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Quantum Field Theory from general principles results in a quantum cellular automaton theory

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Quantum Topology seminar
Department of Mathematics, Statistics, and Computer Science
University of Illinois at Chicago, August 25 2015

Principles for Quantum Theory

- The experience in Quantum Information has led us to look at Quantum Theory (QT) under a completely new angle
- QT is a *theory of information*

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Paolo Perinotti

Selected for a Viewpoint in *Physics*

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Informational derivation of quantum theory

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We derive quantum theory from purely informational principles. Five elementary axioms—causality, perfect distinguishability, ideal compression, local distinguishability, and pure conditioning—define a broad class of theories of information processing that can be regarded as standard. One postulate—purification—singles out quantum theory within this class.

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Principles for Quantum Theory

P1. Causality

P2. Local discriminability

P3. Purification *

P4. Atomicity of composition

P5. Perfect distinguishability

P6. Lossless Compressibility

Book from CUP soon!

Operational Probabilistic Theory

The framework

Logic ⊂ Probability ⊂ OPT

joint probabilities + connectivity

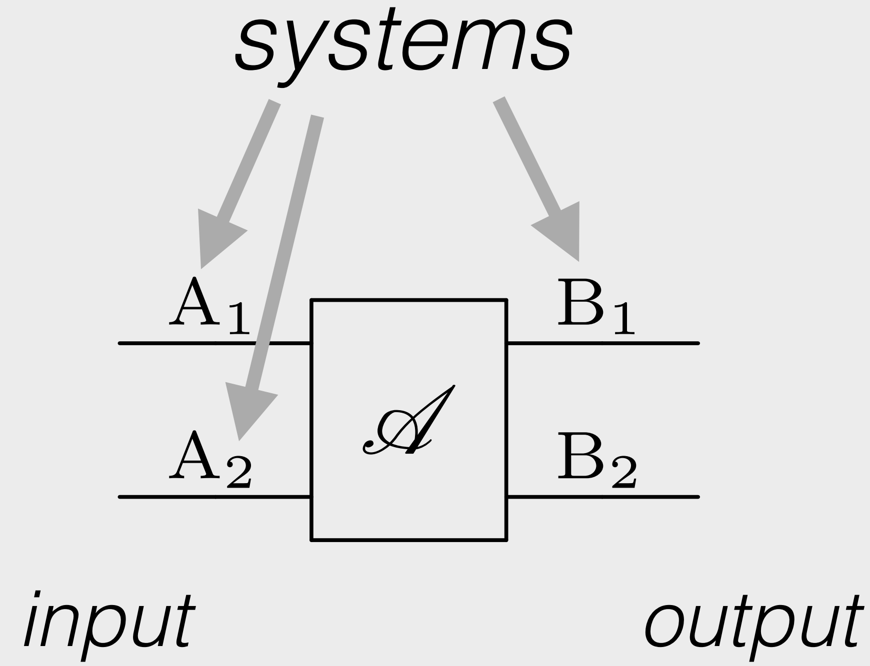
$$p(i, j, k, ...|\text{circuit})$$

Marginal probability

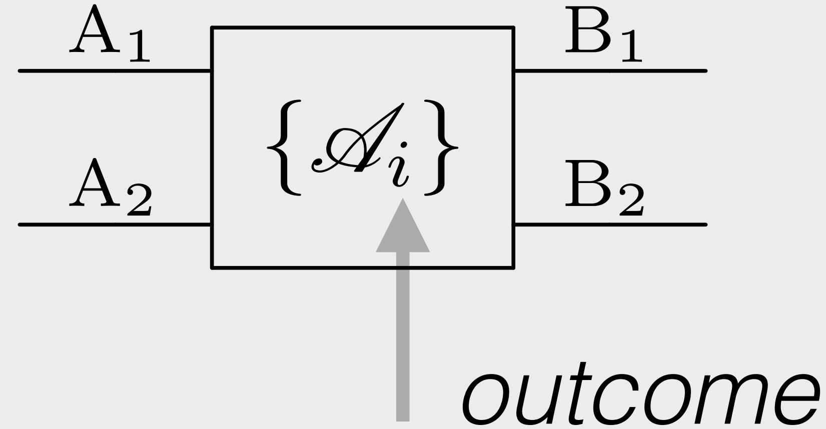
$$\sum_{i,k,...} p(i, j, k, ...|\text{circuit}) =$$

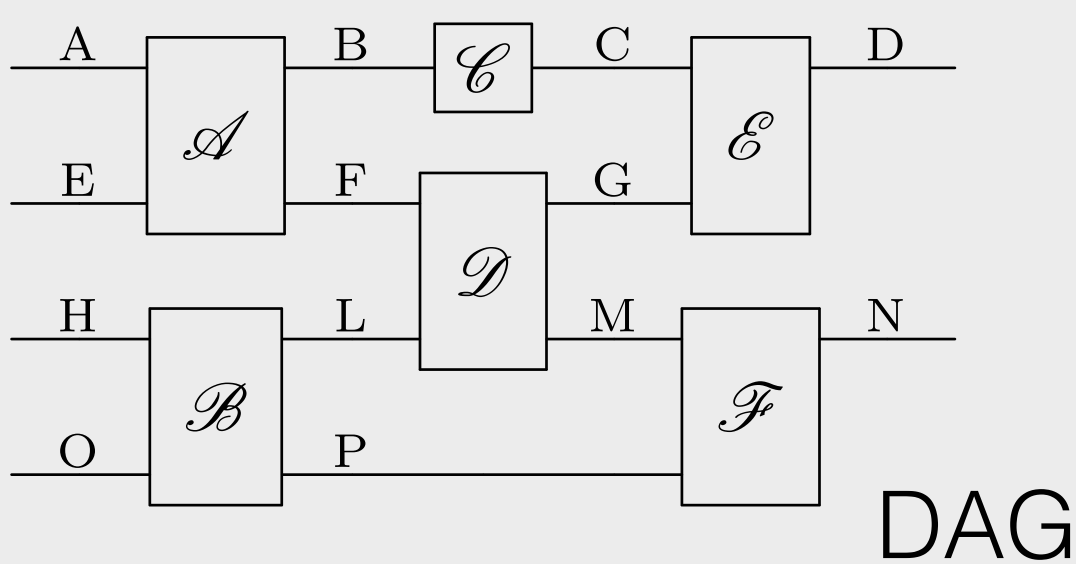
$$p(j|\text{circuit})$$

Event



Test





Operational Probabilistic Theory

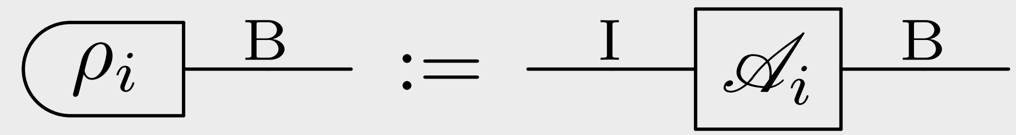
The framework

Logic ⊂ Probability ⊂ OPT

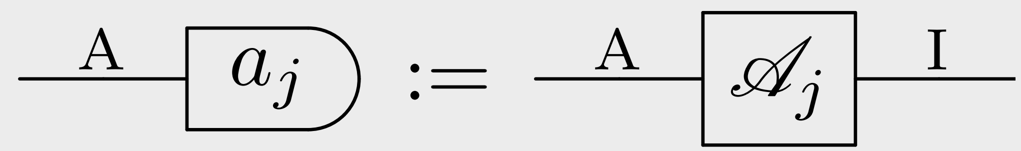
joint probabilities + connectivity

$$p(i, j, k, \ldots | \text{circuit})$$

Notice: the probability of a "preparation" generally depends on the circuit at its output.

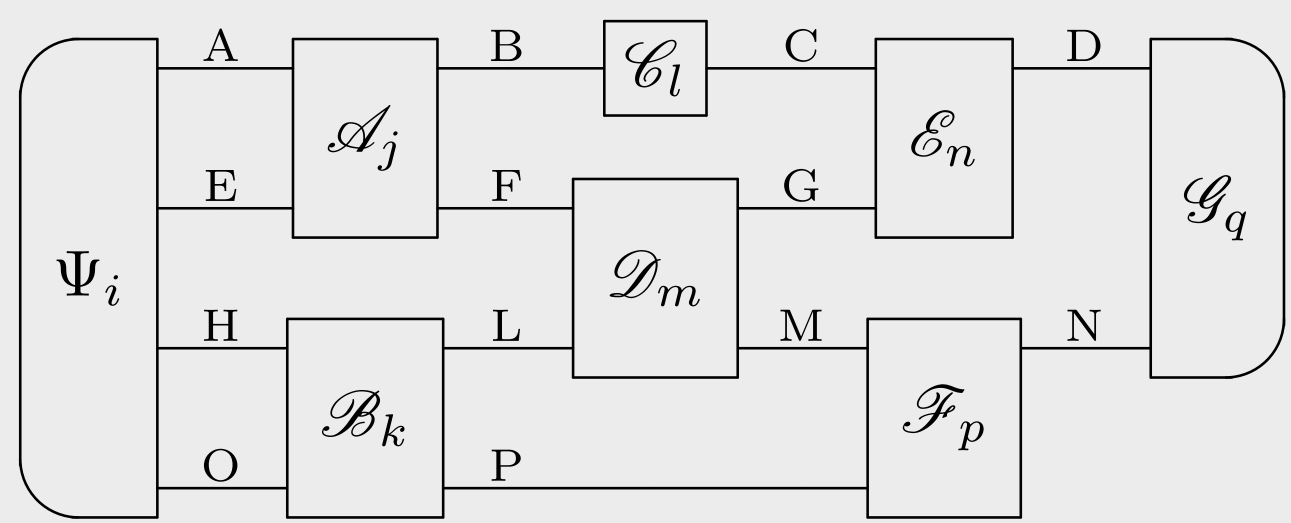


preparation



observation

$$p(i, j, k, l, m, n, p, q | \text{circuit})$$



Operational Probabilistic Theory

The framework

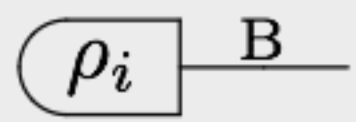
Logic ⊂ Probability ⊂ OPT

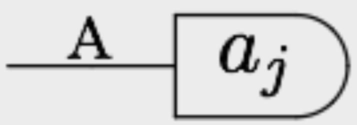
joint probabilities + connectivity

Probabilistic equivalence classes

Notice: the probability of a transformation generally depends on the circuit at its output!!

transformation

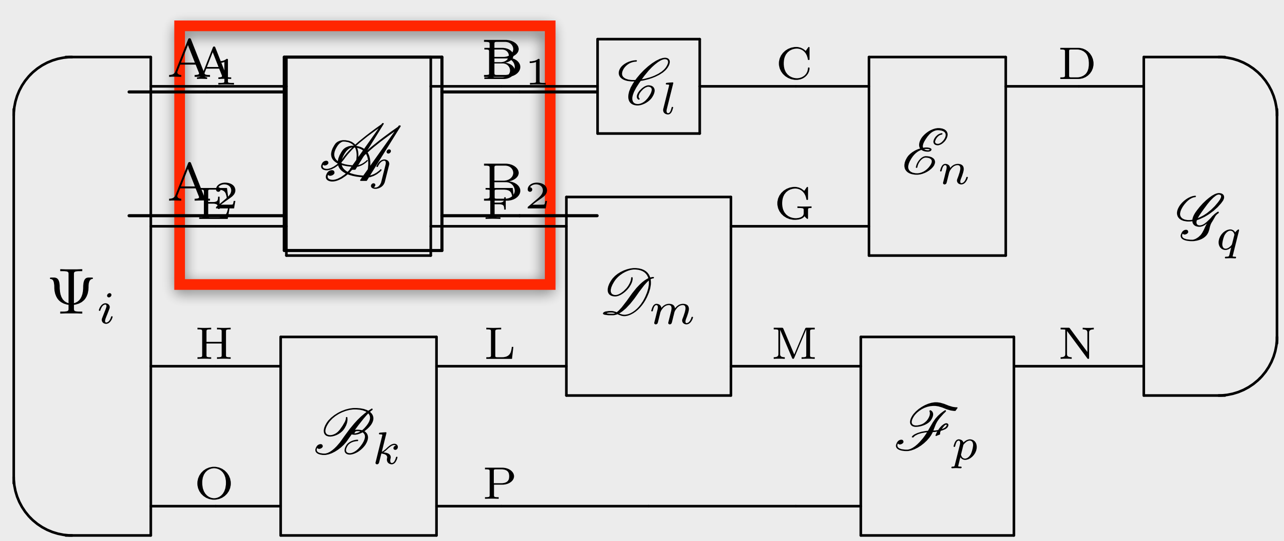




state

effect

$$p(i, j, k, l, m, n, p, q|\text{circuit})$$



Operational Probabilistic Theory

The framework

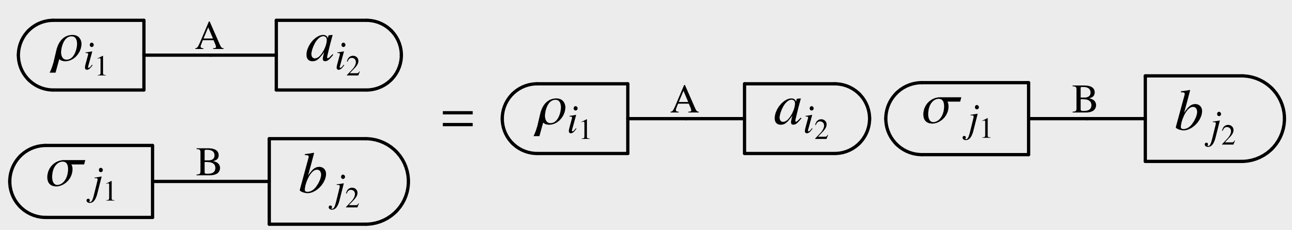
Logic ⊂ Probability ⊂ OPT

joint probabilities + connectivity

Probabilistic equivalence classes

monoidal category theory

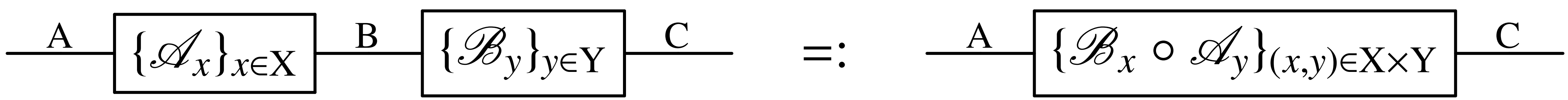
Multiplication of closed circuits



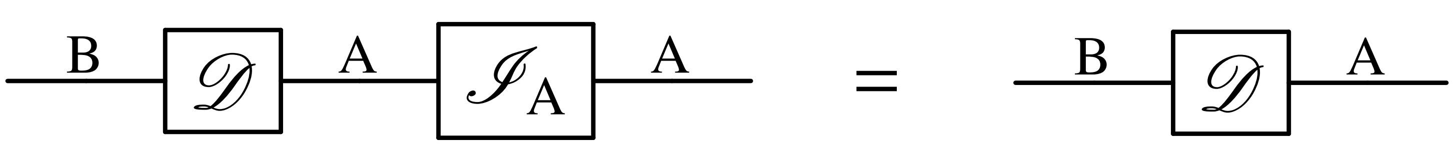
$$= p(i_1, i_2)\, q(j_1, j_2)$$

Operational Probabilistic Theory

Sequential composition (associative)

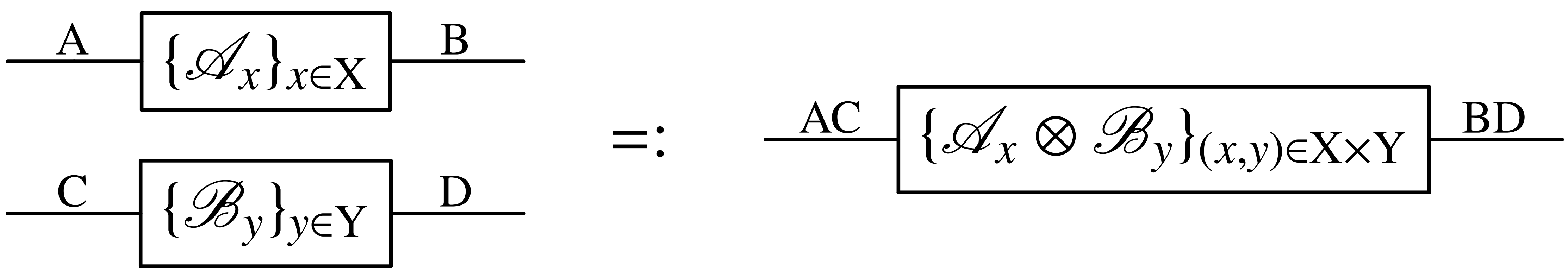


Identity test



Operational Probabilistic Theory

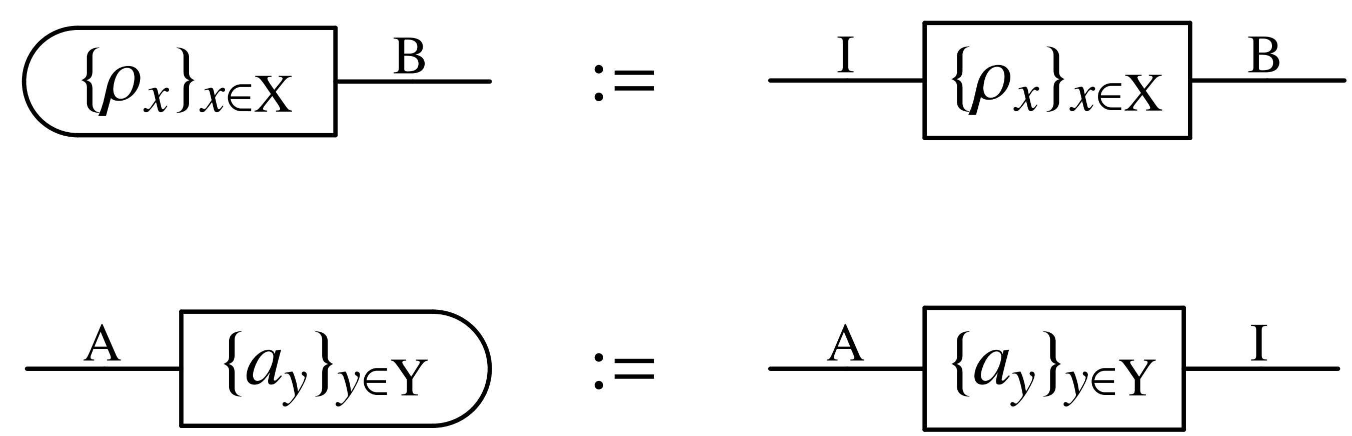
Parallel composition (associative)



$$\mathrm{AB} = \mathrm{BA}$$

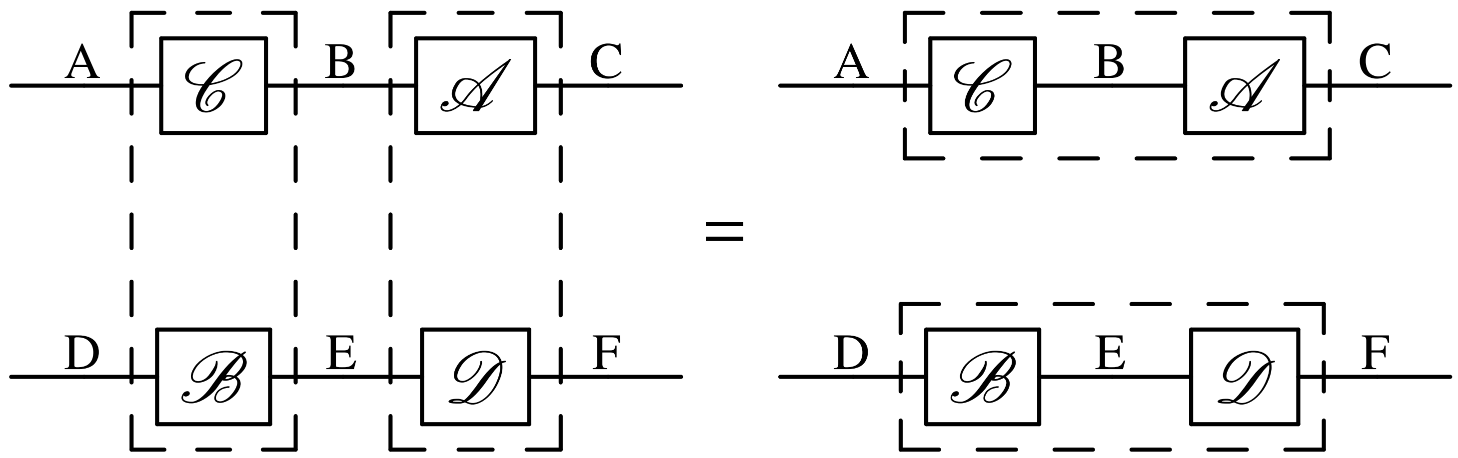
$$\mathrm{AI} = \mathrm{IA} = \mathrm{A}$$

$$\mathrm{A(BC)} = \mathrm{(AB)C}$$

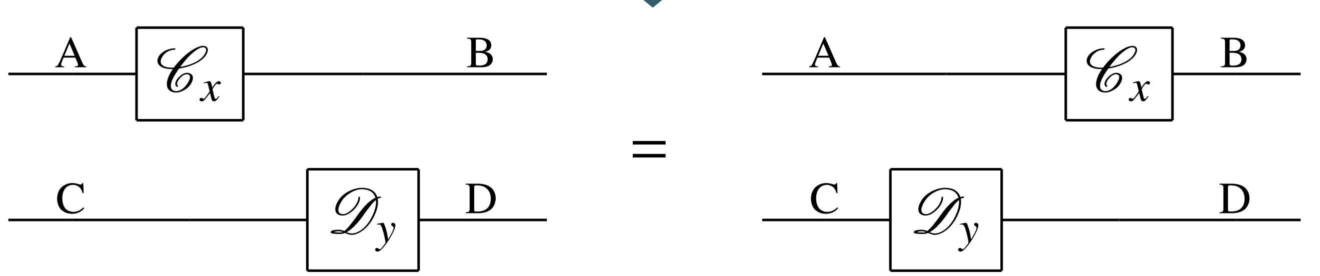


Operational Probabilistic Theory

Sequential and parallel compositions commute



$$(\mathscr{A} \otimes \mathscr{D}) \circ (\mathscr{C} \otimes \mathscr{B}) = (\mathscr{A} \circ \mathscr{C}) \otimes (\mathscr{D} \circ \mathscr{B})$$



wire-stretching

(foliations)

Operational Probabilistic Theory

The framework

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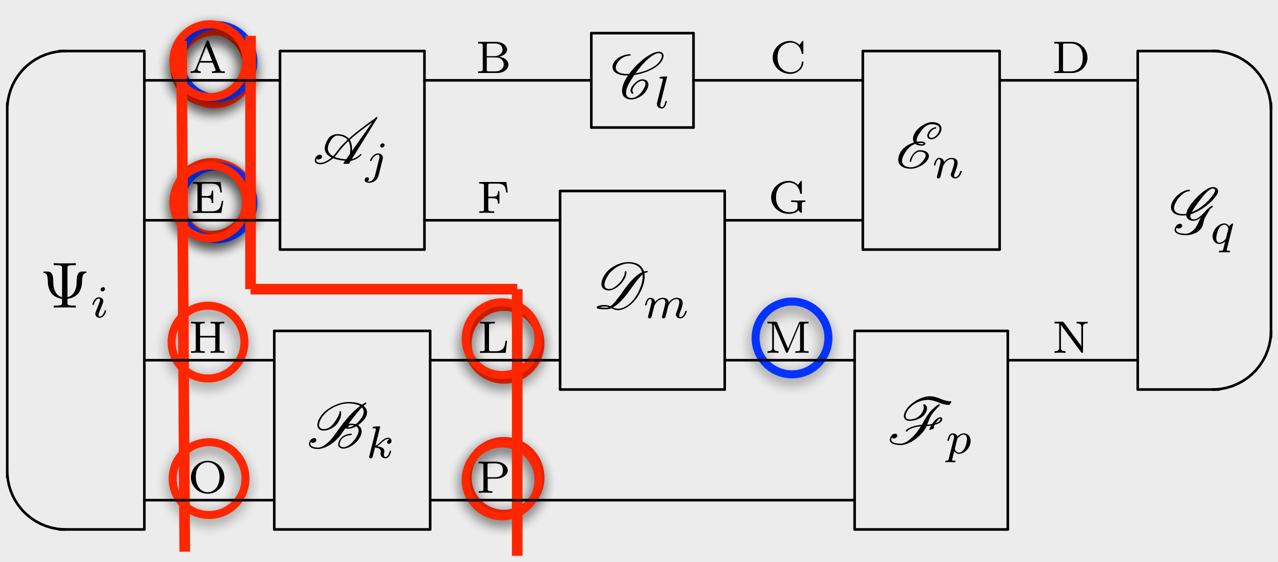
$$p(i, j, k, \ldots | \text{circuit})$$

$$p(i, j, k, l, m, n, p, q | \text{circuit})$$



Maximal set of
NOT independent systems
= *"leaf"*

Operational Probabilistic Theory

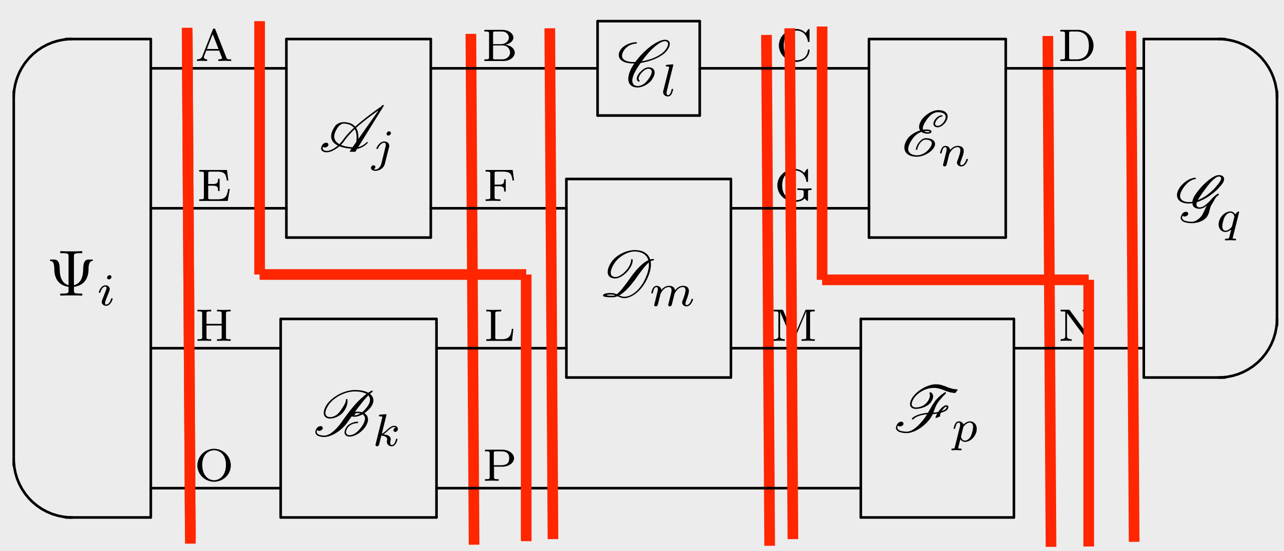
The framework

Logic ⊂ Probability ⊂ OPT

joint probabilities + connectivity

$$p(i, j, k, ...|\text{circuit})$$

$$p(i, j, k, l, m, n, p, q|\text{circuit})$$



Maximal set of independent systems = *"leaf"*

→ *Foliation*

Operational Probabilistic Theory

States are functionals for effects

States are separating for effects

Effects are functionals on states

Effects are separating for states

Embedding in real vector spaces

$\mathsf{St}(\mathrm{A}),\ \mathsf{St}_1(\mathrm{A}),\ \mathsf{St}_{\mathbb{R}}(\mathrm{A})$

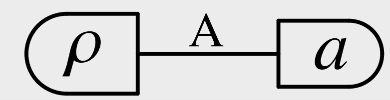
$\mathsf{Eff}(\mathrm{A}),\ \mathsf{Eff}_1(\mathrm{A}),\ \mathsf{Eff}_{\mathbb{R}}(\mathrm{A})$

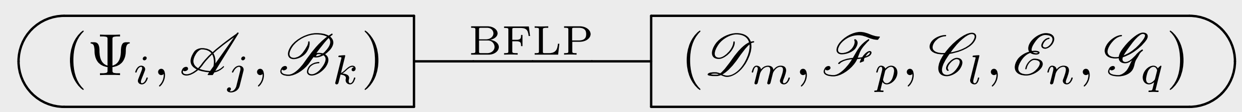
Dimension D_{A}

$\mathsf{Eff}_{\mathbb{R}}(\mathrm{A}) = \mathsf{St}_{\mathbb{R}}(\mathrm{A})^{\vee}$

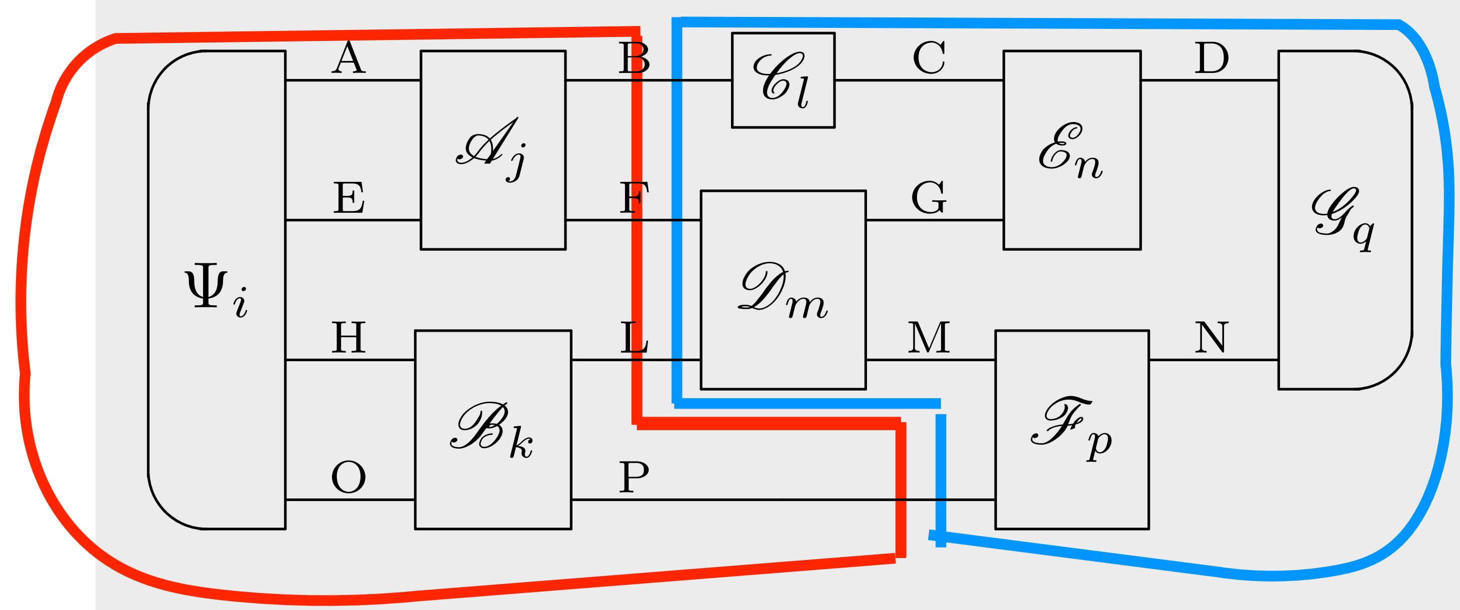
$\mathsf{St}_{\mathbb{R}}(\mathrm{A}) = \mathsf{Eff}_{\mathbb{R}}(\mathrm{A})^{\vee}$

Pairing notation:

$\rho \in \mathsf{St}(\mathrm{A}), a \in \mathsf{Eff}(\mathrm{A}),$  $= (a|\rho)$



$p(i, j, k, l, m, n, p, q|\text{circuit})$



Operational Probabilistic Theory

$$\{\mathscr{T}_i\}_{i\in\{\underbrace{i_1, i_2, \ldots, i_n}_{j_1}, \underbrace{i_{n+1}, i_{n+2}, \ldots}_{j_2}, \underbrace{\ldots}_{\ldots}\}}$$

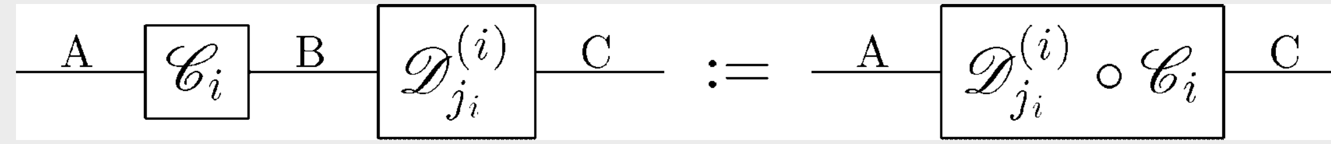
Coarse-graining ↓ ↑ Refinement

$$\{\hat{\mathscr{T}}_j\}_{j\in\{j_1,j_2,\ldots\}}$$

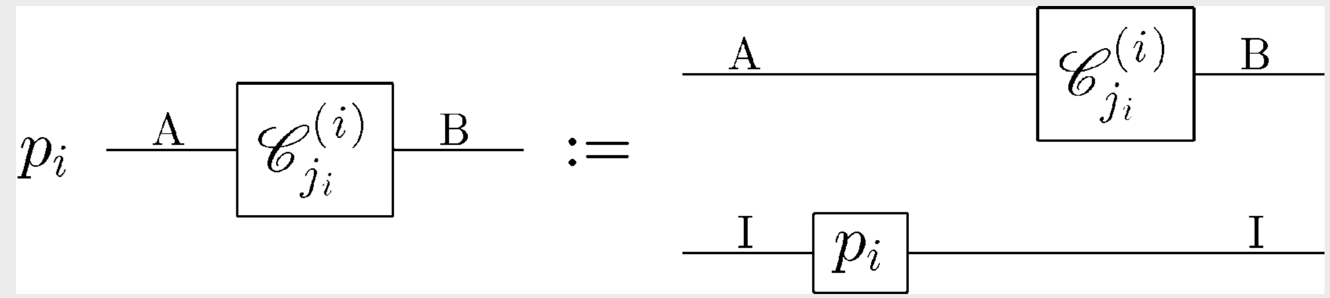
$$\hat{\mathscr{T}}_S = \sum_{i\in S} \mathscr{T}_i$$

Partial ordering

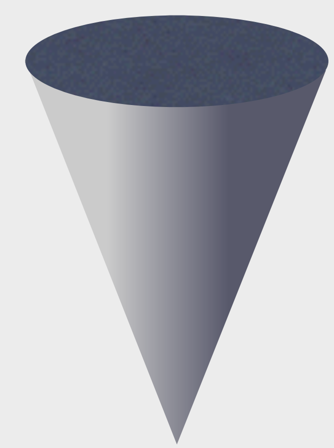
Conditioned test (needs causality)



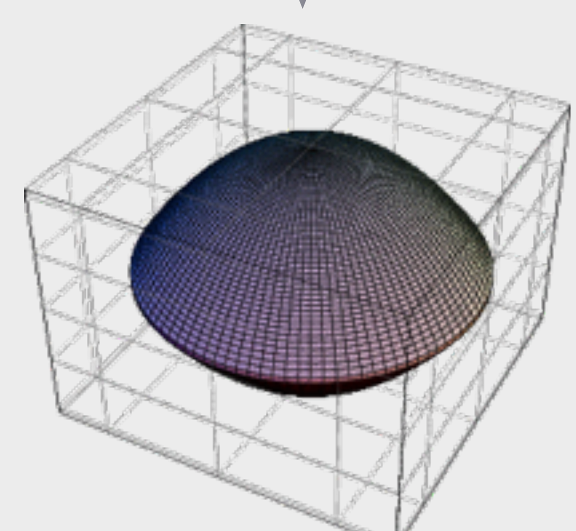
Circuit multiplication: randomize tests



Cone structure



Convex structure



Operational Probabilistic Theory

State tomography

$\{l_i\}_{i\in \mathrm{X}} \subseteq \mathsf{Eff}(\mathrm{A})$ separating for states ⟹ span $\mathsf{Eff}(\mathrm{A})$

⇓

$$\forall a \in \mathsf{Eff}(\mathrm{A}),\ a = \sum_{i\in \mathrm{X}} c_i(a) l_i \qquad c_i \in \mathsf{St}_{\mathbb{R}}(\mathrm{A}).$$

$\{c_i\}_{i\in \mathrm{X}}$ is a *dual set* for $\{l_i\}_{i\in \mathrm{X}}$

$\rho \in \mathsf{St}_1(\mathrm{A})$ deterministic

$$\forall a \in \mathsf{Eff}_{\mathbb{R}}(\mathrm{A}),\ (a|\rho) = \sum_{i\in \mathrm{X}} c_i(a)(l_i|\rho)$$ state-tomography

⇓

$\{l_i\}_{i\in \mathrm{X}}$ *informationally complete* for states

Principles for Quantum Theory

$\{\rho_0, \rho_1\} \subseteq \mathsf{St}(\mathrm{A})$ preparation test

$\{a_0, a_1\}$ observation test

success probability of discrimination

$$p_{\mathrm{succ}} = (a_0|\rho_0) + (a_1|\rho_1)$$
$$= (a|\rho_0) + (a_1|\rho_1 - \rho_0)$$
$$= (a|\rho_1) + (a_0|\rho_0 - \rho_1)$$
$$= \frac{1}{2}[1 + (a_1 - a_0|\rho_1 - \rho_0)]$$

$$a := a_0 + a_1$$

Metric

$$p_{\mathrm{succ}}^{(\mathrm{opt})} = \frac{1}{2}[1 + ||\rho_1 - \rho_0||]$$

$$||\delta|| := \sup_{\{a_0, a_1\}} (a_0 - a_1|\delta)$$

$$||\delta|| = \sup_{a_0 \in \mathsf{Eff}(\mathrm{A})} (a_0|\delta) - \inf_{a_1 \in \mathsf{Eff}(\mathrm{A})} (a_1|\delta)$$

monotonicity

$$\mathscr{C} \in \mathsf{Transf}_1(\mathrm{A}, \mathrm{B})$$

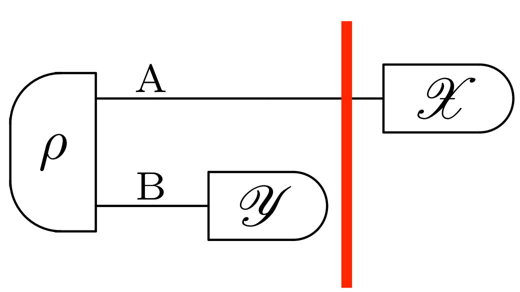
$$||\mathscr{C}\delta||_{\mathrm{B}} \leq ||\delta||_{\mathrm{A}}$$

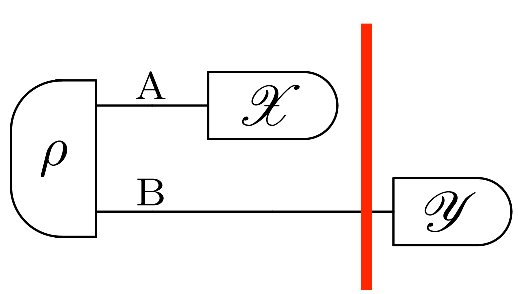
Principles for Quantum Theory

P1. Causality
P2. Local discriminability
P3. Purification
P4. Atomicity of composition
P5. Perfect distinguishability
P6. Lossless Compressibility

The probability of preparations is independent of the choice of observations

no signaling without interaction

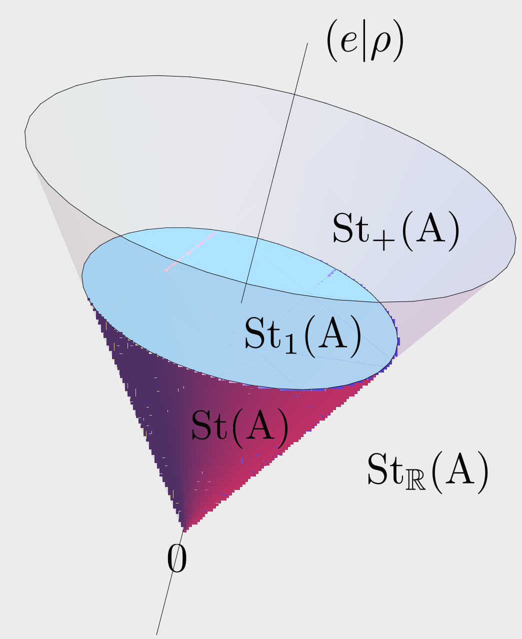


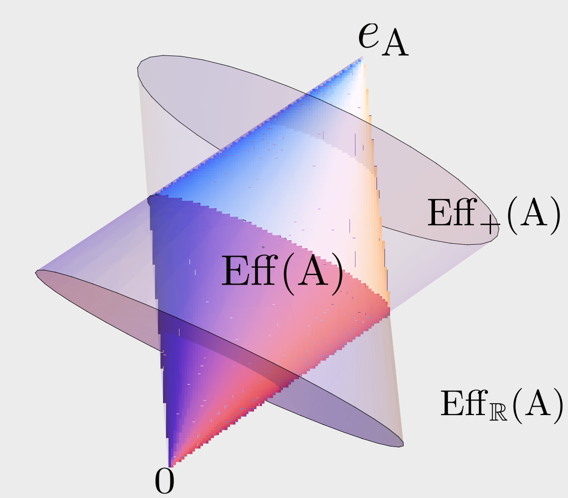


$$p(i,j|\mathscr{X},\mathscr{Y}) := (a_j|\rho_i)$$

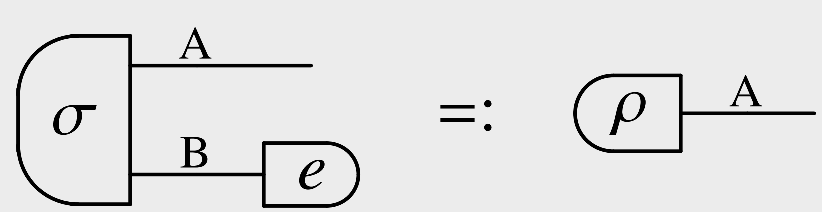
$$p(i|\mathscr{X},\mathscr{Y}) = p(i|\mathscr{X},\mathscr{Y}') = p(i|\mathscr{X})$$

Iff conditions: a) the deterministic effect is unique; b) states are "normalizable"





marginal state

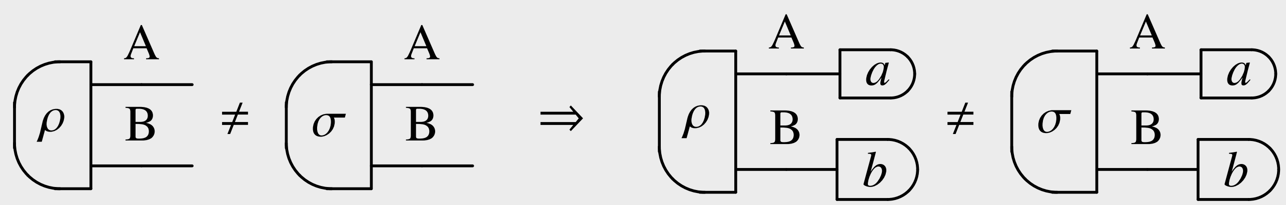


Principles for Quantum Theory

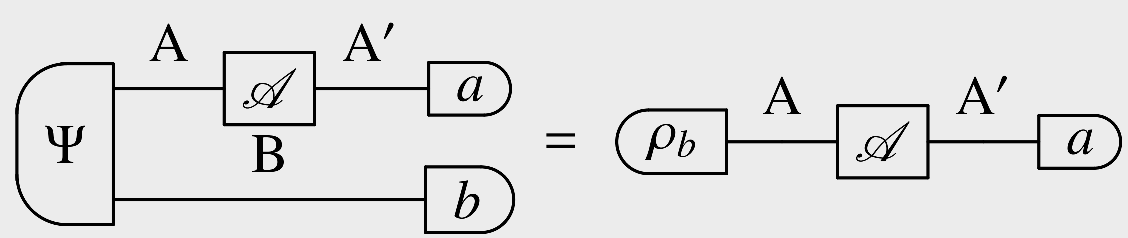
P1. Causality
P2. Local discriminability
P3. Purification
P4. Atomicity of composition
P5. Perfect distinguishability
P6. Lossless Compressibility

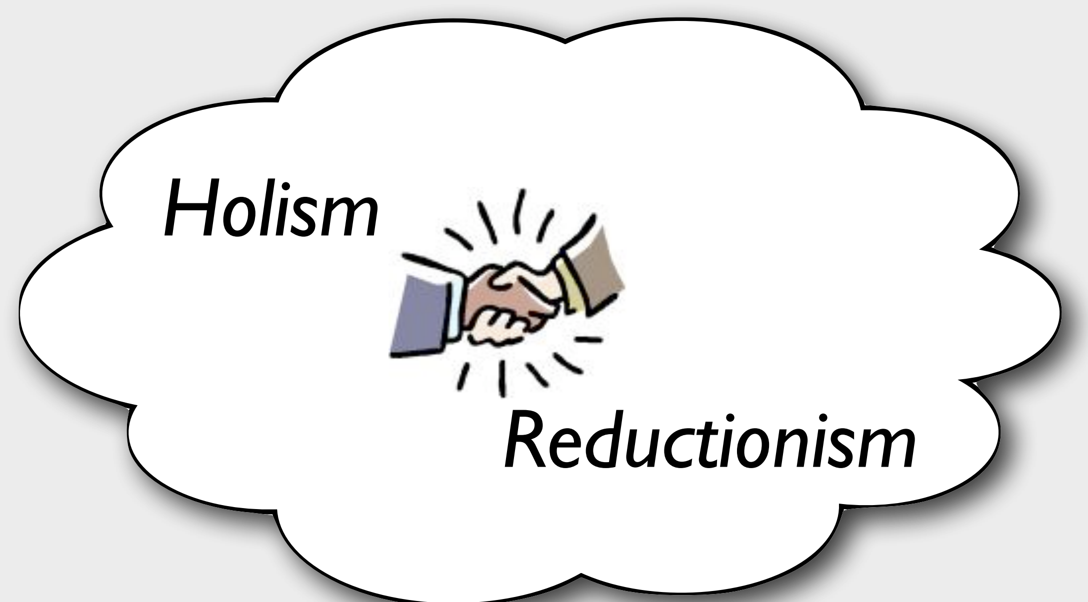
It is possible to discriminate any pair of states of composite systems using only local measurements.

Origin of the complex tensor product



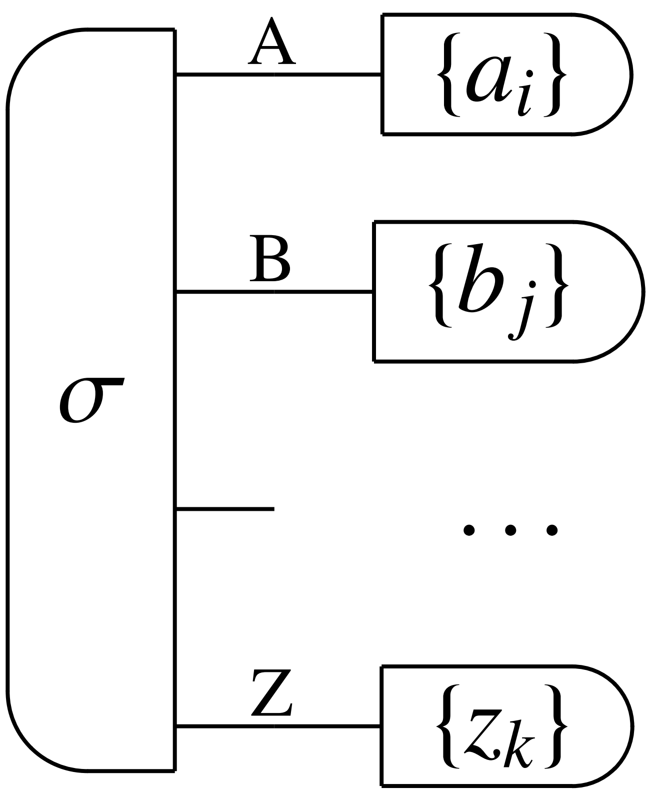
Local characterization of transformations



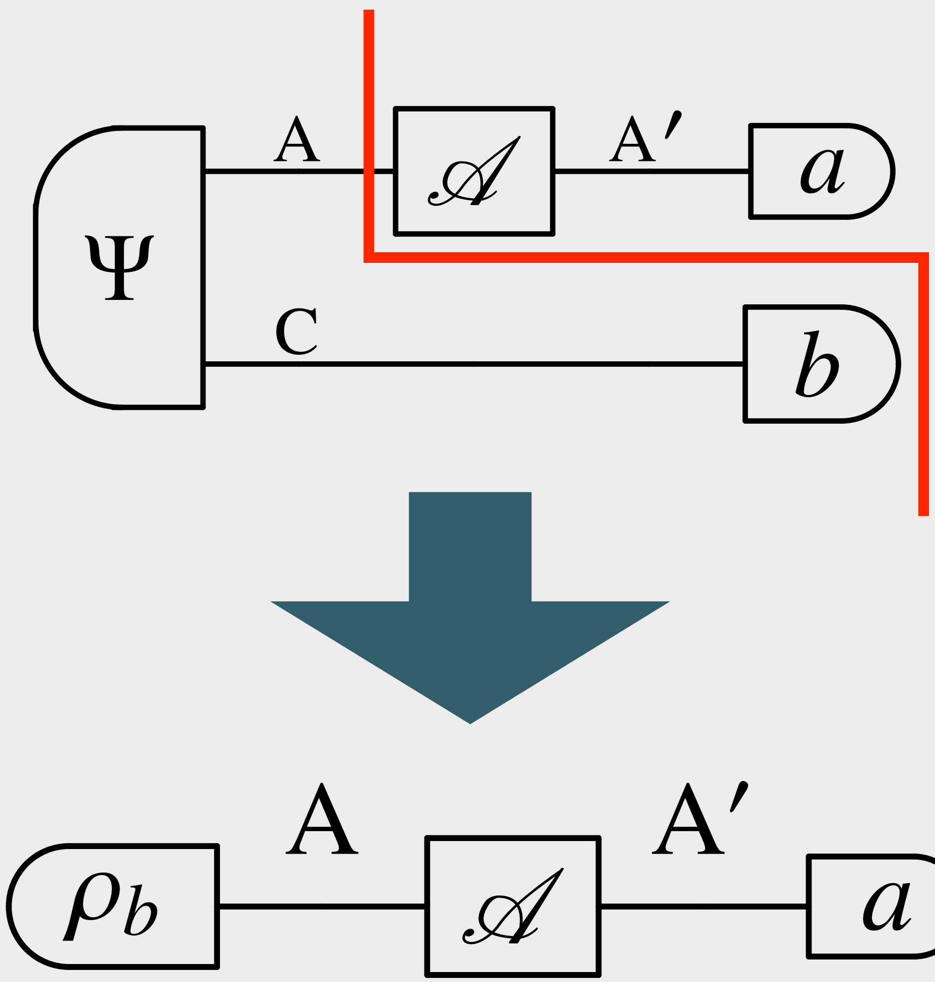


Principles for Quantum Theory

Local effects are separating for joint states



Tomography



Counter-examples: Real QT, Fermionic QT

Principles for Quantum Theory

P1. Causality
P2. Local discriminability
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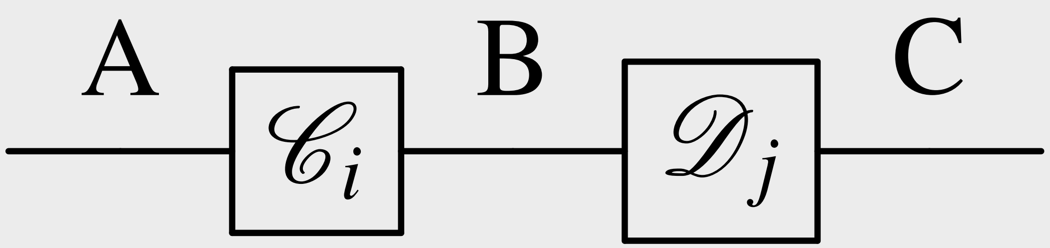
P4. Atomicity of composition
P5. Perfect distinguishability
P6. Lossless Compressibility

The composition of two atomic transformations is atomic

⬇

Complete information can be accessed on a step-by-step basis



Principles for Quantum Theory

P1. Causality
P2. Local discriminability
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P4. Atomicity of composition
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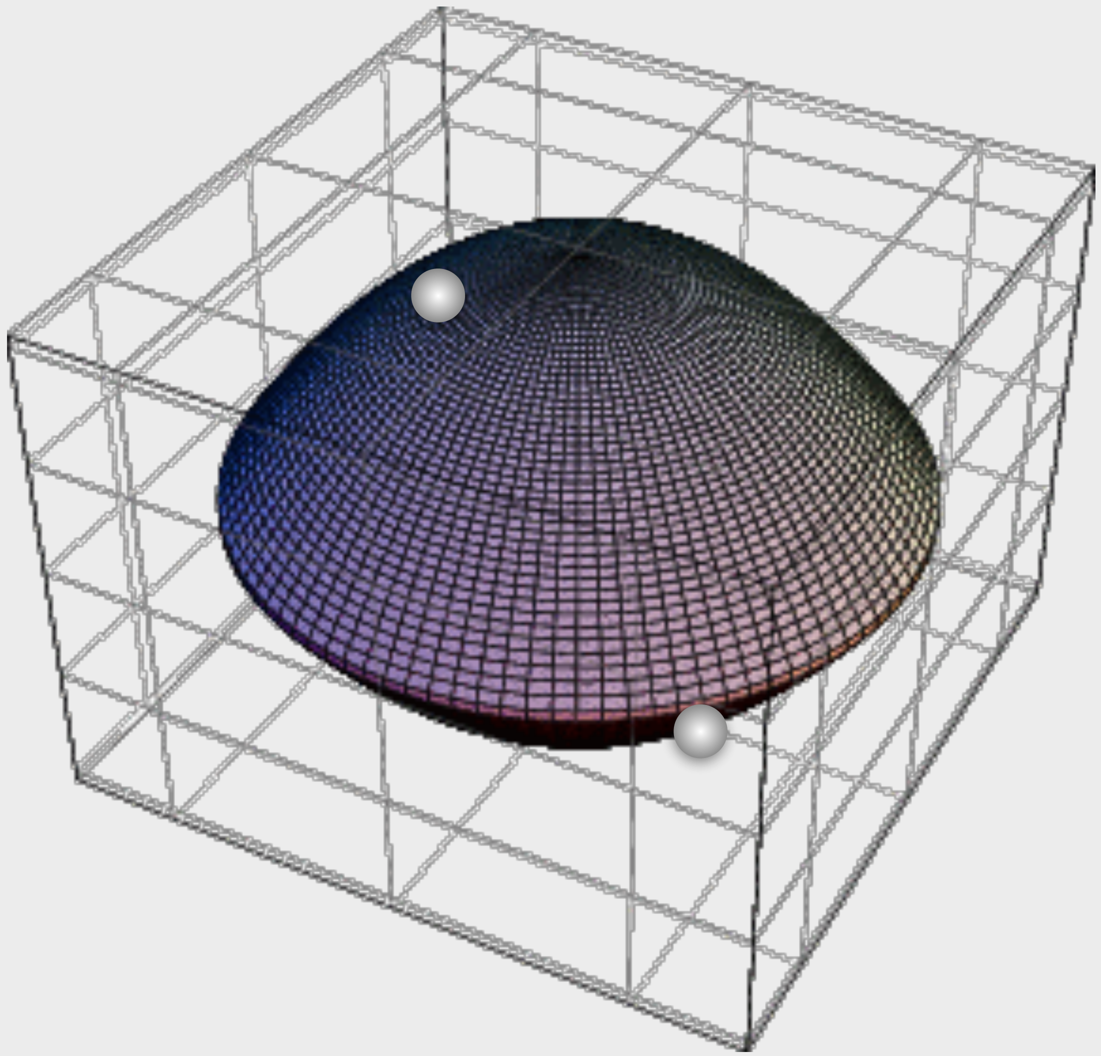
P6. Lossless Compressibility

Every state that is not completely mixed (i.e. on the boundary of the convex) can be perfectly distinguished from some other state.

Falsifiability of the theory



Principles for Quantum Theory

P1. Causality
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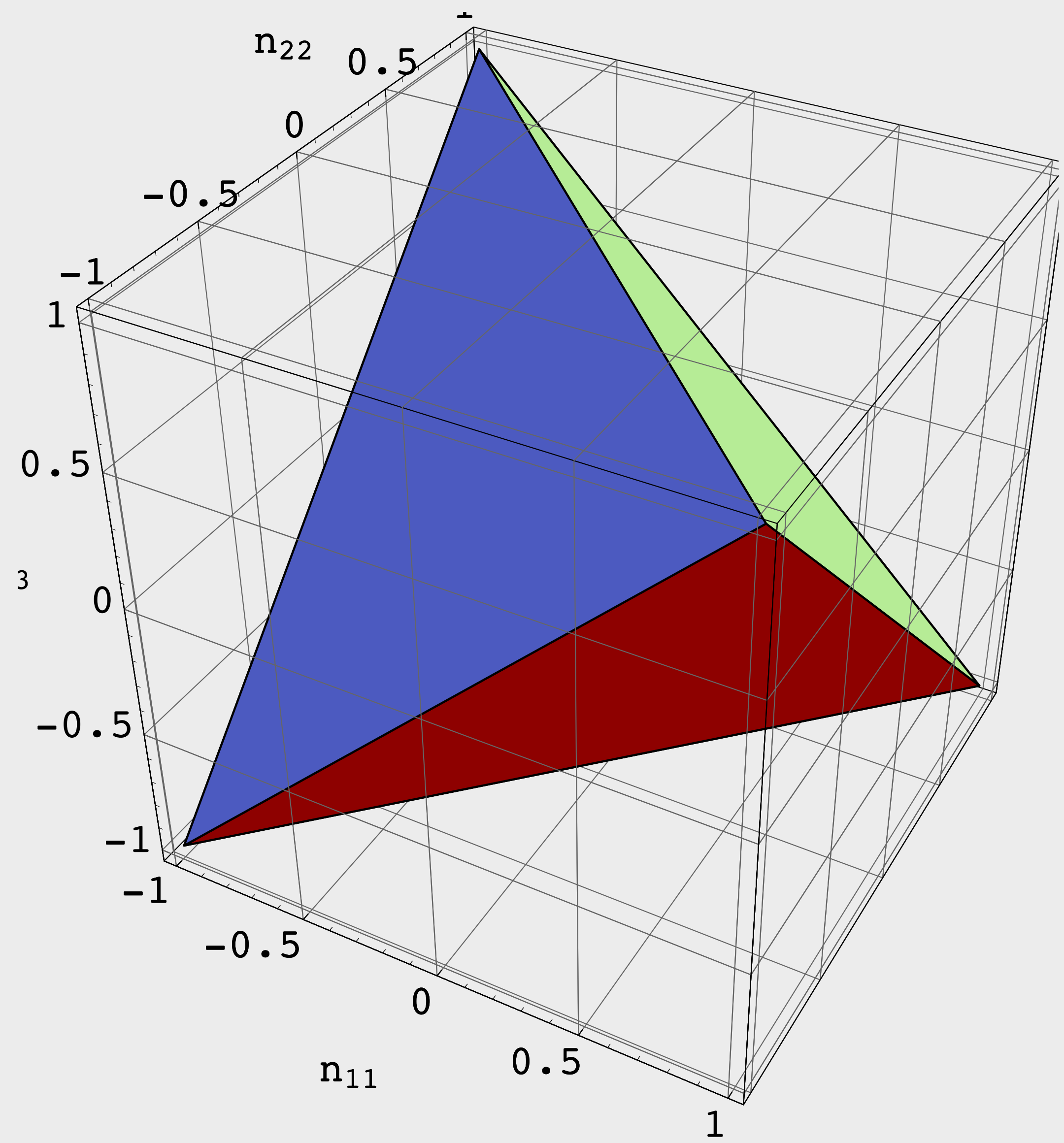
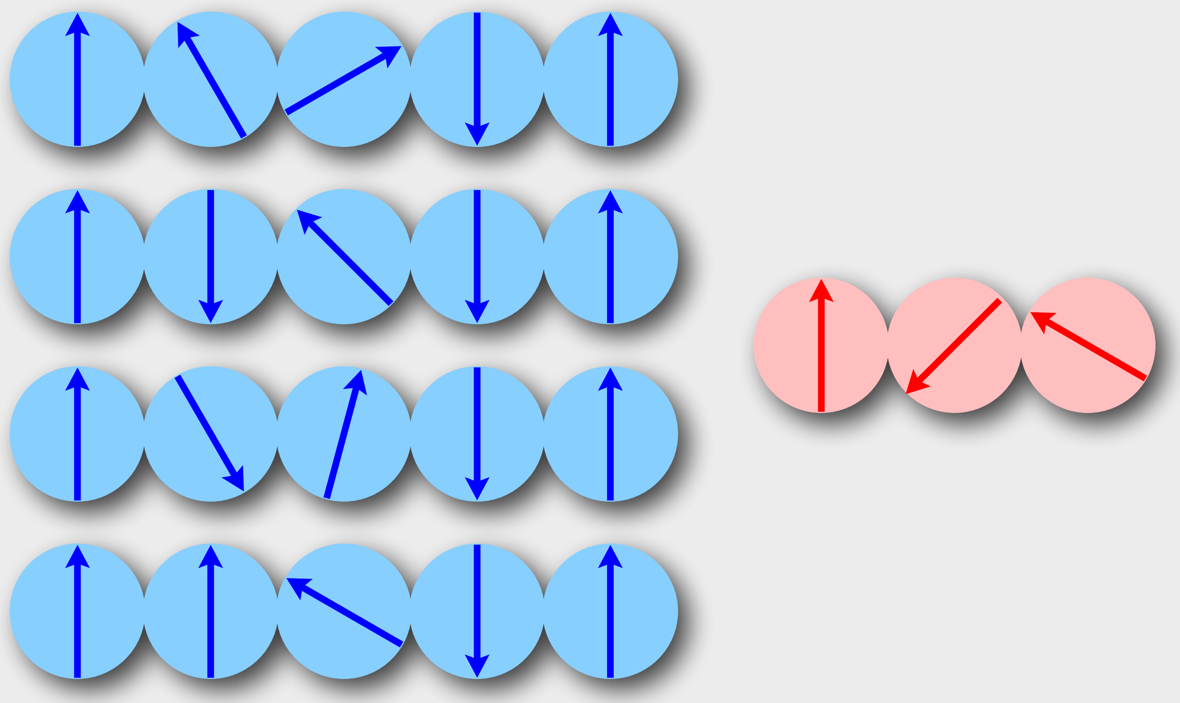
For states that are not completely mixed there exists an ideal compression scheme

Any face of the convex set of states is the convex set of states of some other system





Principles for Quantum Theory

P1. Causality
P2. Local discriminability
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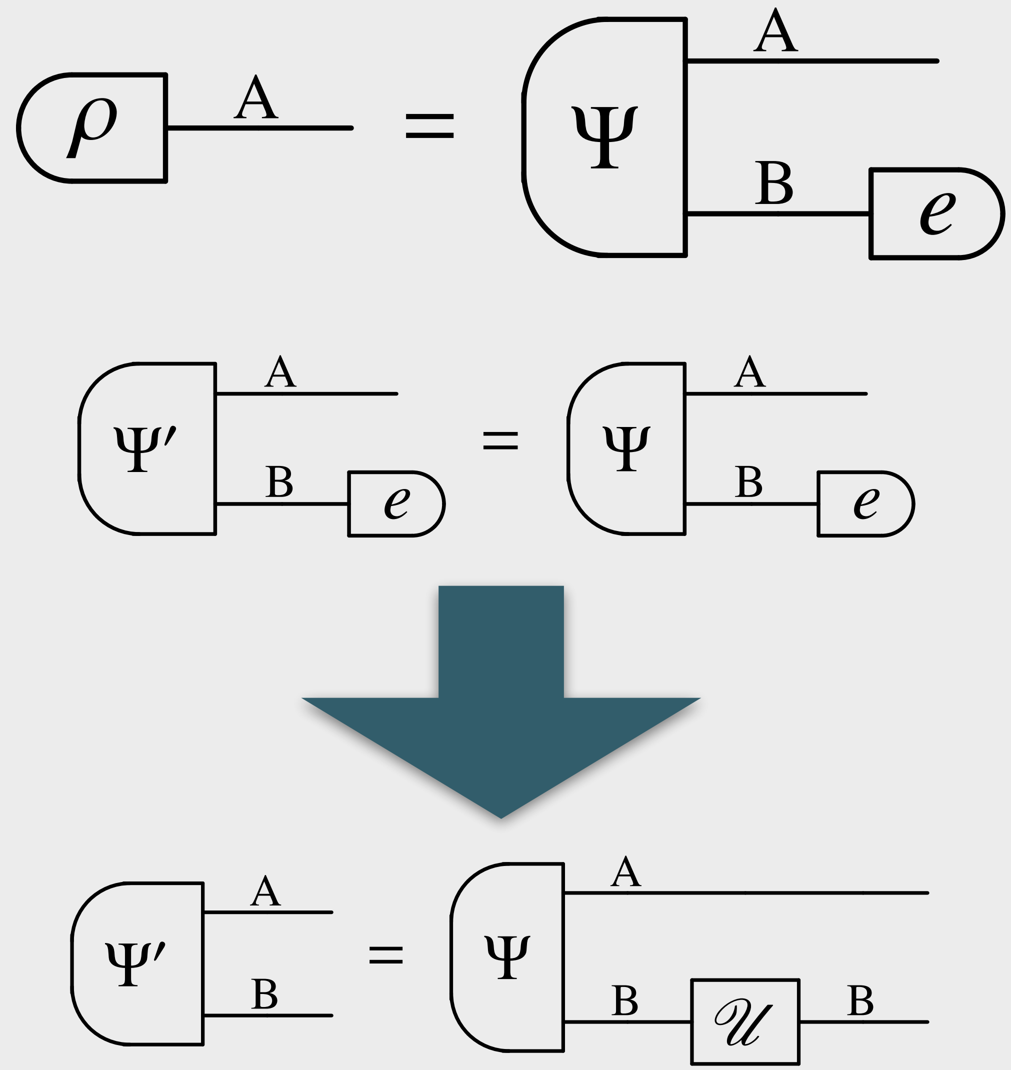
P6. Lossless Compressibility

Every state has a purification. For fixed purifying system, every two purifications of the same state are connected by a reversible transformation on the purifying system



Principles for Quantum Theory

P1. Causality
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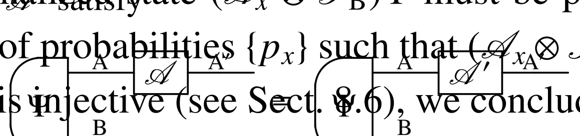
Every state has a purification. For fixed purifying system, every two purifications of the same state are connected by a reversible transformation on the purifying system

Purification establishes an interesting correspondence between transformations and states. This is easy to see: let us take a set of states $\{\alpha_x \mid x \in \mathrm{X}\}$ that span the whole state space of system A and a set of positive probabilities $\{p_x\}_{x\in\mathrm{X}}$. Then, take a purification of the mixed state $\rho = \sum_x p_x \alpha_x$—say $\Psi \in \mathrm{PurSt(AB)}$. Now, if two transformations $\mathscr{A}$ and $\mathscr{A}'$ satisfy



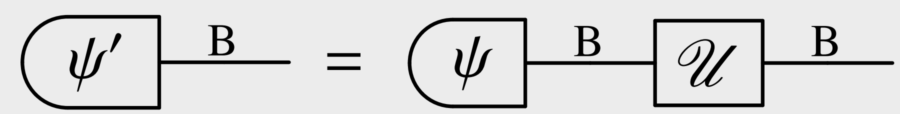
it is clear that $\mathscr{A}$ must be equal to $\mathscr{A}'$, namely the correspondence $\mathscr{A} \mapsto (\mathscr{A} \otimes \mathscr{I}_{\mathrm{B}})\Phi$ is injective.

different. If we take a pure state $\Psi \in \mathrm{PurSt(AB)}$ that can be used for process tomography, then the no-disturbance condition implies $\sum_x (\mathscr{A}_x \otimes \mathscr{I}_{\mathrm{B}})\Psi = \Psi$. But Ψ is pure: hence, each unnormalized state $(\mathscr{A}_x \otimes \mathscr{I}_{\mathrm{B}})\Psi$ must be proportional to Ψ. Precisely, there must be a set of probabilities $\{p_x\}$ such that $(\mathscr{A}_x \otimes \mathscr{I}_{\mathrm{B}})\Psi = p_x \Psi$. Since the map $\mathscr{A} \mapsto (\mathscr{A} \otimes \mathscr{I}_{\mathrm{B}})\Psi$ is injective (see Sect. 4.6), we conclude that $\mathscr{A}_x = p_x \mathscr{I}_A$. In other

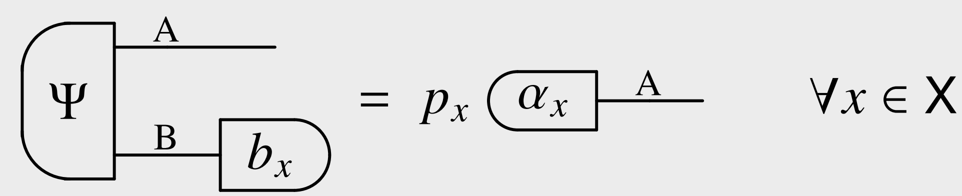
Consequences

1. ***Existence of entangled states:***
the purification of a mixed state is an entangled state;
the marginal of a pure entangled state is a mixed state;

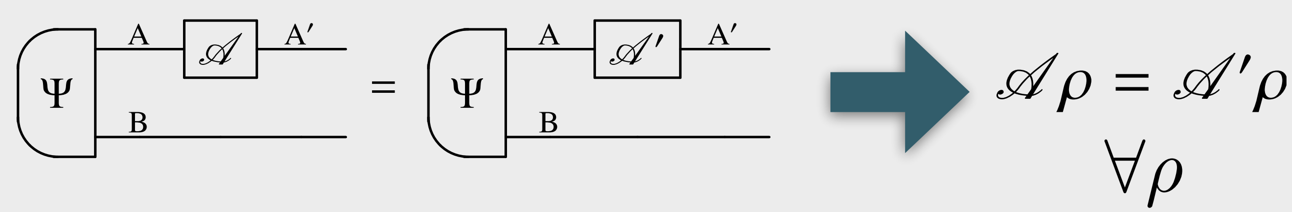
2. *Every two normalized pure states of the same system are connected by a reversible transformation*



3. ***Steering:*** Let **Ψ** purification of ρ. The for every ensemble decomposition $\rho=\sum_x p_x \alpha_x$ there exists a measurement $\{b_x\}$, such that



4. ***Process tomography (faithful state):***



5. ***No information without disturbance***

Principles for Quantum Theory

P1. Causality

P2. Local discriminability

P3. Purification

P4. Atomicity of composition

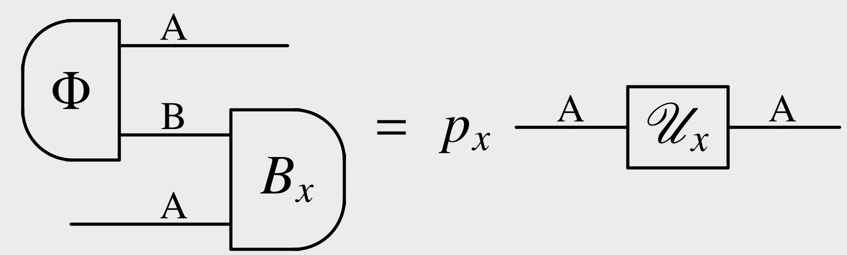
P5. Perfect distinguishability

P6. Lossless Compressibility

Every state has a purification. For fixed purifying system, every two purifications of the same state are connected by a reversible transformation on the purifying system

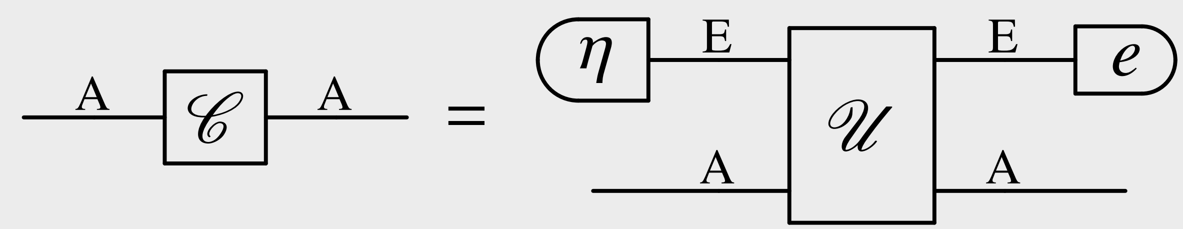
Consequences

6. ***Teleportation***

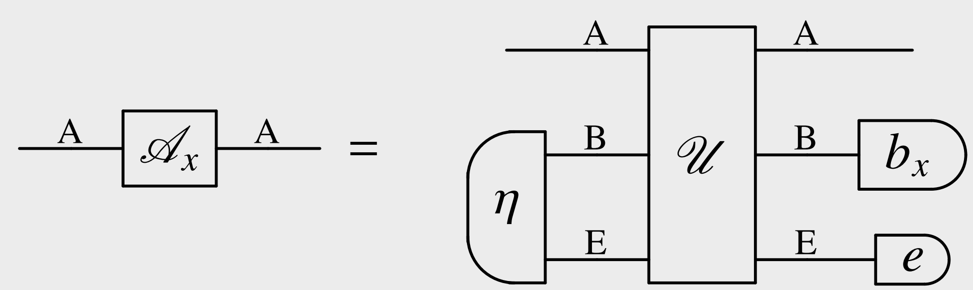


$\forall x \in$ X

7. ***Reversible dilation of "channels"***



8. ***Reversible dilation of "instruments"***

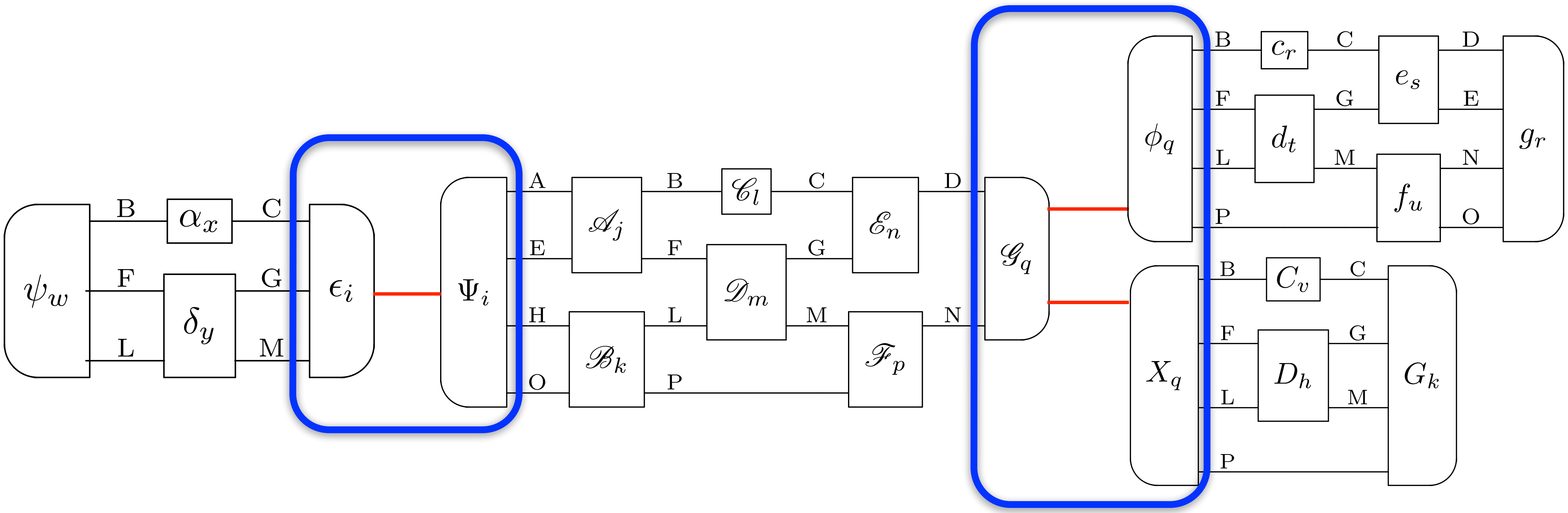


$\forall x \in$ X

9. ***State-transformation cone isomorphism***

10. ***Rev. transform. for a system make a compact Lie group***

On the von Neumann postulate



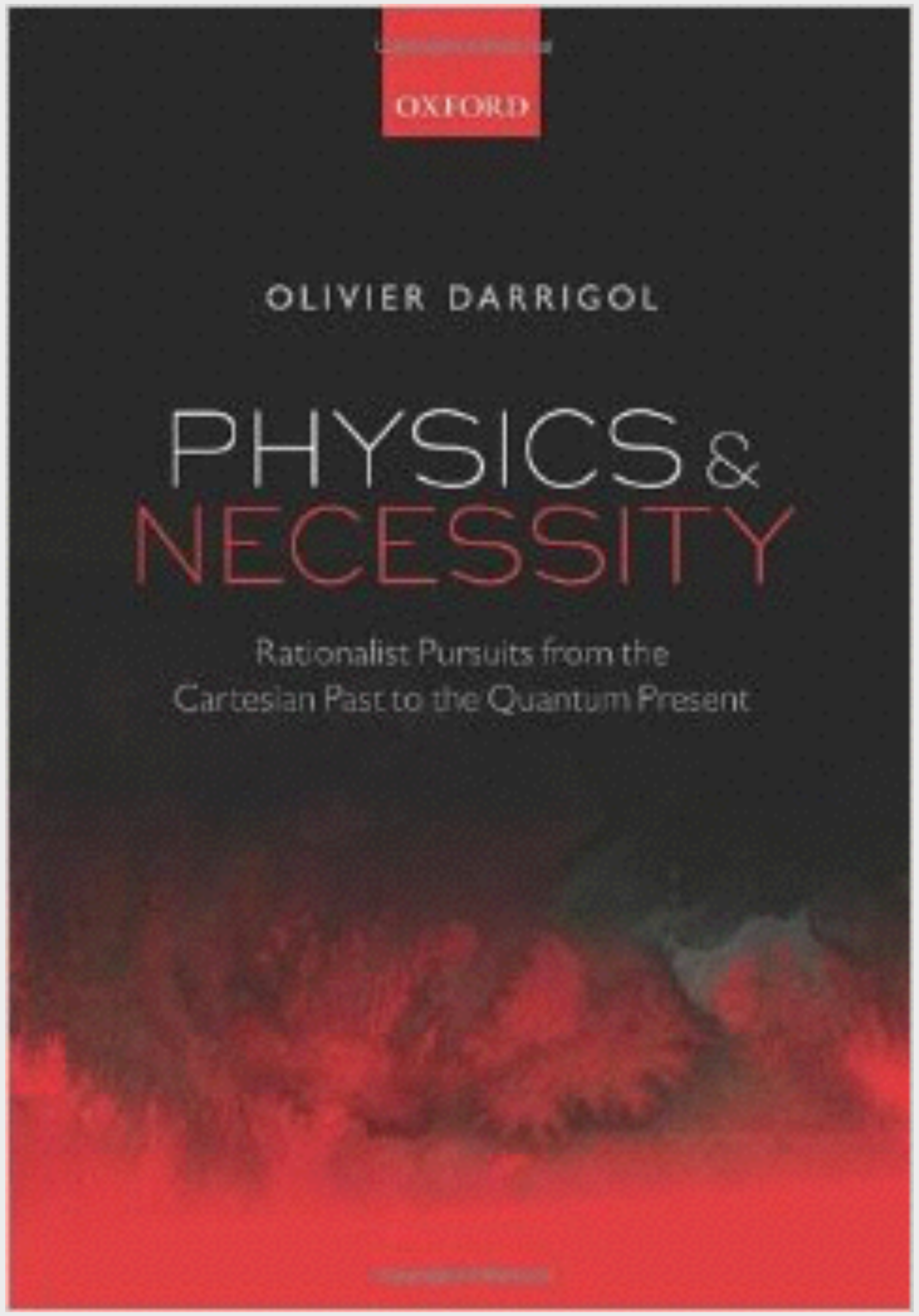
Principles for Quantum Theory

P1. Causality
P2. Local discriminability
P3. Purification
P4. Atomicity of composition
P5. Perfect distinguishability
P6. Lossless Compressibility

Epistemological principles

Are they *necessary*?

Fermionic quantum theory?



Informationalism: Principles for QFT

- *Mechanics (QFT) derived in terms of countably many quantum systems in interaction*

add principles

Min algorithmic complexity principle

- homogeneity
- locality
- reversibility

- linearity
- isotropy
- minimal-dimension
- Cayley qi-embedded in R^d

} Restrictions

Cells labeled by $g \in G$, $|G| \leq \aleph$; $\psi_g \in \mathbb{C}^{s_g}$, $0 < s_g < \infty$

linearity	The interaction between systems is described by $s_{g'} \times s_g$ *transition matrices* $A_{gg'}$ with evolution from step t to step $t+1$ given by $$\psi_g(t+1) = \sum_{g' \in G} A_{gg'} \psi_{g'}(t)$$
unitarity	$$\sum_{g'} A_{gg'} A^\dagger_{g''g'} = \sum_{g'} A^\dagger_{gg'} A_{g''g'} = \delta_{gg''} I_{s_g}$$
locality	$A_{gg'} \neq 0 \iff A_{g'g} \neq 0$: g' and g are *interacting* $|S_g| \leq k < \infty$ for every $g \in G$, where $S_g \subseteq G$ set of cells g' interacting with g
homogeneity	All cells $g \in G$ are equivalent $\implies |S_g|$, s_g, $\{A_{gg'}\}_{g' \in S_g}$ independent of g Identify the matrices $A_{gg'} = A_h$ for some $h \in S$ with $|S| = |S_g|$ Define $gh := g'$ if $A_{gg'} = A_h$ and define $A_{g'g} := A_{h^{-1}}$ A sequence of transitions $A_{h_N} A_{h_{N-1}} \ldots A_{h_1}$ connects g to itself, i.e. $gh_1h_2 \ldots h_N = g$, then it must also connect any other $g' \in G$ to itself, i.e. $g'h_1h_2 \ldots h_N = g'$
all 4 principles together	The following operator over the Hilbert space $\ell^2(G) \otimes \mathbb{C}^s$ is unitary $$A = \sum_{h \in S} T_h \otimes A_h,$$ where T is the right-regular representation of G on $\ell^2(G)$ acting as $T_g |g'\rangle = |g'g^{-1}\rangle$

Informationalism: Principles for QFT

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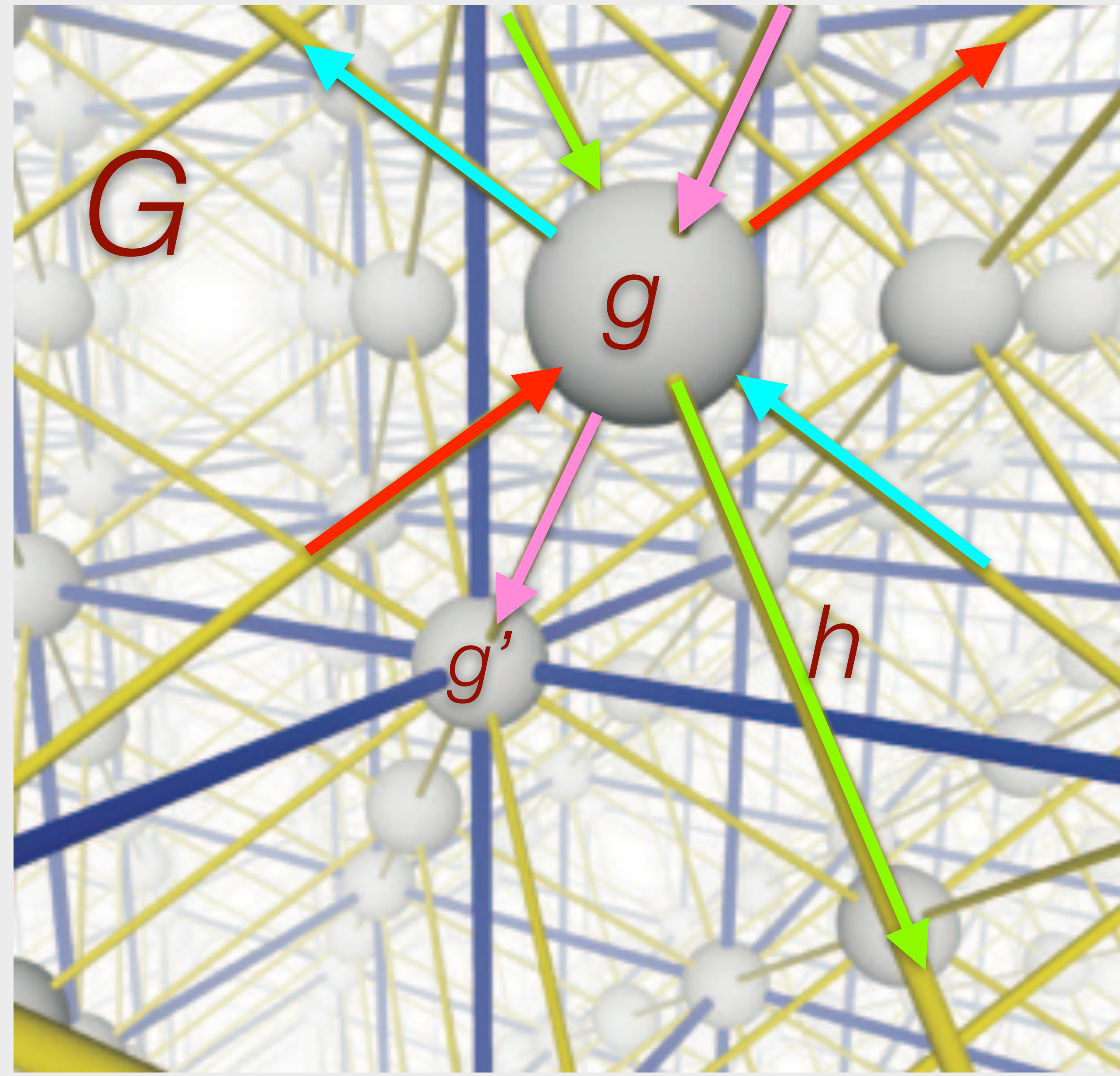
} Quantum Cellular Automata on the Cayley graph of a group *G*

- linearity
- isotropy
- minimal-dimension
- Cayley qi-embedded in R^d

} Restrictions

G virtually Abelian

(geometric group theory)



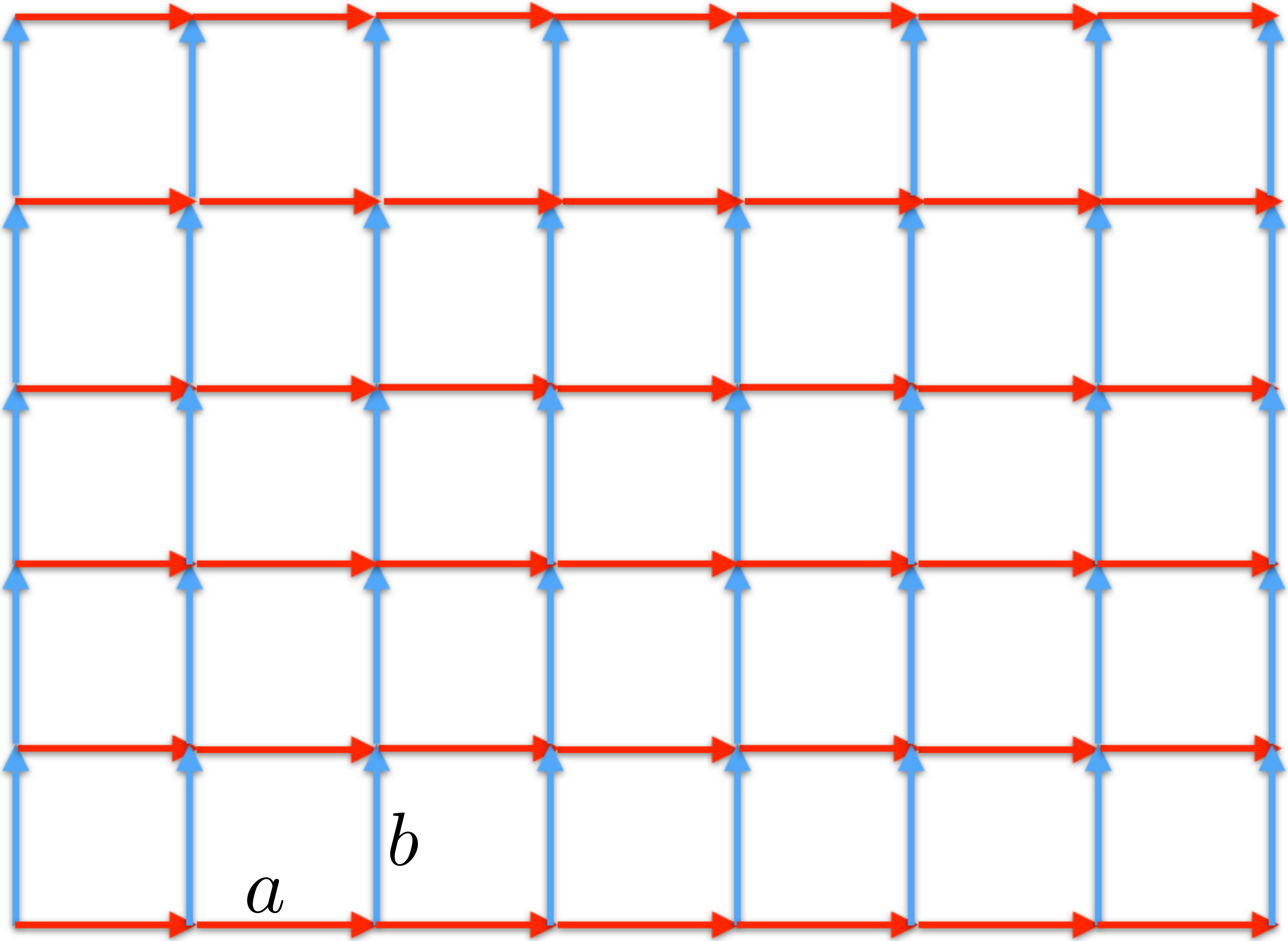
$$G = \langle h_1, h_2, \ldots | r_1, r_2, \ldots \rangle =: \langle S_+ | R \rangle$$

Sketch of derivation of QW on Cayley

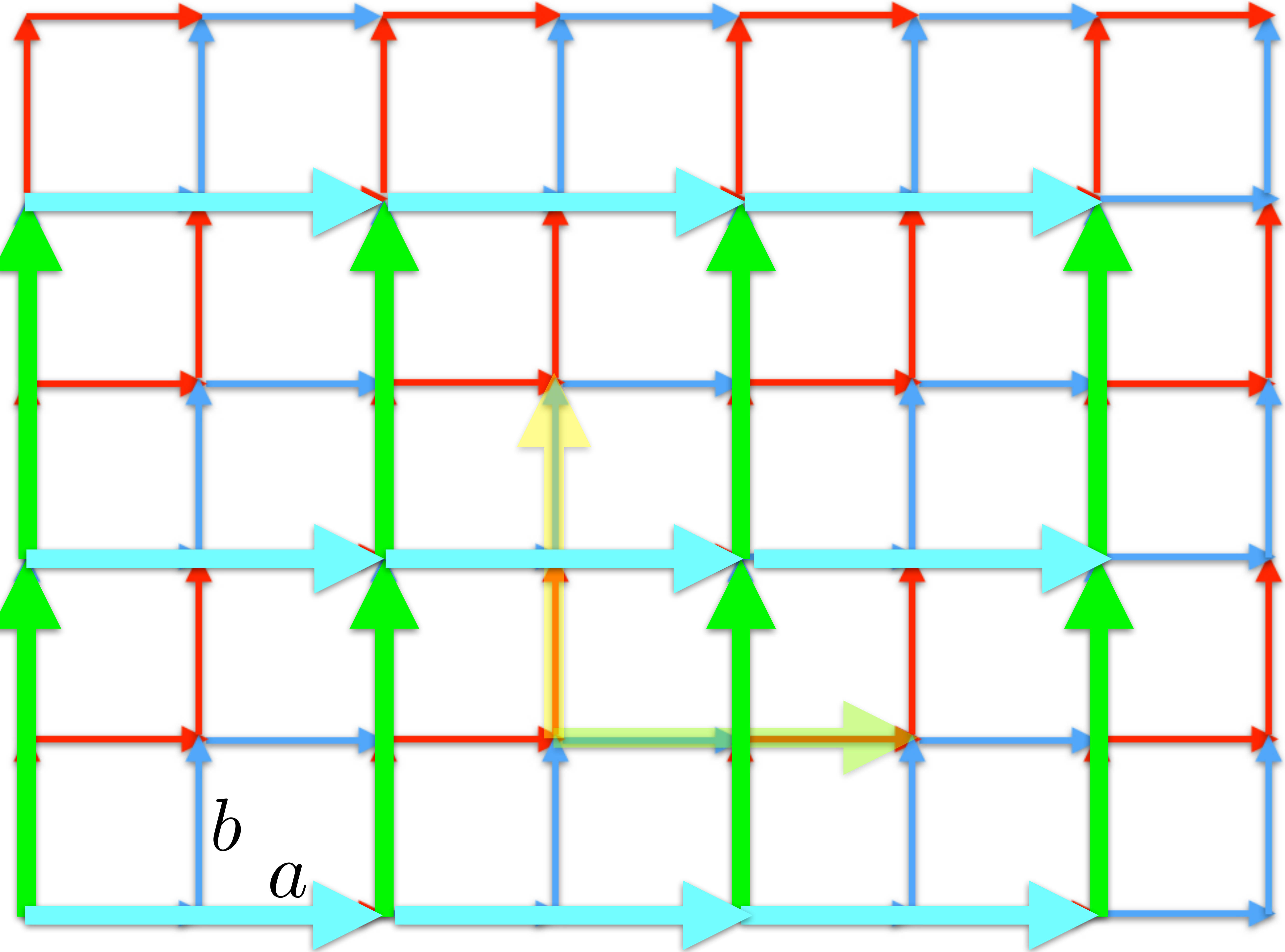
The homogeneity requirement means that all the sites $g \in G$ are equivalent. In other words, the evolution must not allow one to discriminate two sites g and g'. In mathematical terms, this requirement has three main consequences. The first one is that the cardinality $|S_g|$ is independent of g. The second one is that the set of matrices $\{A_{gg'}\}_{g' \in S_g}$ is the same for every g, whence we will identify the matrices $A_{gg'} = A_h$ for some $h \in S$ with $|S| = |S_g|$. This allows us to define $gh = g'$ if $A_{gg'} = A_h$. In this case, we also formally write $g = g'h^{-1}$. Since for $A_{gg'} \neq 0$ also $A_{g'g} \neq 0$, clearly if $h \in S$ then also $h^{-1} \in S$. The third consequence is that, whenever a sequence of transitions $h_1 h_2 \cdots h_N$ with $h_i \in S$ connects g to itself, i.e., $g h_1 h_2 \cdots h_N = g$, then it must also connect any other $g' \in G$ to itself, i.e., $g' h_1 h_2 \cdots h_N = g'$.

Sketch of derivation of QW on Cayley

We now define the graph $\Gamma(G,S)$, where the vertices are elements of G, and edges correspond to couples (g,g') with $g' = gh$. The edges can then be colored with $|S|$ colors, in one-to-one correspondence with the transition matrices $\{A_h\}_{h\in S}$. It is now easy to verify that either the graph $\Gamma(G,S)$ is connected or it consists of n disconnected copies of the same connected graph $\Gamma(G_0,S)$. Since the information in G is generally redundant, consisting of n identical and independent copies of the same QCA with cells belonging to G_0, from now on we assume that the graph $\Gamma(G,S)$ is connected. One can now prove that such a graph represents the Cayley graph of a finitely presented group with generators $h \in S$ and relators corresponding to the set R of strings of elements of S corresponding to closed paths. More precisely, we define the free group F of words with letters in S and the free subgroup H generated by words in R; it is easy to check that H is normal in F, thanks to homogeneity. The group G with Cayley graph $\Gamma(G,S)$ coincides with F/N.



$$G = \langle a, b | aba^{-1}b^{-1} \rangle$$



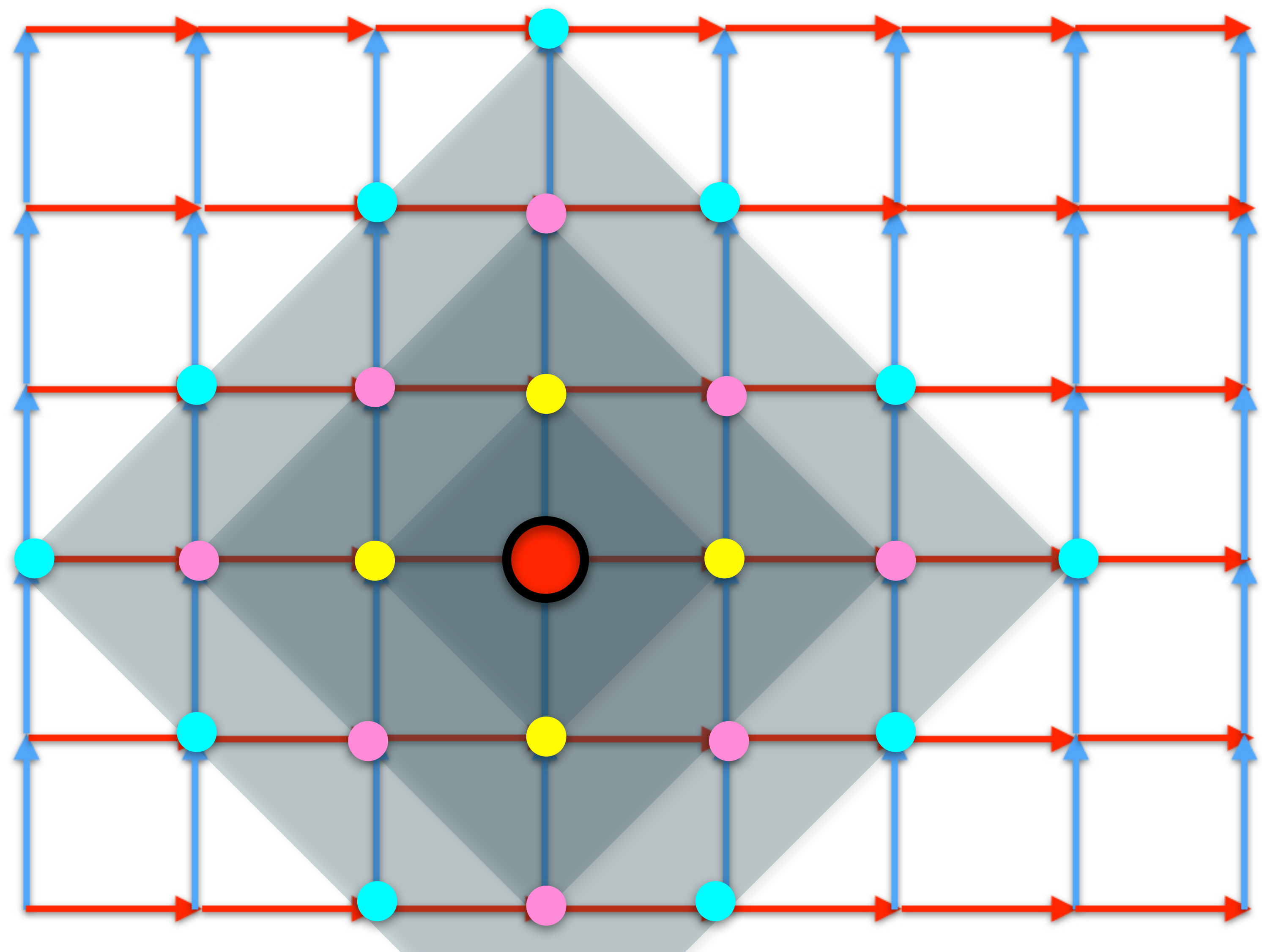
$$G = \langle a, b | a^2 b^{-2} \rangle$$

Quantum walk on Cayley graph

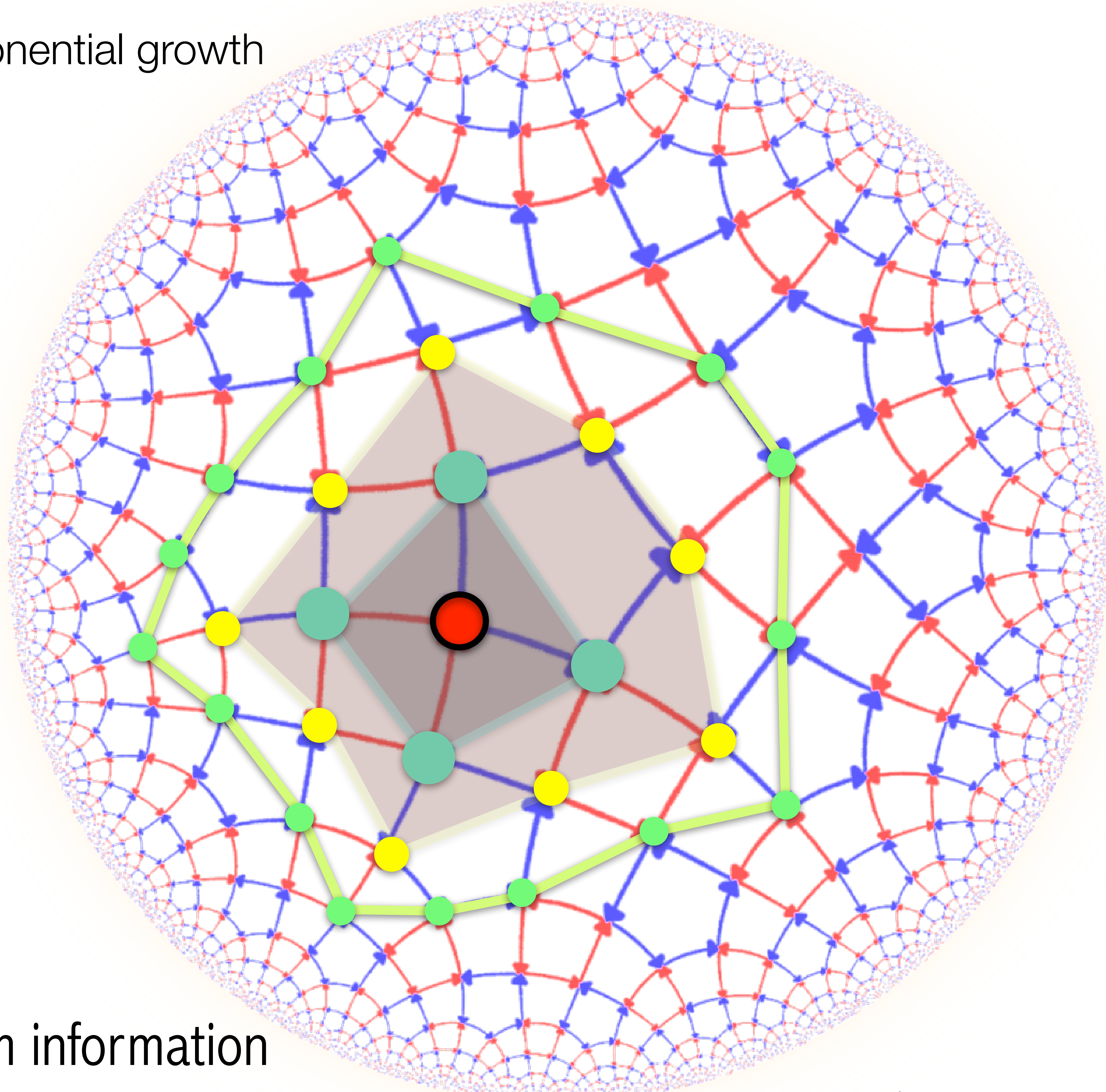
Theorem (Gromov): A group is quasi-isometrically embeddable in R^d iff it is <u>virtually Abelian</u>

Virtually Abelian groups have polynomial growth

points $\sim r^d$



- G hyperbolic → exponential growth



points ~exp(r)

transmitted quantum information decrease as exp(-r)

$$G = \langle a, b | a^5, b^4, (ab)^2 \rangle$$

Informationalism: Principles for QFT

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} Quantum Cellular Automata on the Cayley graph of a group *G*

- linearity
- isotropy
- minimal-dimension
- Cayley qi-embedded in R^d

} Restrictions

Isotropy

- There exists a group L of permutations of S_+, transitive over S_+ that leaves the Cayley graph invariant
- a nontrivial unitary s-dimensional (projective) representation $\{L_l\}$ of L such that:

$$A = \sum_{h \in S} T_h \otimes A_h = \sum_{h \in S} T_{lh} \otimes L_l A_h L_l^\dagger$$

Informationalism: Principles for QFT

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} Quantum Cellular Automata on the Cayley graph of a group *G*

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} Restrictions

- *Relativistic regime (k«1):* free QFT (Weyl, Dirac, and Maxwell)
- *Ultra-relativistic regime (k~1)* [Planck scale]: <u>nonlinear Lorentz</u>

- QFT derived:
  - without assuming Special Relativity
  - without assuming mechanics (quantum ab-initio)

- QCA is a <u>*discrete*</u> theory

Motivations to keep it discrete:

1. Discrete contains continuum as special regime
2. Testing mechanisms in quantum simulations
3. Falsifiable discrete-scale hypothesis
4. Natural scenario for holographic principle
5. Solves all issues in QFT originating from continuum:
   i) uv divergencies
   ii) localization issue
   iii) Path-integral
6. Fully-fledged theory to evaluate cutoffs

Quantum walk on Cayley graph

Definition 2 (Quantum walk on Cayley graph)**.** An s-dimensional quantum walk on the Cayley graph (QWCG) $\Gamma(G, S_+)$ of the finitely presented group G is the quadruple

$$Q = \{G, S_+, s, \{A_h\}_{h\in S}\}, \tag{3}$$

where

(1) $s \in \mathbb{N}$;
(2) $\forall h \in S,\ A_h \in \mathbb{M}_s(\mathbb{C})$ ($s \times s$ complex matrices); A_h are called *transition matrices*.
(3) the following operator is unitary over $\mathcal{H}_Q := \ell^2(G) \otimes \mathbb{C}^s$

$$A_Q = \sum_{h\in S} T_h \otimes A_h\ , \tag{4}$$

Lemma 1. *A is unitary if and only if all the following equations hold*

$$\begin{cases} \displaystyle\sum_{h\in S} A_h^\dagger A_h = \sum_{h\in S} A_h A_h^\dagger = I_s, \\ \displaystyle\forall g \in S^2/\{e\},\ \sum_{h,h'\in S,\, hh'^{-1}=g} A_h^\dagger A_{h'} = \sum_{h,h'\in S,\, hh'^{-1}=g} A_{h'} A_h^\dagger = 0. \end{cases} \tag{5}$$

Lemma 2. *$A_h^\dagger A_{h'} = 0$ if hh' is not a subword of a relator r with $\|r\| = 4$, $\|\cdot\|$ denoting the word metric on G.*

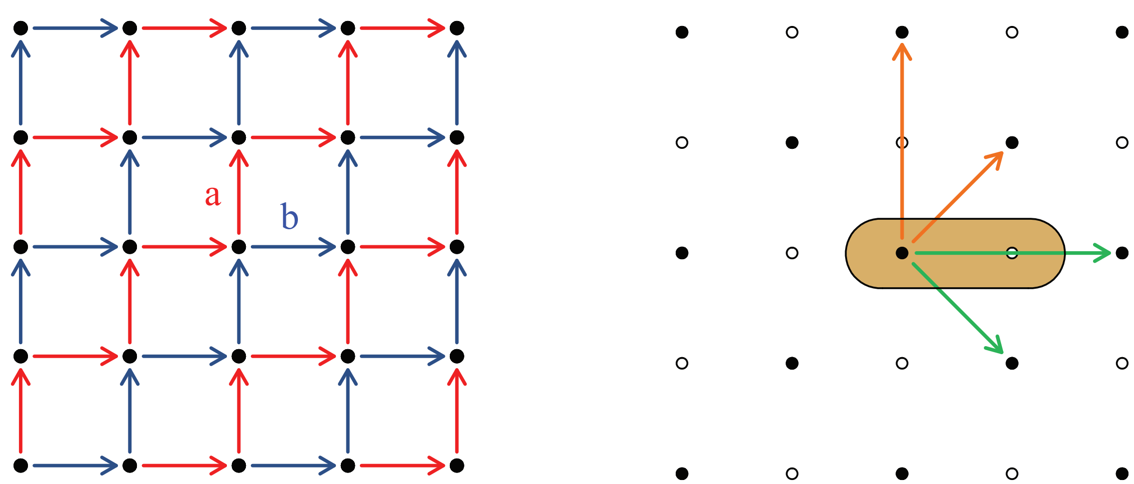
Quantum walk on Cayley graph

Remark 2. *One can prove that for QWCG* $Q = (G, S_+, s, \{A_h\}_{h\in S})$ *with* G *virtually Abelian there exists a quantum walk* $Q' = (H, S_+^H, s\cdot i_H, \{B_h\}_{h\in S^H})$ *with Abelian* $H \subset G$*, with finite index* i_H*, such that*

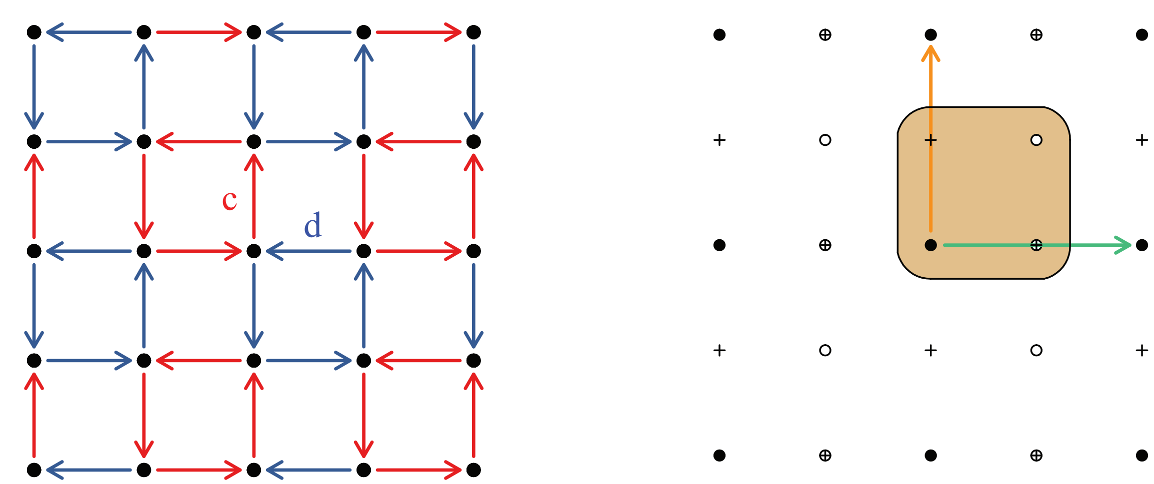
$$A_{Q'} = V A_Q V^\dagger, \quad \text{with } V : u_{g_i a}\otimes\psi \mapsto V u_{g_i a}\otimes\psi = v_a \otimes e_i \otimes \psi, \tag{13}$$

with $\{g_i\}_{i=1,\ldots,i_H}$ *being coset representatives,* v_a *with* $a \in H$ *canonical orthonormal basis of* $\ell^2(H)$*,* $\{e_i\}_{i=1,\ldots,i_H}$ *canonical basis in* $\mathbb{C}^{i_H}$*,* $\psi \in \mathbb{C}^s$*, and* V *isomorphism between* $\ell^2(G)\otimes\mathbb{C}^s$ *and* $\ell^2(H)\otimes\mathbb{C}^{s\cdot i_H}$*.*

$\langle\, a, b \mid a^2 b^{-2} \,\rangle$



$\langle\, c, d \mid c^4, d^4, (cd)^2 \,\rangle$



[Danny Calegary] For isotropic Q with isotropy group L, one can choose H with $i_H \geq |L|$, and consider the orbit of H under the action of L. Then H is still symmetric.

Quantum Cellular Automaton

Linearity ⇒

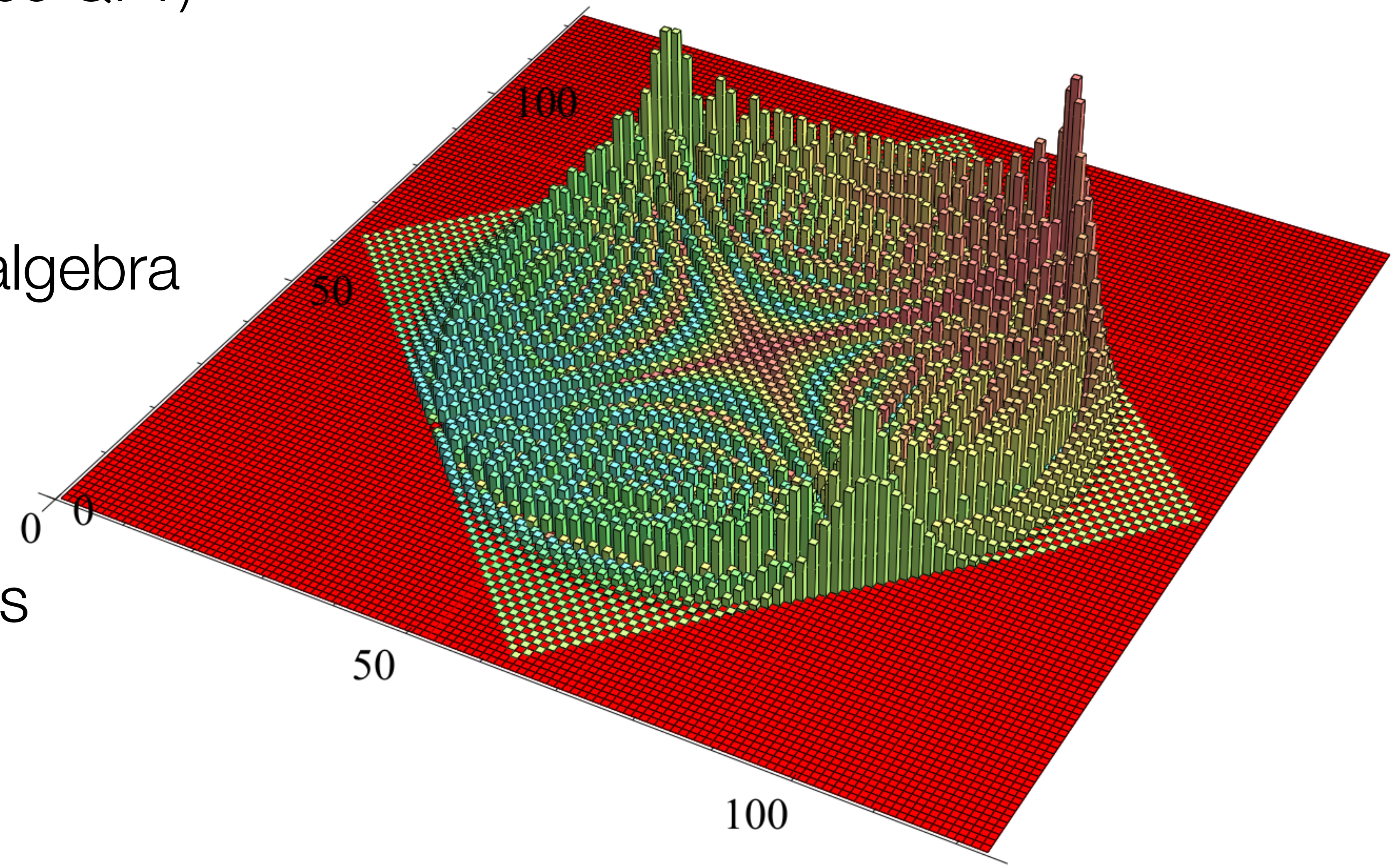
Quantum Cellular Automaton (free QFT)

$$U\psi U^\dagger = A\psi$$

Fock space ⇒ von Neumann algebra

Isotropy ⇒ statistics

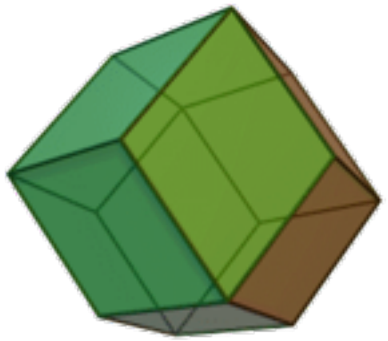
Minimal dimension ⇒ Fermions



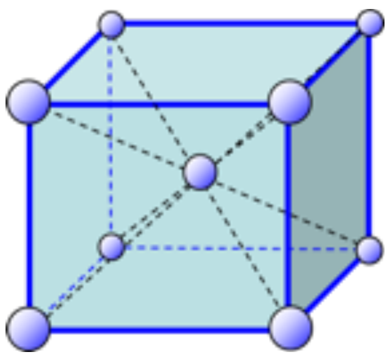
The Weyl QCA

D'Ariano, Perinotti,
PRA **90** 062106 (2014)

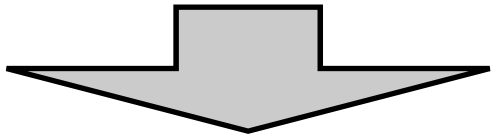
☞ Minimal dimension for nontrivial unitary *A:* s=2



- Unitarity ⇒ for d=3 the only possible *G* is the BCC!!
- Isotropy ⇒ Fermionic ψ (d=3)



Unitary operator: $$A = \int_{\mathrm{B}} \mathrm{d}^3\mathbf{k}\, |\mathbf{k}\rangle\langle\mathbf{k}| \otimes A_{\mathbf{k}}$$



Two QCAs connected by P

$$\begin{aligned} A^{\pm}_{\mathbf{k}} = &- i\sigma_x(s_x c_y c_z \pm c_x s_y s_z) \\ &\mp i\sigma_y(c_x s_y c_z \mp s_x c_y s_z) \\ &- i\sigma_z(c_x c_y s_z \pm s_x s_y c_z) \\ &+ I(c_x c_y c_z \mp s_x s_y s_z) \end{aligned}$$

$$s_\alpha = \sin\frac{k_\alpha}{\sqrt{3}}$$

$$c_\alpha = \cos\frac{k_\alpha}{\sqrt{3}}$$

The Weyl QCA

D'Ariano, Perinotti, PRA **90** 062106 (2014)

$$i\partial_t\psi(t) \simeq \frac{i}{2}[\psi(t+1) - \psi(t-1)] = \frac{i}{2}(A - A^\dagger)\psi(t)$$

$$\begin{aligned}\frac{i}{2}(A^\pm_{\mathbf{k}} - A^{\pm\dagger}_{\mathbf{k}}) = &+ \sigma_x(s_x c_y c_z \pm c_x s_y s_z) \\ &\pm \sigma_y(c_x s_y c_z \mp s_x c_y s_z) \\ &+ \sigma_z(c_x c_y s_z \pm s_x s_y c_z)\end{aligned}$$

"Hamiltonian"

$k \ll 1$ ⇒ $i\partial_t\psi = \frac{1}{\sqrt{3}}\boldsymbol{\sigma}^\pm \cdot \mathbf{k}\,\psi$ ☜ Weyl equation! $\boldsymbol{\sigma}^\pm := (\sigma_x, \pm\sigma_y, \sigma_z)$

Two QCAs connected by P

$$\begin{aligned}A^\pm_{\mathbf{k}} = &- i\sigma_x(s_x c_y c_z \pm c_x s_y s_z) \\ &\mp i\sigma_y(c_x s_y c_z \mp s_x c_y s_z) \\ &- i\sigma_z(c_x c_y s_z \pm s_x s_y c_z) \\ &+ I(c_x c_y c_z \mp s_x s_y s_z)\end{aligned}$$

$$s_\alpha = \sin\frac{k_\alpha}{\sqrt{3}}$$

$$c_\alpha = \cos\frac{k_\alpha}{\sqrt{3}}$$

Exact solution of Dirac Quantum Walk

The analytical solution of the Dirac automaton can also be expressed in terms of Jacobi polynomials $P_k^{(\zeta,\rho)}$ performing the sum over f in Eq. (16) which finally gives

$$\psi(x,t) = \sum_y \sum_{a,b\in\{0,1\}} \gamma_{a,b} P_k^{(1,-t)}\left(1 + 2\left(\frac{m}{n}\right)^2\right) A_{ab}\psi(y,0),$$

$$k = \mu_+ - \frac{a \oplus b + 1}{2},$$

$$\gamma_{a,b} = -(\mathrm{i}^{a\oplus b}) n^t \left(\frac{m}{n}\right)^{2+a\oplus b} \frac{k!\left(\mu_{(-)^{ab}} + \frac{\overline{a\oplus b}}{2}\right)}{(2)_k}, \qquad (18)$$

where $\gamma_{00} = \gamma_{11} = 0$ ($\gamma_{10} = \gamma_{01} = 0$) for $t + x - y$ odd (even) and $(x)_k = x(x+1)\cdots(x+k-1)$.

D'Ariano, Perinotti, PRA **90** 062106 (2014)

Dirac QCA $\oplus$

Local coupling: $A_{\mathbf{k}}$ coupled with its inverse with off-diagonal identity block matrix

$$E^{\pm}_{\mathbf{k}} = \begin{pmatrix} nA^{\pm}_{\mathbf{k}} & imI \\ imI & nA^{\pm\dagger}_{\mathbf{k}} \end{pmatrix}$$

$$n^2 + m^2 = 1$$

$E^{\pm}_{\mathbf{k}}$ CPT-connected!

$$\omega^E_{\pm}(\mathbf{k}) = \cos^{-1}[n(c_x c_y c_z \mp s_x s_y s_z)]$$

Dirac in relativistic limit $k \ll 1$

m≤1: mass

n^{-1}: refraction index

Bisio, D'Ariano, Perinotti, arXiv:1407.6928

Maxwell QCA $\otimes$

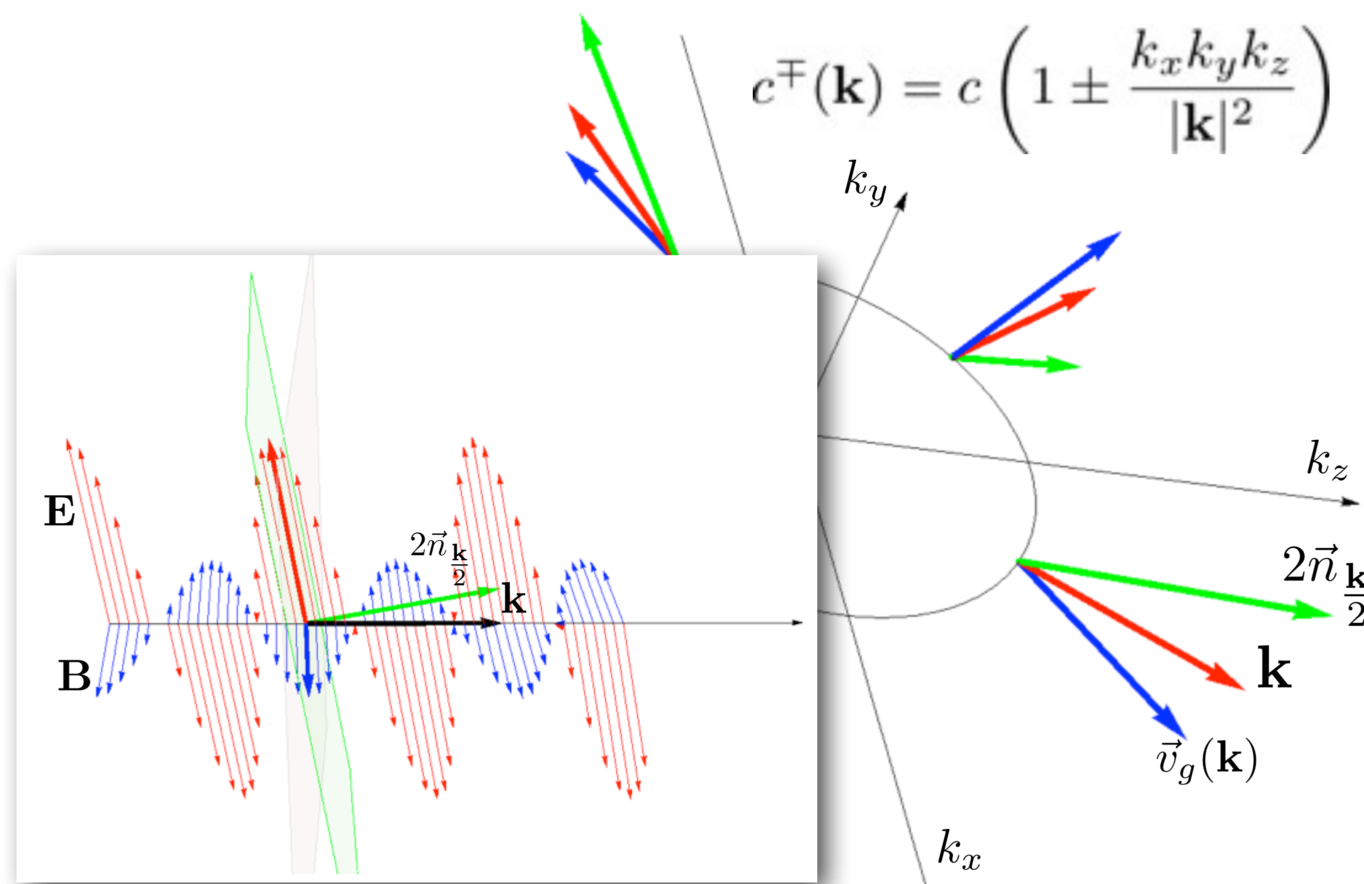
$$M_{\mathbf{k}} = A_{\mathbf{k}} \otimes A^*_{\mathbf{k}}$$

$$F^{\mu}(\mathbf{k}) = \int \frac{\mathrm{d}\,\mathbf{q}}{2\pi} f(\mathbf{q}) \tilde{\psi}(\tfrac{\mathbf{k}}{2} - \mathbf{q}) \sigma^{\mu} \varphi(\tfrac{\mathbf{k}}{2} + \mathbf{q})$$

Maxwell in relativistic limit $k \ll 1$

Boson: emergent from convolution of fermions

(De Broglie neutrino-theory of photon)



The LTM standards of the theory

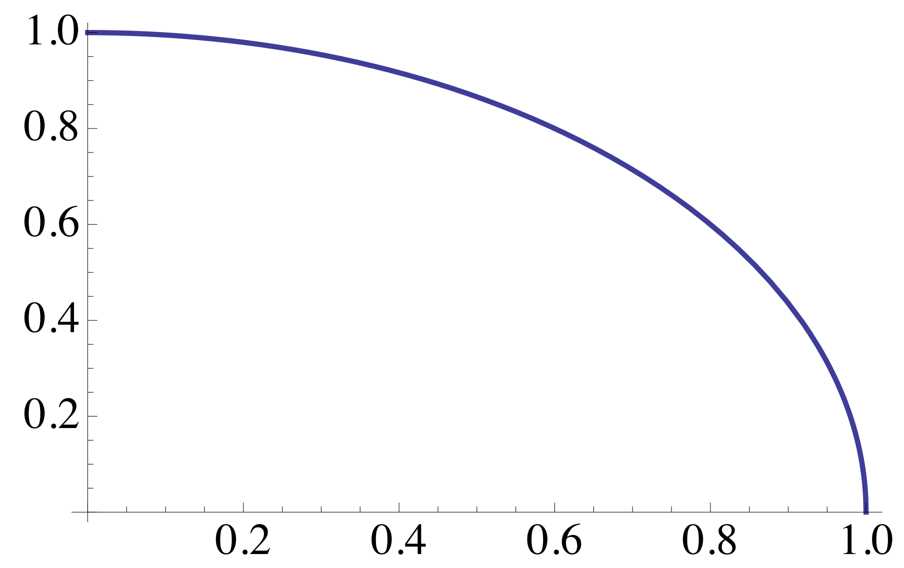
Dimensionless variables

$$x = \frac{x_m}{\mathfrak{a}} \in \mathbb{Z}, \quad t = \frac{t_s}{\mathfrak{t}} \in \mathbb{N}, \quad m = \frac{m_g}{\mathfrak{m}} \in [0, 1]$$

Relativistic limit: $\Rightarrow$ $c = \mathfrak{a}/\mathfrak{t} \qquad \hbar = \mathfrak{m}\mathfrak{a}c$

Measure $\mathfrak{m}$ from mass-refraction-index

$$\Rightarrow \quad n(m_g) = \sqrt{1 - \left(\frac{m_g}{\mathfrak{m}}\right)^2}$$



Measure $\mathfrak{a}$ from light-refraction-index

$$\Rightarrow \quad c^{\mp}(k) = c\left(1 \pm \frac{k}{\sqrt{3}k_{max}}\right)$$

Dirac emerging from the QCA

D'Ariano, Perinotti,
PRA **90** 062106 (2014)

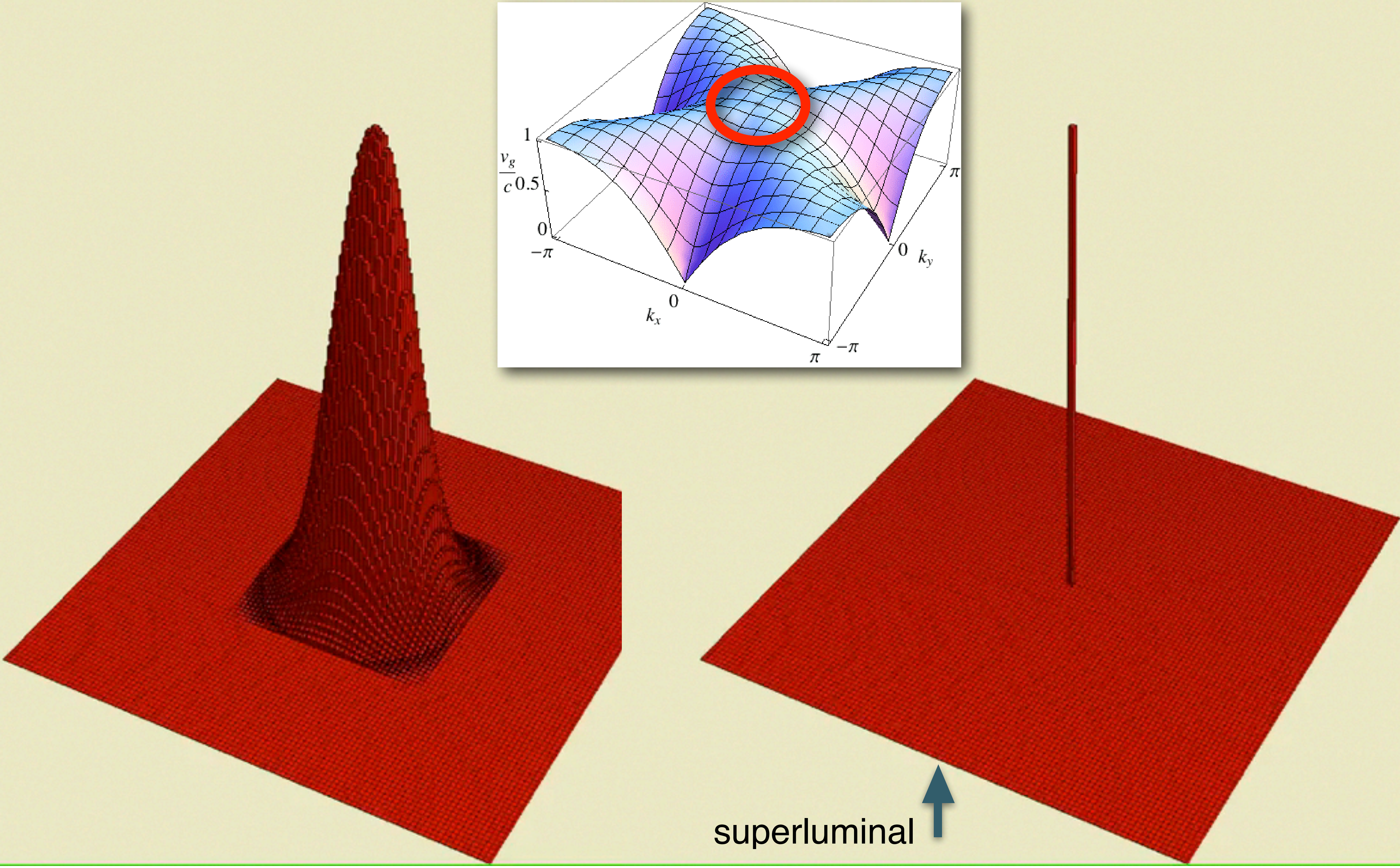
fidelity with Dirac for a narrowband packets in the relativistic limit $k \simeq m \ll 1$

$$F = |\langle \exp[-iN\boldsymbol{\Delta}(\mathbf{k})]\rangle|$$

$$\begin{aligned}\Delta(\mathbf{k}) &:= (m^2 + \tfrac{k^2}{3})^{\frac{1}{2}} - \omega^E(\mathbf{k}) \\ &= \frac{\sqrt{3}k_x k_y k_z}{(m^2 + \frac{k^2}{3})^{\frac{1}{2}}} - \frac{3(k_x k_y k_z)^2}{(m^2 + \frac{k^2}{3})^{\frac{3}{2}}} + \tfrac{1}{24}(m^2 + \tfrac{k^2}{3})^{\frac{3}{2}} + \mathcal{O}(k^4 + N^{-1}k^2)\end{aligned}$$

relativistic proton: $N \simeq m^{-3} = 2.2 * 10^{57} \Rightarrow t = 1.2 * 10^{14}\mathrm{s} = 3.7 * 10^6$ y

UHECRs: $k = 10^{-8} \gg m \Rightarrow N \simeq k^{-2} = 10^{16} \Rightarrow 5 * 10^{-28}$ s



2d automaton

- Evolution of a *narrow-band particle-state*
- Evolution of a *localized state*

Case of study: Relativity Principle without kinematics

$$A = \int_B^{\oplus} d\mathbf{k}\, A_{\mathbf{k}}$$

decomposition into irreps. of *G*

multiplicity (internal symmetries, matter-antimatter)

Dynamics (QCA eigenvalue equation)

$$\mathcal{H}_\omega = \{\psi \in \mathcal{F} : A\psi = e^{i\omega}\psi\} = \mathrm{Ker}[A - e^{i\omega} I] = \int_B^{\oplus} d\mathbf{k}\, \mathrm{Ker}[A_{\mathbf{k}} - e^{i\omega} I]$$

dispersion relations $\omega = \omega_l(\mathbf{k}),\ l = 1, \ldots, r$

Reference-frame: particular decomposition into irreps. preserving dispersion relation

Change of frame (boost, ...) $\mathbf{k}' = f(\mathbf{k})$ → $(\omega', \mathbf{k}') = (\omega(f(\mathbf{k})), f(\mathbf{k}))$

Relativity principle

$$A_{\mathbf{k}} - e^{i\omega} I = \tilde{\Lambda}_f^{-1}(A_{\mathbf{k}'} - e^{i\omega'} I)\Lambda_f, \quad \Lambda_f = \Lambda_f(\omega, \mathbf{k}) \in \mathrm{SL}_{sr}(\mathbb{C})$$

Bisio, D'Ariano, Perinotti, arXiv:1503.01017

Case of study: Relativity Principle without kinematics

Weyl QCA

$$A^{\pm}_{\mathbf{k}} := \lambda^{\pm}(\mathbf{k})I - i\mathbf{n}^{\pm}(\mathbf{k}) \cdot \boldsymbol{\sigma}^{\pm}$$

$$\boldsymbol{\sigma}^{+} = \boldsymbol{\sigma},\ \boldsymbol{\sigma}^{-} = \boldsymbol{\sigma}^{T}$$
$$c_\alpha := \cos(k_\alpha/\sqrt{3})$$
$$s_\alpha := \sin(k_\alpha/\sqrt{3})$$
$$\alpha = x, y, z$$

$$\mathbf{n}^{\pm}(\mathbf{k}) := \begin{pmatrix} s_x c_y c_z \pm c_x s_y s_z \\ c_x s_y c_z \mp s_x c_y s_z \\ c_x c_y s_z \pm s_x s_y c_z \end{pmatrix}$$

$$\lambda^{\pm}(\mathbf{k}) := (c_x c_y c_z \mp s_x s_y s_z)$$

eigenvalue equation $$(\sin\omega I - \mathbf{n}(\mathbf{k}) \cdot \boldsymbol{\sigma})\psi(\mathbf{k}, \omega) = 0$$

⇒ $\sin^2\omega - |\mathbf{n}(\mathbf{k})|^2 = 0$ dispersion relations $(\sin\omega, \mathbf{n}) \in \mathbb{M}^4$

Relativity principle

$$(\sin\omega I - \mathbf{n}(\mathbf{k}) \cdot \boldsymbol{\sigma}) = \Lambda^{\dagger}(\sin\omega' I - \mathbf{n}(\mathbf{k}') \cdot \boldsymbol{\sigma})\Lambda \qquad \Lambda = \Lambda(\mathbf{k}, \omega) \in \mathrm{SL}_2(\mathbb{C})$$

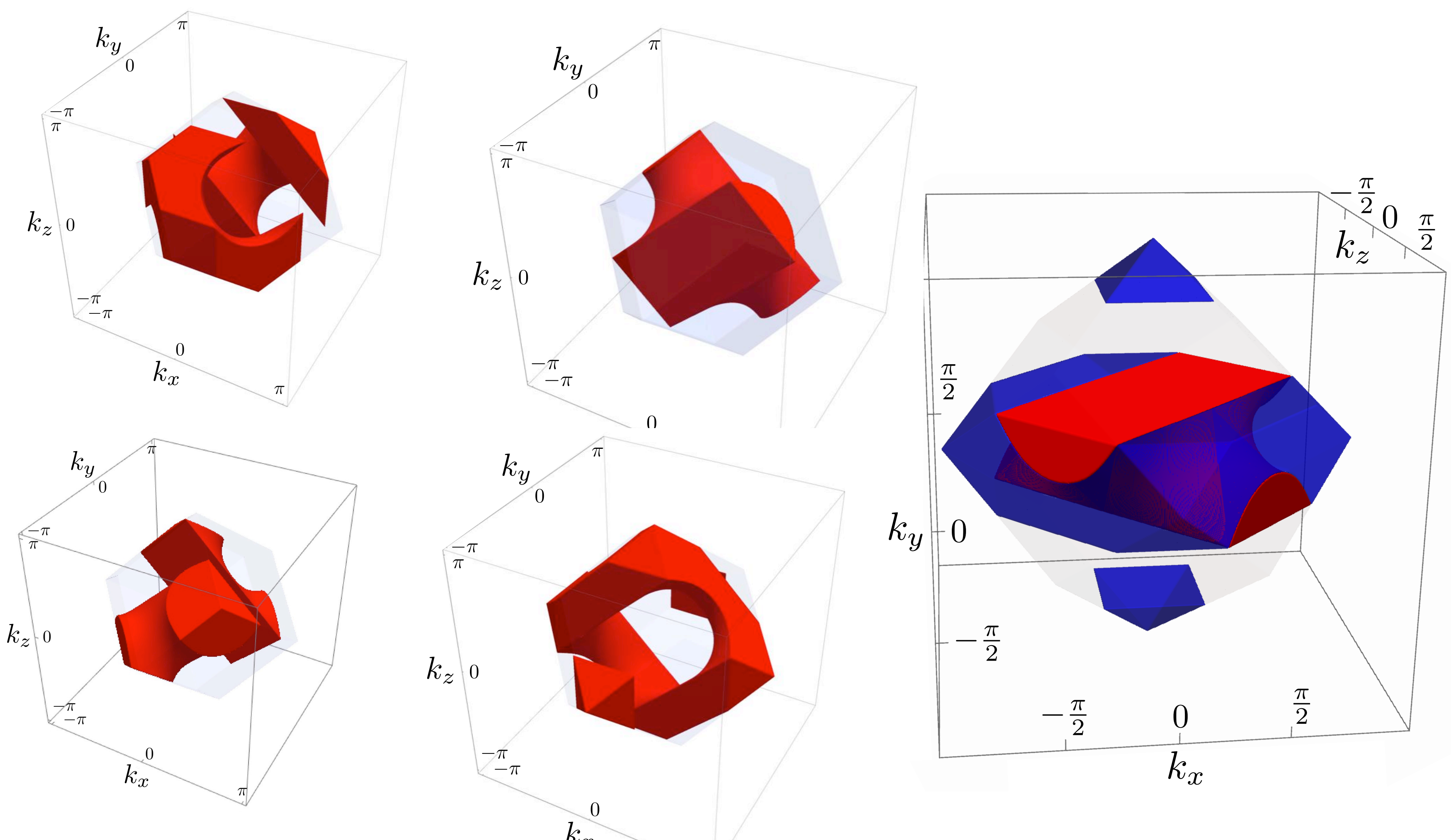
Case of study: Relativity Principle without kinematics

Action on $(\mathbf{k},\omega)$ given by the ***non-linear representation of the Lorentz group***

$$\mathcal{L}_\beta := \mathcal{D}^{-1} \circ L_\beta \circ \mathcal{D} \qquad \mathcal{D}(\omega,\mathbf{k}) := w(\omega,\mathbf{k})(\sin\omega, \mathbf{n}(\mathbf{k}))$$

$$\psi(\mathbf{k},\omega) \mapsto \Lambda\psi(\mathbf{k}',\omega')$$

Λ independent on $\mathbf{k}$ and ω



The Brillouin zone separates into ***four invariant regions*** diffeomorphic to balls, corresponding to four different ***particles***.

Case of study: Relativity Principle without kinematics

Dirac automaton: De Sitter covariance

Covariance for Dirac QCA cannot leave m invariant

invariance of de Sitter norm:

$$\sin^2\omega - (1-m^2)|\mathbf{n}(\mathbf{k})|^2 - m^2 = 0$$

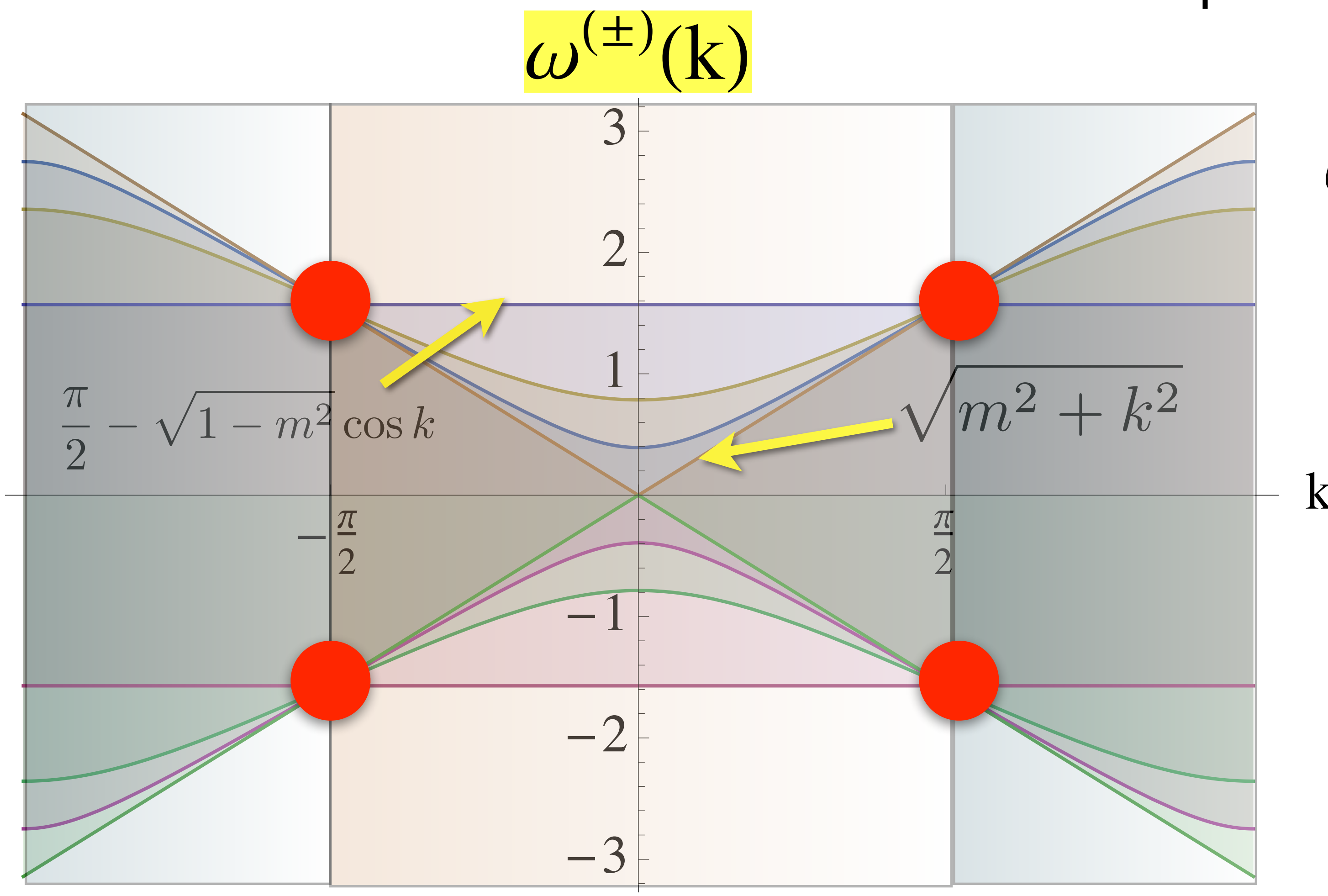
➡ $SO(1,4)$ invariance

$SO(1,4) \longrightarrow SO(1,3)$ for $m \to 0$ $\mathcal{O}(m^2)$

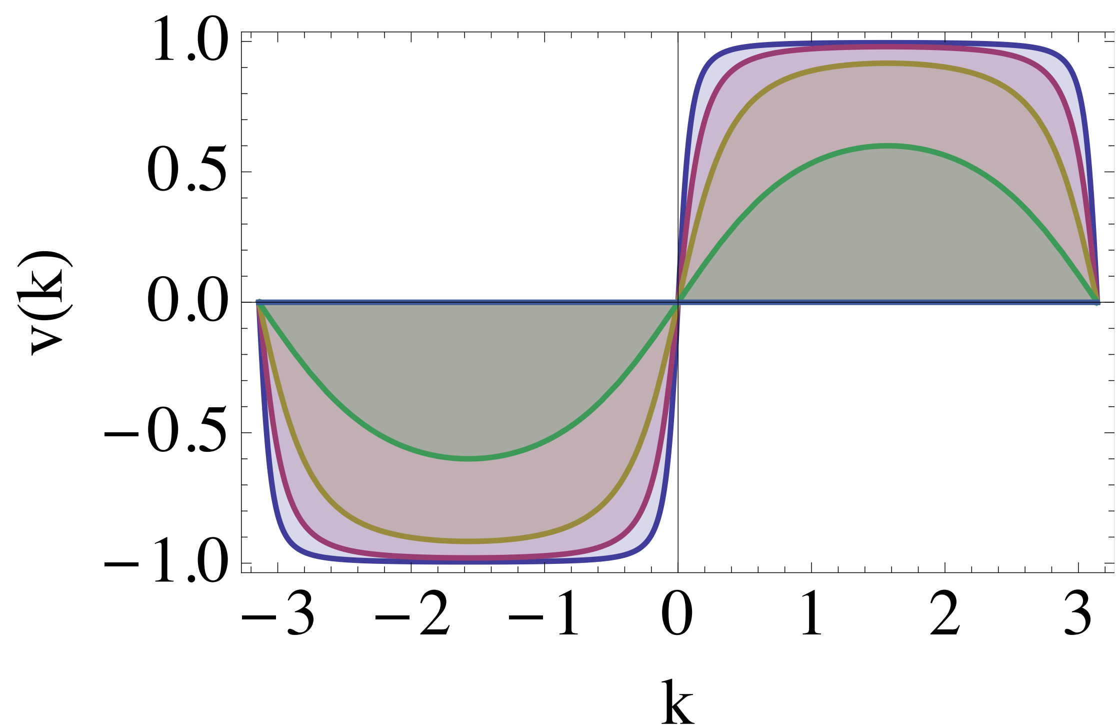
Bibeau-Delisle, Bisio, D'Ariano, Perinotti, Tosini, arXiv:1310.6760

Planck-scale effects: Lorentz covariance distortion

Transformations that leave the dispersion relation invariant



$$\omega_E(k) := \pm\cos^{-1}(\sqrt{1-m^2}\cos k)$$



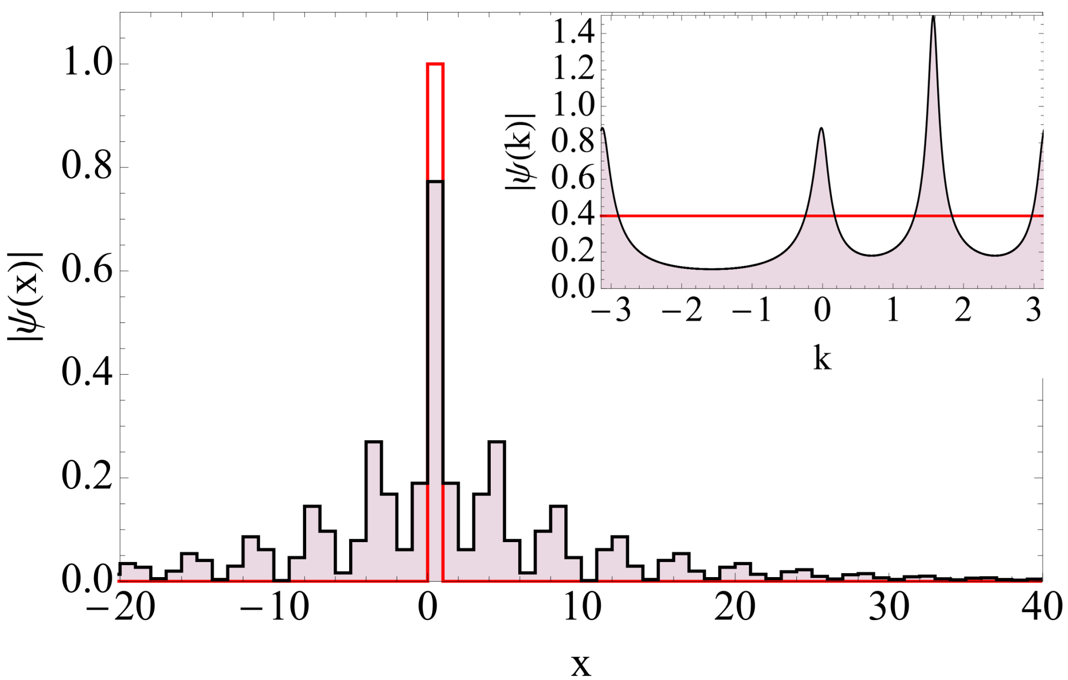
$$\omega' = \arcsin\left[\gamma\left(\sin\omega/\cos k - \beta\tan k\right)\cos k'\right]$$

$$k' = \arctan\left[\gamma\left(\tan k - \beta\sin\omega/\cos k\right)\right]$$

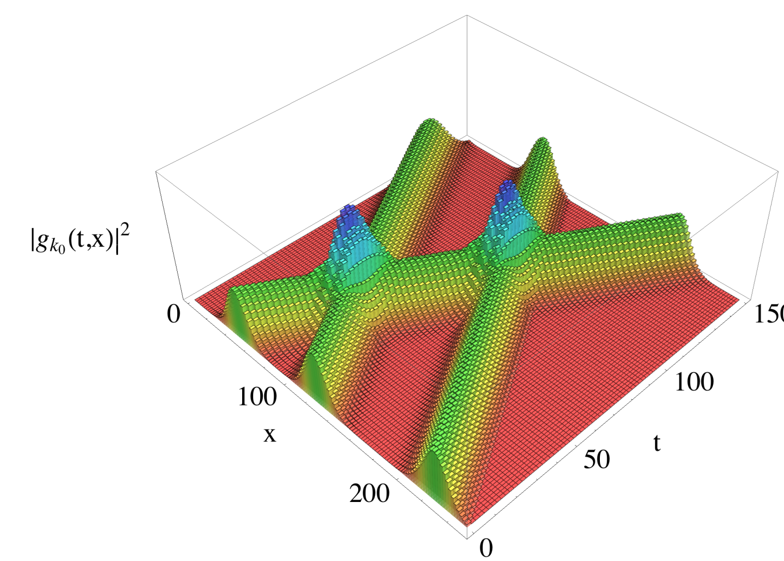
$$\gamma := (1-\beta^2)^{-1/2}$$

Bibeau-Delisle, Bisio, D'Ariano, Perinotti, Tosini, arXiv:1310.6760

Planck-scale effects: Lorentz covariance distortion



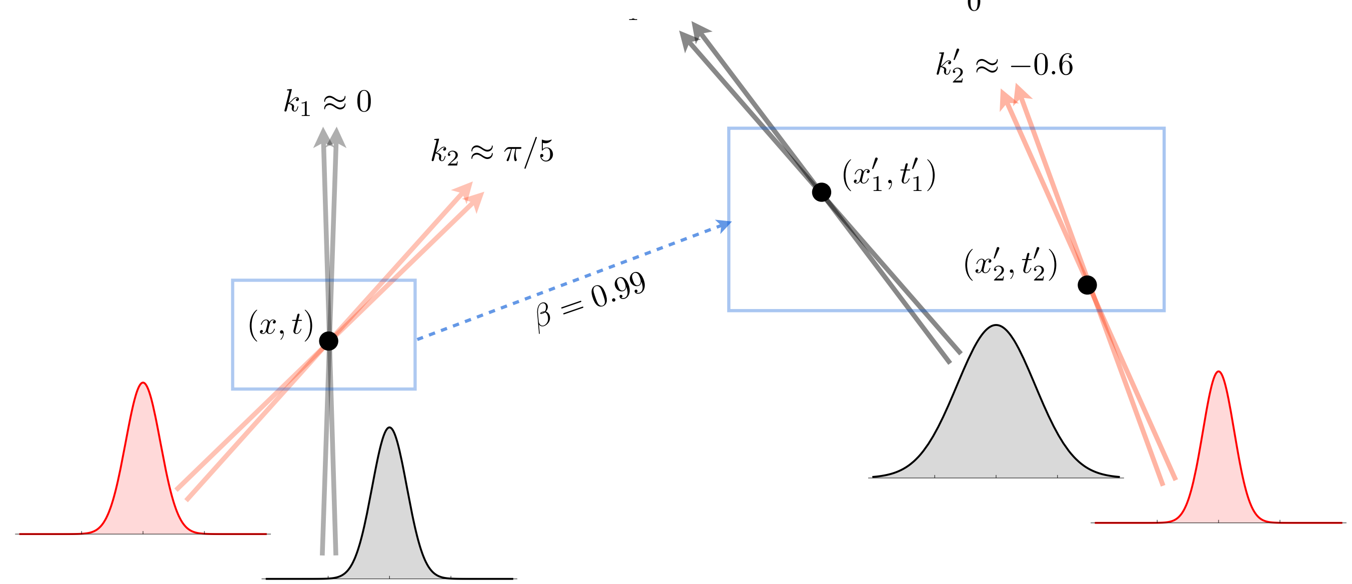
For narrow-band states we can linearize Lorentz transformations around *k=k₀* and we get *k*-dependent Lorentz transformations



Delocalization under boost

$$|\psi\rangle = \int \mathrm{d}k\mu(k)\hat{g}(k)|k\rangle \xrightarrow{L_\beta^D} \int \mathrm{d}k\mu(k)\,\hat{g}(k)|k'\rangle =$$

$$= \int \mathrm{d}k\mu(k')\,\hat{g}(k(k'))|k'\rangle$$



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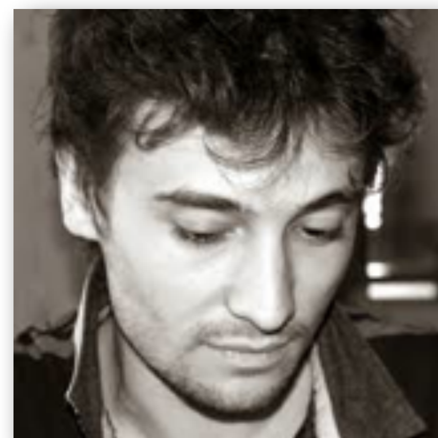
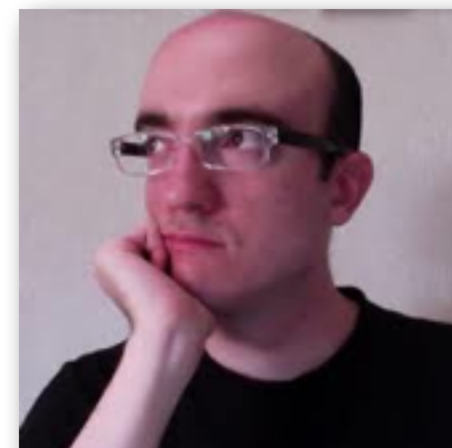

Franco Manessi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