A QUANTUM-DIGITAL UNIVERSE: A QCA APPROACH TO FIELD THEORY

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Speakable in quantum mechanics: atomic, nuclear and subnuclear physics tests
ECT, Trento, 30 August 2011
Selected for a Viewpoint in Physics

PHYSICAL REVIEW A 84, 012311 (2011)

Informational derivation of quantum theory

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(Received 29 November 2010; published 11 July 2011)

We derive quantum theory from purely informational principles. Five elementary axioms—causality, perfect distinguishability, ideal compression, local distinguishability, and pure conditioning—define a broad class of theories of information processing that can be regarded as standard. One postulate—purification—singles out quantum theory within this class.

DOI: 10.1103/PhysRevA.84.012311

PACS number(s): 03.67.Ac, 03.65.Ta

I. INTRODUCTION

than 80 years after its formulation, quantum theory ysterious. The theory has a solid mathematical foun-

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I. INTRODUCTION

More than 80 years after its formulation, quantum theory continues to astonish. The theory has a solid mathematical foundation, addressed by Hilbert, von Neumann, and von Neumann himself expressed his dissatisfaction with his mathematical formulation of quantum theory with the surprising words “I don’t believe in Hilbert space anymore,” reported by Birkhoff in
Q-DIGITALIZATION PROGRAM

QCA AS A PLANCK-SCALE THEORY
Q-DIGITALIZATION PROGRAM

QCA AS A PLANCK-SCALE THEORY

PROBLEMS WITH QFT
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QCA AS A PLANCK-SCALE THEORY

PROBLEMS WITH QFT

• renormalization
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QCA AS A PLANCK-SCALE THEORY

PROBLEMS WITH QFT

* renormalization
* violation (?) of Einstein causality
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QCA AS A PLANCK-SCALE THEORY

PROBLEMS WITH QFT

* renormalization
* violation (?) of Einstein causality
* localization → measurement
**PROBLEMS WITH QFT**

- renormalization
- violation (?) of Einstein causality
- localization $\rightarrow$ measurement
- Quantum Gravity
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CURE

* space-time emergence from events
* *homogeneous* causal networks
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ISSUES

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1+1 DIMENSIONS

* covariance is automatic
**Q-DIGITALIZATION PROGRAM**

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* emergence of physics
* elimination of q-fields for qubits

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### 1+1 DIMENSIONS

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### >1+1 DIMENSIONS

**ISSUES**

* violation of Lorentz covariance and dispersion relations  
* dimensional conundrum

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# Q-Digitalization Program

## QCA as a Planck-Scale Theory

### Problems with QFT

- renormalization
- violation (?) of Einstein causality
- localization → measurement
- Quantum Gravity

**CURE**

- space-time emergence from events
- *homogeneous* causal networks

### Issues

- violation of Lorentz covariance and dispersion relations
- dimensional conundrum

### 1+1 Dimensions

- covariance is automatic
- emergence of physics
- elimination of q-fields for qubits

### >1+1 Dimensions

- elimination of q-fields → Majorana

---

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* elimination of q-fields \rightarrow Majorana
* dim. conundrum \rightarrow quantumness

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>1+1 DIMENSIONS

* elimination of q-fields → Majorana
* dim. conundrum → quantumness

... AND MORE

* quantization vs classicalization

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Causal network
Causal network

event
event

causal link

Causal network

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event
subroutine
transformation

EFFECT
readout
measurement

STATE
initialization
preparation

Causal network

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A QCA FIELD THEORY

QUANTUM CELLULAR AUTOMATA

\[
\begin{align*}
&n-2 & n-1 & n & n+1 & n+2 & \ldots \\
&+ & - & + & - & + & - & + & - \\
\end{align*}
\]
A QCA FIELD THEORY

QUANTUM CELLULAR AUTOMATA

Translational invariance
A QCA FIELD THEORY
QUANTUM CELLULAR AUTOMATA

Locality of interactions

\[ n-2 \quad n-1 \quad n \quad n+1 \quad n+2 \quad \ldots \]
\[ + \quad - \quad + \quad - \quad + \quad - \quad + \quad - \quad + \quad - \quad + \quad - \quad + \quad - \quad \]

Translational invariance
A QCA FIELD THEORY

QUANTUM CELLULAR AUTOMATA

Locality of interactions

\[ n-2 \quad n-1 \quad n \quad n+1 \quad n+2 \quad \ldots \]
\[ + \quad - \quad + \quad - \quad + \quad - \quad + \quad - \quad + \quad - \quad + \quad - \]

Translational invariance

Margolus scheme
A QCA FIELD THEORY
QUANTUM CELLULAR AUTOMATA

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A QCA FIELD THEORY
TRANSLATIONAL INVARIANCE

Physical law
Homogeneous network topology
Homogeneous network topology
CAUSAL NETWORKS

THE PHYSICAL LAW: UNDRESSING TOPOLOGY

* Homogeneous network topology
Homogeneous network topology
CAUSAL NETWORKS

GRAPH DIMENSION

* Homogeneous network topology

* Space-time dimension: graph-dimension = d+1
CAUSAL NETWORKS

FEW POSSIBLE LATTICES

* Homogeneous network topology
* graph-dimension = d+1

few possible causal networks
CAUSAL NETWORKS

FEW POSSIBLE LATTICES

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EMERGENCE OF SPACE-TIME FROM CN

CAUSAL DEPENDENCE
EMERGENCE OF SPACE-TIME FROM CN

CAUSAL DEPENDENCE

Causally dependent events

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EMERGENCE OF SPACE-TIME FROM CN

CAUSAL DEPENDENCE

Causally dependent events

Causally independent events
EMERGENCE OF SPACE-TIME FROM CN

FOLIATION: TIME AS A COMPUTER CLOCK
Time is a computer clock for synchronizing the calls to subroutines in a distributed parallel calculus.
EMERGENCE OF SPACE-TIME FROM CN
THE COMPUTATIONAL TOMONAGA-SCHWINGER

BOOSTED FRAME
EMERGENCE OF SPACE-TIME FROM CN

BOOSTED FRAME

THE COMPUTATIONAL TOMONAGA-SCHWINGER

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same circuit topology, different synchronization

BOOSTED FRAME

REST FRAME
EMERGENCE OF SPACE-TIME FROM CN

TIME-DILATION AND SPACE-CONTRACTION

BOOSTED FRAME

REST FRAME
EMERGENCE OF SPACE-TIME FROM CN

TIME-DILATION AND SPACE-CONTRACTION

BOOSTED FRAME

REST FRAME
EMERGENCE OF SPACE-TIME FROM CN

CONSTRUCTION OF THE COORDINATE SYSTEM
EMERGENCE OF SPACE-TIME FROM CN

CONSTRUCTION OF THE COORDINATE SYSTEM

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EMERGENCE OF SPACE-TIME FROM CN

DIGITAL LORENTZ TRANSFORMATIONS
EMERGENCE OF SPACE-TIME FROM CN

DIGITAL LORENTZ TRANSFORMATIONS

\[ t^1 = \chi_{12} \frac{t^2 + v^{12}s^2}{\sqrt{1 - (v^{12})^2}}, \]
\[ s^1 = \chi_{12} \frac{s^2 + v^{12}t^2}{\sqrt{1 - (v^{12})^2}}, \]
\[ \chi_{12} := \sqrt{\alpha^{12} \beta^{12}} \]
\[ v_{13} = \frac{v_{12} + v_{23}}{1 + v_{12}v_{23}} \]
EMERGENCE OF SPACE-TIME FROM CN

DIGITAL LORENTZ TRANSFORMATIONS

\[ t^1 = \chi_{12} \frac{t^2 + v_{12} s^2}{\sqrt{1 - (v_{12})^2}}, \]
\[ s^1 = \chi_{12} \frac{s^2 + v_{12} t^2}{\sqrt{1 - (v_{12})^2}}, \]

\[ \chi_{12} := \sqrt{\alpha_{12} \beta_{12}} \]
\[ \frac{1}{2} (\alpha_{12} + \beta_{12}) \]

\[ v_{13} = \frac{v_{12} + v_{23}}{1 + v_{12} v_{23}} \]
Anisotropy of max-speed of information (no-digital-go theorem by Tobias Fritz)
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EMERGENCE OF SPACE-TIME FROM CN

DIMENSIONAL CONUNDRUM

* Anisotropy of max-speed of information (no-digital-go theorem by Tobias Fritz)
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EMERGENCE OF SPACE-TIME FROM CN

DIMENSIONAL CONUNDRUM

* Anisotropy of max-speed of information (no-digital-go theorem by Tobias Fritz)

Possible solution:

quantum nature of the CN!
INFORMATION FLOW IN 1+1: LORENTZ COVARIANCE IS A BONUS!
THE FREE FLOW OF INFORMATION

i.e. the DIRAC EQUATION (1+1 dimensions)
THE FREE FLOW OF INFORMATION

i.e. the DIRAC EQUATION (1+1 dimensions)

Information can flow only in two directions
THE FREE FLOW OF INFORMATION

i.e. the DIRAC EQUATION (1+1 dimensions)

Information can flow only in two directions and at fixed direction only at max speed
THE FREE FLOW OF INFORMATION

Information can flow only in two directions and at fixed direction only at max speed.

i.e. the DIRAC EQUATION (1+1 dimensions)

- luminal
- subluminal
- superluminal
THE FREE FLOW OF INFORMATION

i.e. the DIRAC EQUATION (1+1 dimensions)

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Information can flow only in two directions and at fixed direction only at max speed

\[ c = \frac{a}{\tau} \]
THE FREE FLOW OF INFORMATION

*i.e. the DIRAC EQUATION (1+1 dimensions)*

Information can flow only in two directions and at fixed direction only at max speed

\[
\hat{\partial}_t \begin{bmatrix} \phi^+ \\ \phi^- \end{bmatrix} = \begin{bmatrix} c\hat{\partial}_x & 0 \\ 0 & -c\hat{\partial}_x \end{bmatrix} \begin{bmatrix} \phi^+ \\ \phi^- \end{bmatrix}
\]
THE FREE FLOW OF INFORMATION

\[ \frac{\hat{\partial}_t}{\partial t} \begin{bmatrix} \phi^+ \\ \phi^- \end{bmatrix} = \begin{bmatrix} c\hat{\partial}_x & 0 \\ 0 & -c\hat{\partial}_x \end{bmatrix} \begin{bmatrix} \phi^+ \\ \phi^- \end{bmatrix} \]

Information can flow only in two directions and at fixed direction only at max speed

Slower speed = periodic change of direction
Slower speed = periodic change of direction coupling between \( \phi^+ \) and \( \phi^- \) by an imaginary constant

\[
\hat{\partial}_t \begin{bmatrix} \phi^+ \\ \phi^- \end{bmatrix} = \begin{bmatrix} c\hat{\partial}_x & 0 \\ 0 & -c\hat{\partial}_x \end{bmatrix} \begin{bmatrix} \phi^+ \\ \phi^- \end{bmatrix}
\]

Information can flow only in two directions and at fixed direction only at max speed

The free flow of information i.e. the Dirac equation (1+1 dimensions)

No need of imposing relativistic invariance!

(spinless) Dirac equation!
Slower speed = periodic change of direction

coupling between $\phi^+$ and $\phi^-$ by an imaginary constant

$$
\hat{\partial}_t \begin{bmatrix} \phi^+ \\ \phi^- \end{bmatrix} = 
\begin{bmatrix}
 c\hat{\partial}_x & 0 \\
 0 & -c\hat{\partial}_x
\end{bmatrix}
\begin{bmatrix} \phi^+ \\ \phi^- \end{bmatrix}
$$

and at fixed direction only at max speed

Information can flow only in two directions

THE FREE FLOW OF INFORMATION

**i.e. the DIRAC EQUATION**

\[ \text{(spinless) } \textbf{Dirac equation!} \]

... a kinematical definition of inertial mass ...

No need of imposing relativistic invariance!

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THE FREE FLOW OF INFORMATION

**i.e. the DIRAC EQUATION**

Information can flow only in two directions and at fixed direction only at max speed.

\[
\hat{\partial}_t \begin{bmatrix} \phi^+ \\ \phi^- \end{bmatrix} = \begin{bmatrix} c \hat{\partial}_x & 0 \\ 0 & -c \hat{\partial}_x \end{bmatrix} \begin{bmatrix} \phi^+ \\ \phi^- \end{bmatrix}
\]

Slower speed = periodic change of direction

**... a kinematical definition of inertial mass ...**

**... an informational meaning for \( \hbar \)**

(conversion info-mass - kg-mass)

\[
\begin{align*}
\lambda &= \frac{\hbar}{mc} \\
\omega &= \text{mass (informational) in s}^{-1} \\
m &= \frac{1}{c^2} \hbar \omega
\end{align*}
\]

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FREE INFORMATION FLOW

DIRAC EQUATION

spin: circuit
“undressing”

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spin: circuit “undressing”
THE NETWORK BECOMES QUANTUM: QCA
The coupling between left and right fields leads to a renormalization of the field speed due to unitarity. 

\[ c \rightarrow \zeta c, \quad \zeta = \zeta(m) \]

\[ \zeta(m) = \sqrt{1 - \left(\frac{m}{M}\right)^2} \]

Information halt at the Planck mass
PHYSICS EMERGING FROM THE COMPUTATION
Hermiticity is a consequence of the universality of the physical law.
FIELDS REPLACED BY QUBITS

Jordan-Wigner construction

\[
\gamma_n := \sigma_n^+ \prod_{l=-\infty}^{n-1} \sigma_k^z \quad [\gamma_n, \gamma_m] = 0, \quad [\gamma_n^\dagger, \gamma_m] = \delta_{mn}
\]
FIELDS REPLACED BY QUBITS

Jordan-Wigner construction

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\gamma_n := \sigma_n^+ \prod_{l=-\infty}^{n-1} \sigma_k^\sim \quad [\gamma_n, \gamma_m] = 0, \quad [\gamma_n^+, \gamma_m] = \delta_{mn}
\]

\[
\gamma_n \gamma_{n-1} := \sigma_n^+ \sigma_{n-1}^-
\]
\[ \gamma_n := \sigma_n^+ \prod_{l=-\infty}^{n-1} \sigma_k^z \quad [\gamma_n, \gamma_m] = 0, \quad [\gamma_n^\dagger, \gamma_m] = \delta_{mn} \]

\[ \gamma_n^\dagger \gamma_{n-1} := \sigma_n^+ \sigma_{n-1}^- \]

\[ \gamma_n^\dagger \gamma_{n-k-1} := \sigma_n^+ \sigma_{n-1}^z \cdots \sigma_{n-k}^z \sigma_{n-k-1}^- \]
### FIELDS REPLACED BY QUBITS

**Dirac in 1+1 d**

#### Fields are eliminated!

\[
A = \exp \left\{ i\theta \left[ \phi_{n-1}^+ \phi_n^- + \phi_n^- \phi_{n-1}^+ \right] \right\}
\]

\[
B = \exp \left\{ i \frac{\pi}{2} \left[ \phi_{n-1}^+ \phi_n^- + \phi_n^- \phi_{n-1}^+ \right] \right\}
\]

<table>
<thead>
<tr>
<th>Commuting</th>
<th>Anticommuting</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Harmonic oscillator</strong></td>
<td><strong>Jordan-Wigner</strong></td>
</tr>
</tbody>
</table>
| \[ [a_l, a_k^\dagger] = \delta_{lk} \] | \[
\phi_n^+ = \sigma_{2n}^- \prod_{k=2}^{n-1} \sigma_{2k+1}^- \sigma_{2k}^Z
\]
| \[
\phi_n^+ = a_{2n}, \quad \phi_n^- = a_{2n+1}
\] | \[
\phi_n^- = \sigma_{2n+1}^- \sigma_{2n}^Z \prod_{k=-\infty}^{n-1} \sigma_k^Z
\]

**Gates act on local qubits only!**

\[
A = \exp \left[ -i\theta \left( \sigma_{2n-1}^- \sigma_{2n}^+ + \sigma_{2n-1}^+ \sigma_{2n}^- \right) \right]
\]

\[
B = \exp \left[ -i \frac{\pi}{2} \left( \sigma_{2n}^+ \sigma_{2n+1}^- + \sigma_{2n}^- \sigma_{2n+1}^+ \right) \right]
\]
FIELDS REPLACED BY QUBITS

Dirac in > 1+1 d!!

* Jordan-Wigner transformation for d+1>2
FIELDS REPLACED BY QUBITS

Dirac in $> 1+1$ d!!

* Jordan-Wigner transformation for $d+1 > 2$
FIELDS REPLACED BY QUBITS

Dirac in $1+1$ d!!

* Jordan-Wigner transformation for $d+1 > 2$

Dirac field
FIELDS REPLACED BY QUBITS

Dirac in $> 1+1 \text{ d!!}$

- Jordan-Wigner transformation for $d+1 > 2$

- Possible solution: add a Majorana field!
**FIELDS REPLACED BY QUBITS**

Dirac in $> 1+1$ d!!

* Jordan-Wigner transformation for $d+1 > 2$

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---


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<th>qubits</th>
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<td>$(\sigma_{k,l}^x \sigma_{k,l+1}^x + \sigma_{k,l+1}^y \sigma_{k,l}^y)(-)^{l+1} \tilde{\sigma}<em>{k,l}^x \tilde{\sigma}</em>{k,l+1}^y$</td>
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<td>$\gamma_{k,l}^\dagger \gamma_{k+1,l} + \gamma_{k+1,l}^\dagger \gamma_{k,l}$</td>
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</tr>
</tbody>
</table>

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**FIELDS REPLACED BY QUBITS**

* Dirac in $> 1+1$ d!!

* Jordan-Wigner transformation for $d+1 > 2$

* Possible solution: add a Majorana field!

---


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Weyl, Dirac, and Maxwell equations on a lattice as unitary cellular automata

Iwo Bialynicki-Birula

Centrum Fizyki Teoretycznej, Polska Akademia Nauk, Lotników 32/46, 02-668 Warsaw, Poland*
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Robert-Mayer-Strasse 8-10, Frankfurt am Main, Germany
(Received 27 September 1993; revised manuscript received 27 December 1993)

Very simple unitary cellular automata on a time evolution of the wave functions for spinor value of the wave function at a given site depend discretized evolution is also unitary and preserved is studied in detail, and it is shown that ever under some natural assumptions, leads in the context of histories is evaluated and is shown to reproduce Generalizations to include massive particles (D higher-spin particles are also described.

PACS number(s): 03.65.Pm, 02.70.–c, 11.15.Hi

II. WEYL EQUATION ON A LATTICE

I shall start with a lattice description of the wave equation for a massless spin-1/2 particle and extend it later to massive particles and to higher spins. In my quantum cellular automaton the two-component wave function $\phi(i, j, k, t)$ is defined on a cubic lattice and it is updated at each time increment $\Delta t$ according to the local algorithm

$$
\phi(i, j, k, t + \Delta t) = W_{+++}\phi(i + 1, j + 1, k + 1, t) \\
+ W_{+++}\phi(i + 1, j + 1, k - 1, t) + \cdots \\
+ W_{---}\phi(i - 1, j - 1, k - 1, t), \quad (1)
$$
Weyl, Dirac, and Maxwell equations on a lattice as unitary cellular automata

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Very simple unitary cellular automata on a time evolution of the wave functions for spinor value of the wave function at a given site depend discretized evolution is also unitary and preserved is studied in detail, and it is shown that ever and by identifying the evolution operator $T_A$ with the generic evolution operator $U_\Delta$ introduced before. The exact evolution operator in the continuum limit is recovered from the Lie-Trotter product formula (cf., for example, Ref. [39]), when $N = t/\Delta t$ tends to infinity,

$$\lim_{N \to \infty} \left[ \exp(a \sigma_x \partial_x) \exp(a \sigma_y \partial_y) \exp(a \sigma_z \partial_z) \right]^N$$

$$= \exp(c \sigma \cdot \nabla \Delta t). \quad (18)$$

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FIELDS REPLACED BY QUBITS

CLASSICALIZATION vs QUANTIZATION
FIELDS REPLACED BY QUBITS

CLASSICALIZATION vs QUANTIZATION

- locally interacting qubits
- Jordan Wigner
- Locally interacting quantum fields
- emergent Hamiltonian
- classical fields

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FIELDS REPLACED BY QUBITS

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FIELDS REPLACED BY QUBITS

CLASSICALIZATION vs QUANTIZATION

-fields

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Dirac QCA: First Quantization

Single particle state
Dirac QCA: First Quantization

Single particle state
Dirac QCA: First Quantization

Single particle state
Dirac QCA: First Quantization

Single particle state
First Quantization: two-particle states
First Quantization: two-particle states
First Quantization: two-particle states
IS REALITY QUANTUM-DIGITAL?

SOME INTERESTING POINTS FOR DISCUSSION

* Emergent physics:
  * Minkowski space-time
  * Hamiltonian
  * inertial mass
  * Planck constant
  * classical mechanics
  * quantization/dequantization
  * gravitation...

* Violations:
  * Lorentz covariance,
  * dispersion relations ...

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IS REALITY QUANTUM-DIGITAL?

SOME INTERESTING POINTS FOR DISCUSSION

- Emergent physics:
  - Minkowski space-time
  - Hamiltonian
  - inertial mass
  - Planck constant
  - classical mechanics
  - quantization/dequantization
  - gravitation...

- Violations:
  - Lorentz covariance,
  - dispersion relations...

THANK YOU!

martedì 30 agosto 2011