Relativity principle without space-time

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Is quantum theory exact?
The endeavor for the theory beyond standard quantum mechanics.
Second Edition FQT2015

Frascati September 23-25 2015
Program

Derive the whole Physics from principles

Physics as an axiomatic theory

with thorough physical interpretation
Principles for Quantum Theory

I Selected for a Viewpoint in Physics
PHYSICAL REVIEW A 84, 012311 (2011)

Informational derivation of quantum theory

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(Received 29 November 2010; published 11 July 2011)

We derive quantum theory from purely informational principles. Five elementary axioms—causality, perfect distinguishability, ideal compression, local distinguishability, and pure conditioning—define a broad class of theories of information processing that can be regarded as standard. One postulate—purification—singles out quantum theory within this class.

DOI: 10.1103/PhysRevA.84.012311
PACS number(s): 03.67.Ac, 03.65.Ta

Principles for Quantum Theory

P1. Causality
P2. Local discriminability
P3. Purification
P4. Atomicity of composition
P5. Perfect distinguishability
P6. Lossless Compressibility

Book from CUP soon!

• Mechanics (QFT) derived in terms of countably many quantum systems in interaction

Min algorithmic complexity principle

• homogeneity
• locality
• reversibility
Principles for Quantum Field Theory

- QFT derived in terms of countably many quantum systems in interaction

**add principles**

- homogeneity
- locality
- reversibility

Quantum Cellular Automata on the Cayley graph of a group $G$

Restrictions

\[ G = \langle h_1, h_2, \ldots | r_1, r_2, \ldots \rangle := \langle S_+ | R \rangle \]
Principles for Quantum Field Theory

- QFT derived in terms of countably many quantum systems in interaction

add principles

- homogeneity
- locality
- reversibility
- linearity
- isotropy
- minimal-dimension
- Cayley qi-embedded in $R^d$

Min algorithmic complexity principle

Quantum Cellular Automata on the Cayley graph of a group $G$

Restrictions

- homogeneity
- locality
- reversibility

(geometric group theory)

G virtually Abelian

Linearity (free QFT)

Quantum Cellular Automaton $\Rightarrow$ Quantum Walk

$$U\psi U^\dagger = A\psi$$

don Neumann algebra $\Rightarrow$ Fock space

Isotropy

- There exists a group $L$ of permutations of $S_+$, transitive over $S_+$ that leaves the Cayley graph invariant
- a nontrivial unitary $s$-dimensional (projective) representation $\{L_l\}$ of $L$ such that:

$$A = \sum_{h \in S} T_h \otimes A_h = \sum_{h \in S} T_{lh} \otimes L_l A_h L_l^\dagger$$
\[ G = \langle a, b | aba^{-1} b^{-1} \rangle \equiv \mathbb{Z} \times \mathbb{Z} \]
Proposition 2. All the $A_h$ (with $h \in S$) are full rank.

Proof. The unitarity condition $P_h P_h^\dagger = I$ leads to $A_h A_h^\dagger = 0$. Then either $A_h$ is full rank and $A_h^\dagger = 0$ (against hypothesis) or both $A_h$ and $A_h^\dagger$ are not full rank. □

Proposition 3. For $s = 2$, if isotropy holds then the $A_h$ with $h^2 \in S$ and $|S| + |S| = d$ belong to a ring/group/algebra made of at most $d^2$ elements.

Proof. Being $s = 2$, the $A_h$ have rank equals to 1. Then a generic $A_h$ can be written as $A_h = |\alpha h| \cdot |\beta h|$. The composition of two arbitrary $A_h, A_k$ leads to $A_h A_k = |\alpha h| \cdot |\beta k| = |\alpha k| \cdot |\beta h|$. Thanks to isotropy we have $|\beta k| = c$ for every $\alpha, \beta$. □

Remark 2. For $s = 2$, for a virtually Abelian QW the theorem: every virtually Abelian QW with cell dimension $s$ is equivalent to an Abelian QW with quantum cell dimension multiple of $s$. 
Quantum walk on Cayley graph

**Theorem**: A group is quasi-isometrically embeddable in $\mathbb{R}^d$ iff it is **virtually Abelian**

Virtually Abelian groups have polynomial growth (Gromov)

# points $\sim r^d$
• $G$ hyperbolic

\[ G = \langle a, b | a^5, b^4, (ab)^2 \rangle \]
• $G$ hyperbolic $\rightarrow$ exponential growth

$$G = \langle a, b | a^5, b^4, (ab)^2 \rangle$$

# points $\sim \exp(r)$

information “transmitted” over the graph decreases as $\exp(-r)$
Informationalism: Principles for QFT

- QFT derived in terms of countably many quantum systems in interaction

QCA is a discrete theory
- Ultra-relativistic regime ($k \sim 1$) [Planck scale]: nonlinear Lorentz
  - Relativistic regime ($k \ll 1$): free QFT (Weyl, Dirac, and Maxwell)

QFT derived:
- without assuming Special Relativity
- without assuming mechanics (quantum ab-initio)

Motivations to keep it discrete:
1. Discrete contains continuum as special regime
2. Testing mechanisms in quantum simulations
3. Falsifiable discrete-scale hypothesis
4. Natural scenario for holographic principle
5. Solves all issues in QFT originating from continuum:
   - i) uv divergencies
   - ii) localization issue
   - iii) Path-integral
6. Fully-fledged theory to evaluate cutoffs

Min algorithmic complexity principle

- homogeneity
- locality
- reversibility
- linearity
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- Cayley qi-embedded in $\mathbb{R}^d$

Quantum Cellular Automata on the Cayley graph of a group $G$

Restrictions
- homogeneity
- locality
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- Cayley qi-embedded in $\mathbb{R}^d$

$G$ virtually Abelian
The Weyl QCA

- Minimal dimension for nontrivial unitary Abelian QW is s=2

Qi-embeddability in $\mathbb{R}^3$
- Uni+Iso $\Rightarrow$ the only possible Cayley is the BCC!!
- Iso $\Rightarrow$ Fermionic $\psi$ ($d=3$)

Unitary operator:  
\[
A = \int_B \Theta d\mathbf{k} \ A_\mathbf{k}
\]

- Two QWs connected by $P$

\[
A_{\mathbf{k}}^\pm = -i\sigma_x(s_x c_y c_z \pm c_x s_y s_z) \\
+ i\sigma_y(c_x s_y c_z \mp s_x c_y s_z) \\
- i\sigma_z(c_x c_y s_z \pm s_x s_y c_z) \\
+ I(c_x c_y c_z \mp s_x s_y s_z)
\]

$s_\alpha = \sin \frac{k_\alpha}{\sqrt{3}}$
$c_\alpha = \cos \frac{k_\alpha}{\sqrt{3}}$
The Weyl QCA

\[ i\partial_t \psi(t) \simeq \frac{i}{2}[\psi(t + 1) - \psi(t - 1)] = \frac{i}{2}(A - A^\dagger)\psi(t) \]

\[ \frac{i}{2}(A_k^{\pm} - A_k^{\pm\dagger}) = + \sigma_x(s_x c_y c_z \pm c_x s_y s_z) \quad \text{"Hamiltonian"} \]
\[ \pm \sigma_y(c_x s_y c_z \mp s_x c_y s_z) \]
\[ + \sigma_z(c_x c_y s_z \pm s_x s_y c_z) \]

\[ k \ll 1 \quad \text{\Rightarrow} \quad i\partial_t \psi = \frac{1}{\sqrt{3}}\sigma^{\pm} \cdot k \psi \quad \text{\& Weyl equation!} \quad \sigma^{\pm} := (\sigma_x, \pm \sigma_y, \sigma_z) \]

Two QCAs connected by \( P \)

\[ A_k^{\pm} = -i\sigma_x(s_x c_y c_z \pm c_x s_y s_z) \]
\[ \mp i\sigma_y(c_x s_y c_z \mp s_x c_y s_z) \]
\[ - i\sigma_z(c_x c_y s_z \pm s_x s_y c_z) \]
\[ + I(c_x c_y c_z \mp s_x s_y s_z) \]

\[ s_\alpha = \sin \frac{k_\alpha}{\sqrt{3}} \]
\[ c_\alpha = \cos \frac{k_\alpha}{\sqrt{3}} \]
**Dirac QCA**

Local coupling: $A_k$ coupled with its inverse with off-diagonal identity block matrix

$$E_{\pm}^k = \begin{pmatrix} n A_{\pm}^k & i m I \\ i m I & n A_{\pm}^{\dagger} \end{pmatrix}$$

$n^2 + m^2 = 1$

$E_{\pm}^k$ CPT-connected!

$$\omega_{\pm}(k) = \cos^{-1}[n(c_x c_y c_z \mp s_x s_y s_z)]$$

Dirac in relativistic limit $k \ll 1$

$m \leq 1$: mass

$n^{-1}$: refraction index

**Maxwell QCA**

$$M_k = A_k \otimes A_k^*$$

$$F^\mu(k) = \int \frac{dq}{2\pi} f(q) \tilde{\psi}(\frac{k}{2} - q)\sigma^\mu \varphi(\frac{k}{2} + q)$$

Maxwell in relativistic limit $k \ll 1$

Boson: emergent from entangled Fermions

(De Broglie neutrino-theory of photon)

1. Vacuum birefringence
2. Vacuum dispersion
3. Fermionic saturation
The LTM standards of the theory

Dimensionless variables

\[ x = \frac{x[m]}{a} \in \mathbb{Z}, \quad t = \frac{t[sec]}{t} \in \mathbb{N}, \quad m = \frac{m[kg]}{m} \in [0, 1] \]

Relativistic limit:

\[ c = \frac{a}{t} \quad \hbar = mac \]

Measure \( m \) from mass-refraction-index

\[ n(m[kg]) = \sqrt{1 - \left( \frac{m[kg]}{m} \right)} \]

Measure \( a \) from light-refraction-index

\[ c^\mp(k) = c \left( 1 \pm \frac{k}{\sqrt{3k_{max}}} \right) \]
The relativity principle
Symmetries and Relativity Principle

Looking for changes of reference-frames that leaves the dynamics invariant
Change of reference-frame = special change of representation

VA QWs

\[ A = \int_B^\oplus d\mathbf{k} A_k \]

\[ \mathbf{n}(\mathbf{k}) \cdot \mathbf{T} := \frac{i}{2} (A_k - A_k^\dagger) \] “Hamiltonian”

\[ \mathbf{n}(\mathbf{k}) \text{ analytic in } \mathbf{k} \]

\[ (I, \mathbf{T}) = (T^\mu) \] Hermitian basis for Lin(\(\mathbb{C}^s\))

Dynamics: eigenvalue equation

\[ A_k \psi(\mathbf{k}, \omega) = e^{i\omega} \psi(\mathbf{k}, \omega) \]

\[ (\sin \omega I - \mathbf{n}(\mathbf{k}) \cdot \mathbf{T}) \psi(\mathbf{k}, \omega) = 0 \]

For each value of \( \mathbf{k} \) there are at most \( s \) eigenvalues \( \{\omega_l(\mathbf{k})\} \)
\n\( \mathbf{n}(\mathbf{k}) \) analytic in \( \mathbf{k} \) + finite-dim irreps.
\n\( \omega_l(\mathbf{k}) \) continuous dispersion relations branches
Symmetries and Relativity Principle

**Change of reference-frame:** $(\omega, \mathbf{k}) \rightarrow (\omega', \mathbf{k}') = \mathcal{L}_\beta(\omega, \mathbf{k})$

$\mathcal{L}_\beta$ invertible (generally non continuous) over $[-\pi, \pi] \times B$

$\{\mathcal{L}_\beta\}_{\beta \in G}$ group (including space-inversion, charge conjugation, …)

**Symmetry of the dynamics:**

there exists a pair of invertible matrices $\Gamma_\beta$ and $\tilde{\Gamma}_\beta$

such that the following identity holds:

$$(\sin \omega I - \mathbf{n}(\mathbf{k}) \cdot \mathbf{T}) = \tilde{\Gamma}_\beta^{-1} (\sin \omega' I - \mathbf{n}(\mathbf{k}') \cdot \mathbf{T}) \Gamma_\beta$$

$\Gamma_\beta$ and $\tilde{\Gamma}_\beta$ continuous functions of $(\omega, \mathbf{k})$

$G_0$ (id-component of $G$) preserves the branches

change of reference-frame $= \mathbf{k} \rightarrow \mathbf{k}'(\mathbf{k})$

change of reference-frame $= \text{reshuffling } \mathbf{k} \rightarrow \mathbf{k}'(\mathbf{k})$ of irreps. holds for the whole class of VA QW

$k \rightarrow k'(k)$

$L_\beta(\omega, \mathbf{k}) = (\omega(k'), k'(k))$

$G_0, G$ depend on the QW!
Relativity Principle for Weyl QW

\[ p^{(f)} := f(\omega, k)(\sin \omega, n(k)) \]

\[ p_{\mu} p^{\mu} = 0 \text{ on } \text{Disp}(A) \]

\[ p^{(f)}_{\mu} \sigma^\mu \psi(k, \omega) = 0 \text{ “4-momentum”} \]

**Non-linear Lorentz group**

\[ L^{(f)}_\beta := D^{(f)}^{-1} L_\beta D^{(f)} \]

\[ D^{(f)} : (\omega, k) \mapsto p^{(f)}(\omega, k) \]

acting on \([-\pi, \pi] \times B\)

\[ \text{Disp}(A) \text{ invariant} \]

\[ L_\beta \text{ Lorentz} \]

Relativistic covariance of dynamics

\[ (\sin \omega I - n(k) \cdot \sigma) = \tilde{\Lambda}_\beta^\dagger (\sin \omega' I - n(k') \cdot \sigma) \Lambda_\beta \]

\[ \Lambda_\beta \in \text{SL}_2(\mathbb{C}) \text{ independent of } (k_\mu) \]
Relativity Principle for Weyl QW

Includes the group of “translations” of the Cayley graph: \( G_0 \) is the Poincaré group.

The Brillouin zone separates into four invariant regions diffeomorphic to balls, corresponding to four different particles.
Relativity Principle for Dirac QW

Dirac automaton: De Sitter covariance (non linear)

Covariance for Dirac QCA cannot leave $m$ invariant

invariance of de Sitter norm:

$$\text{Disp}(A): \quad \sin^2 \omega - (1 - m^2)|\mathbf{n}(\mathbf{k})|^2 - m^2 = 0$$

$\Rightarrow \quad SO(1, 4) \text{ invariance}$

$SO(1, 4) \longrightarrow SO(1, 3) \quad \text{for} \quad m \to 0 \quad O(m^2)$
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