($d=3$)

$$A = \int_B d\mathbf{k} |\mathbf{k}\rangle \langle \mathbf{k}| \otimes A_{\mathbf{k}}$$

$$A_{\pm\mathbf{k}} = -i\sigma_x(s_{x}c_{y}c_{z} \pm c_{x}s_{y}s_{z})$$
$$-i(\pm\sigma_y)(c_{x}s_{y}c_{z} \mp s_{x}c_{y}s_{z})$$
$$-i\sigma_z(c_{x}c_{y}s_{z} \pm s_{x}s_{y}c_{z})$$
$$+I(c_{x}c_{y}c_{z} \mp s_{x}s_{y}s_{z})$$

$s_{\alpha} = \sin \frac{k_{\alpha}}{\sqrt{3}}$
$c_{\alpha} = \cos \frac{k_{\alpha}}{\sqrt{3}}$
**Dirac QCA**

Local coupling: $A_k$ coupled with inverse with off-diagonal identity block matrix

$$E_{k}^{\pm} = \begin{pmatrix} nA_{k}^{\pm} & imI \\ imI & nA_{k}^{\pm \dagger} \end{pmatrix}$$

$$n^2 + m^2 = 1$$

$E_{k}^{\pm}$ CPT-connected!

$$\omega_{E}^{\pm}(k) = \cos^{-1}[n(c_x c_y c_z \pm s_x s_y s_z)]$$

Dirac in relativistic limit $k \ll 1$

$m \leq 1$: mass

$n^{-1}$: refraction index

**Maxwell QCA**

$$M_k = A_{k}^{\dagger} \otimes A_k$$

$$F^\mu(k) = \int \frac{dq}{2\pi} f(q) \tilde{\psi}(\frac{k}{2} - q) \sigma^\mu \varphi(\frac{k}{2} + q)$$

Maxwell in relativistic limit $k \ll 1$

Boson: emergent from convolution of fermions *(De Broglie neutrino-theory of photon)*

$m \leq 1$: mass

$n^{-1}$: refraction index
Universal constants of QCA theory

Conversion to dimensional units

\[
\begin{array}{ccc}
    l_P & t_P & m_P \\
\end{array}
\]

\[m_g = m \cdot m_P\]

\[p = \frac{\hbar k}{\sqrt{3}l_P}\]

\[c := \frac{l_P}{t_P}\]

\[\hbar = m_P l_P c\]

\[G = \frac{l_P c^2}{m_P}\]
Dirac emerging from the QCA

fidelity with Dirac evolution for a narrowband packet in the relativistic limit \( k \approx m \ll 1 \)

\[
F = \left| \langle \exp \left[ -iN \Delta(k) \right] \rangle \right| \quad \omega^E(k) = \sqrt{k^2 + m^2}
\]

\[
\Delta(k) := \left( m^2 + \frac{k^2}{3} \right)^{\frac{1}{2}} - \omega^E(k)
\]

\[
= \frac{\sqrt{3}k_x k_y k_z}{(m^2 + \frac{k^2}{3})^{\frac{1}{2}}} - \frac{3(k_x k_y k_z)^2}{(m^2 + \frac{k^2}{3})^{\frac{3}{2}}} + \frac{1}{24} \left( m^2 + \frac{k^2}{3} \right)^{\frac{3}{2}} + O(k^4 + N^{-1} k^2)
\]

relativistic proton: \( N \approx m^{-3} = 2.2 \times 10^{57} \Rightarrow t = 1.2 \times 10^{14} \text{s} = 3.7 \times 10^6 \text{y} \)

UHECRs: \( k = 10^{-8} \gg m \Rightarrow N \approx k^{-2} = 10^{16} \Rightarrow 5 \times 10^{-28} \text{s} \)
2d automaton

- Evolution of a narrow-band particle-state
- Evolution of a localized state
superluminal

Dirac QCA

\[ k = \left( \frac{\pi}{10}, 0 \right), \ m = .1, \ N = 120 \]

\[ \Delta_x^2 = 10^2, \ \Delta_y^2 = \frac{1}{2} \Delta_x^2 \]

Particle state: \( k_0 = 0, \ m = 0.15, \ \sigma = 40 \). Oscillation frequency \( \nu = 0.048 \)
The general dispersive Schrödinger equation

\[ i \partial_t \psi(k, t) = s[\omega(k) - \omega_0] e^{-ik_0 \cdot x + i\omega_0 t} \psi(k, t) \]

\[ i \partial_t \tilde{\psi}(k, t) = s[\omega(k) - \omega_0] \tilde{\psi}(k, t) \]

\[ i \partial_t \tilde{\psi}(x, t) = s[v \cdot \nabla + \frac{1}{2} D \cdot \nabla \nabla] \tilde{\psi}(x, t) \]

\[ v = (\nabla_k \omega)(k_0) \]

\[ D = (\nabla_k \nabla_k \omega)(k_0) \]
Planck-scale effects: Lorentz covariance distortion

Transformations that leave the dispersion relation invariant

\[ \omega^{(\pm)}(k) \]

\[ \omega_E(k) := \pm \cos^{-1}\left(\sqrt{1 - m^2 \cos k}\right) \]

\[ \omega' = \arcsin\left[\gamma \left(\sin \omega / \cos k - \beta \tan k\right) \cos k'\right] \]

\[ k' = \arctan\left[\gamma \left(\tan k - \beta \sin \omega / \cos k\right)\right] \]

\[ \gamma := (1 - \beta^2)^{-1/2} \]
Planck-scale effects: Lorentz covariance distortion

For narrow-band states we can linearize Lorentz transformations around \( k=k_0 \) and we get \( k \)-dependent Lorentz transformations.

\[
|\psi\rangle = \int dk \mu(k) \hat{g}(k)|k\rangle \xrightarrow{L_B^{D}} \int dk \mu(k) \hat{g}(k)|k'\rangle = \int dk \mu(k') \hat{g}(k(k'))|k'\rangle
\]

Delocalization under boost

Relative locality

Modifications of QM

- Complete positivity
- Unitarity
- Schrödinger equation
- Linearity of QT (not of QFT)
- Convexity
- Causality
- QFT
- GRW

Please don't waste time with interpretations!!
Problems with GRW

- Lorentz covariance
- Indistinguishable particles
- QFT

A. Rimini (private comm.)

A proposed solution

GRW of $\psi^\dagger \psi(g)$ homogeneous and isotropic

emergence

GRW for particle position

- Lorentz covariance in the relativistic limit
- GRW for QFT
THANK YOU!

A Quantum-Digital Universe (ID: 43796)

Paolo Perinotti  Alessandro Bisio  Alessandro Tosini

Alexandre Bibeau  Franco Manessi  Nicola Mosco  Marco Erba
Items for discussion

- QT is a theory of information
- The Weyl, Dirac, and Maxwell equations are derived from information-theoretic principles only, without assuming SR
- Only denumerable quantum systems in interaction
- QCA theory to be regarded as a theory unifying scales from Planck to Fermi (no continuum limit!)
- QFT is recovered in the relativistic limit ($k \ll 1$)
- In the ultra-relativistic limit (Planck scale) Lorentz covariance is an approximate symmetry, and one has the Doubly Special Relativity of Amelino-Camelia/Smolin/Magueijo

Proposed solution for GRW

- GRW of $\psi^\dagger \psi(g)$ homogeneous and isotropic
- GRW for particle position
- Lorentz covariance in the relativistic limit
- GRW for QFT

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