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# SEEKING A PRINCIPLE OF QUANTUMNESS

Giacomo Mauro D'Ariano

Pavia University

*Quantum Theory: Reconsideration of Foundations, 5  
June 17th, Växjö University*

**arXiv:0807.4383**: in *Philosophy of Quantum Information and Entanglement*, Eds A. Bokulich and G. Jaeger (Cambridge University Press, Cambridge UK, in press)

Problem: to derive QM as  
a probabilistic theory from  
some operational principle:  
the principle of *Quantumness*

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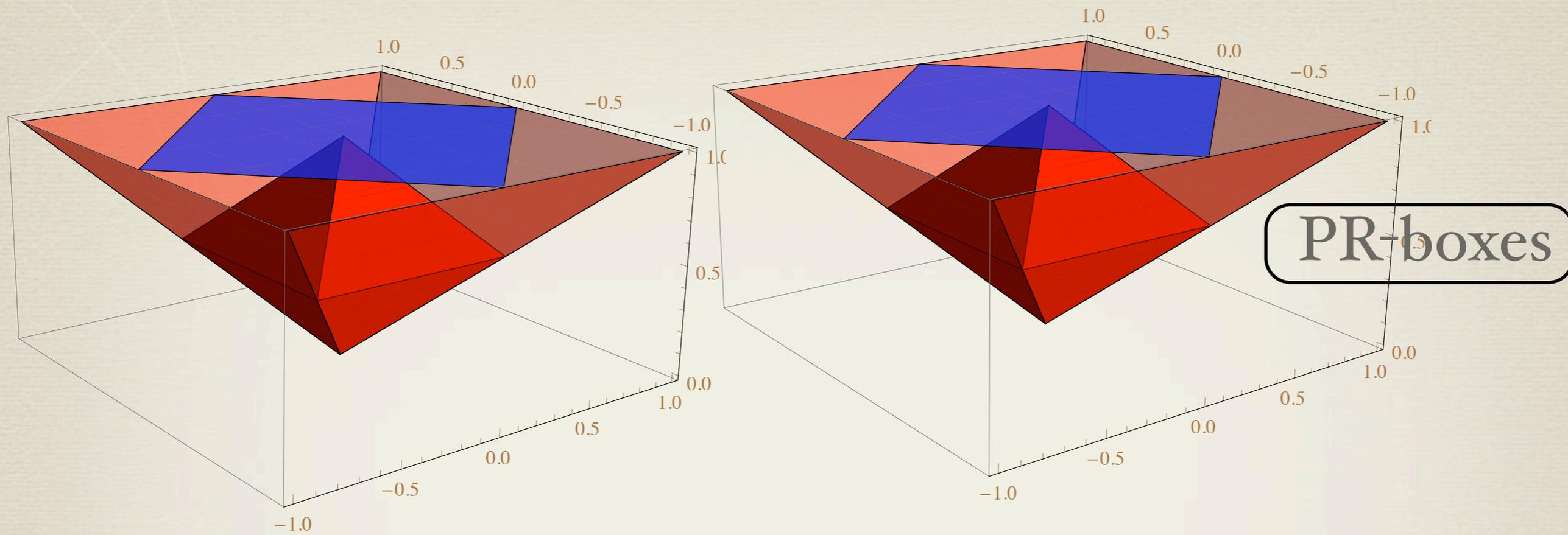


G.M. D'Ariano

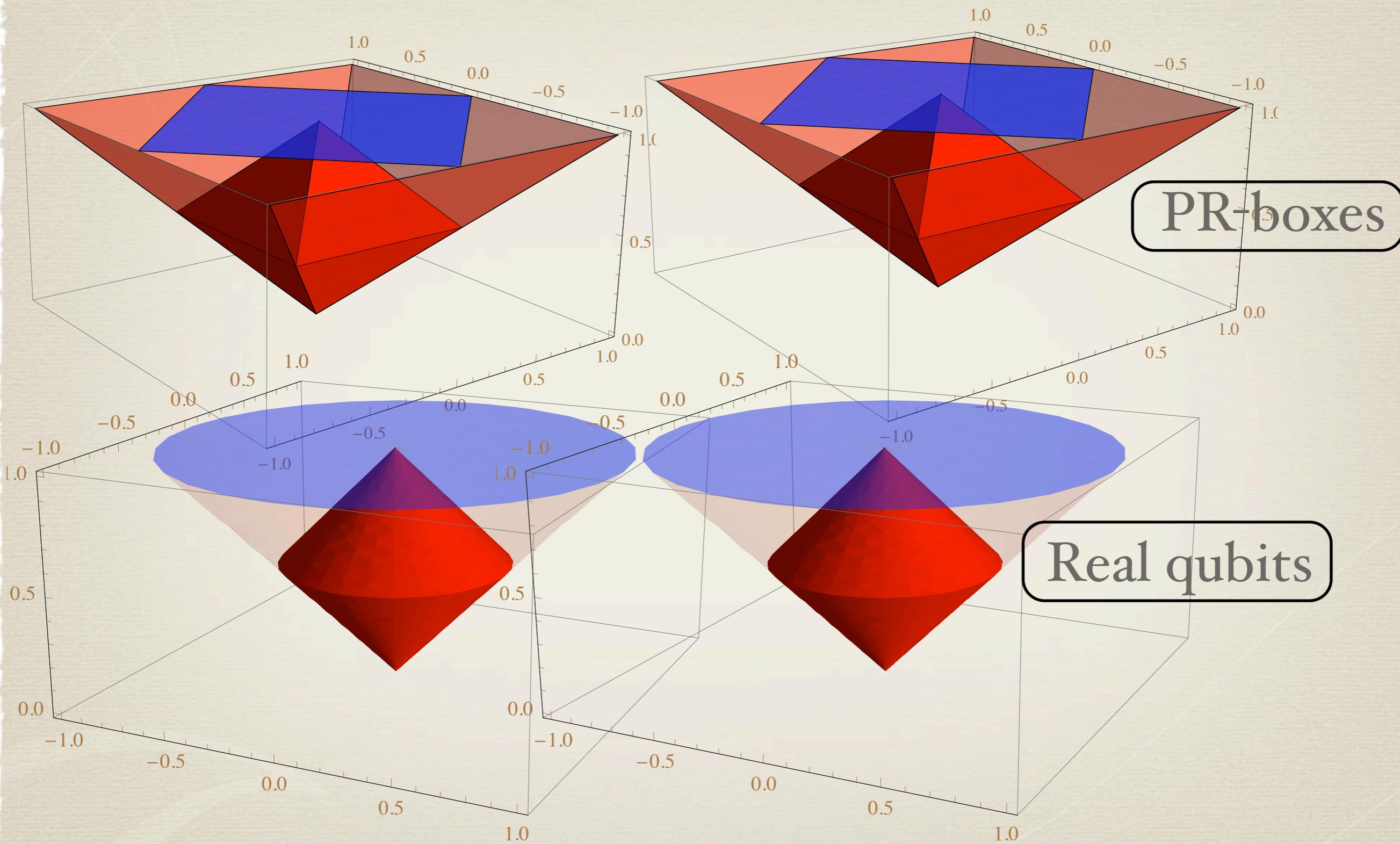
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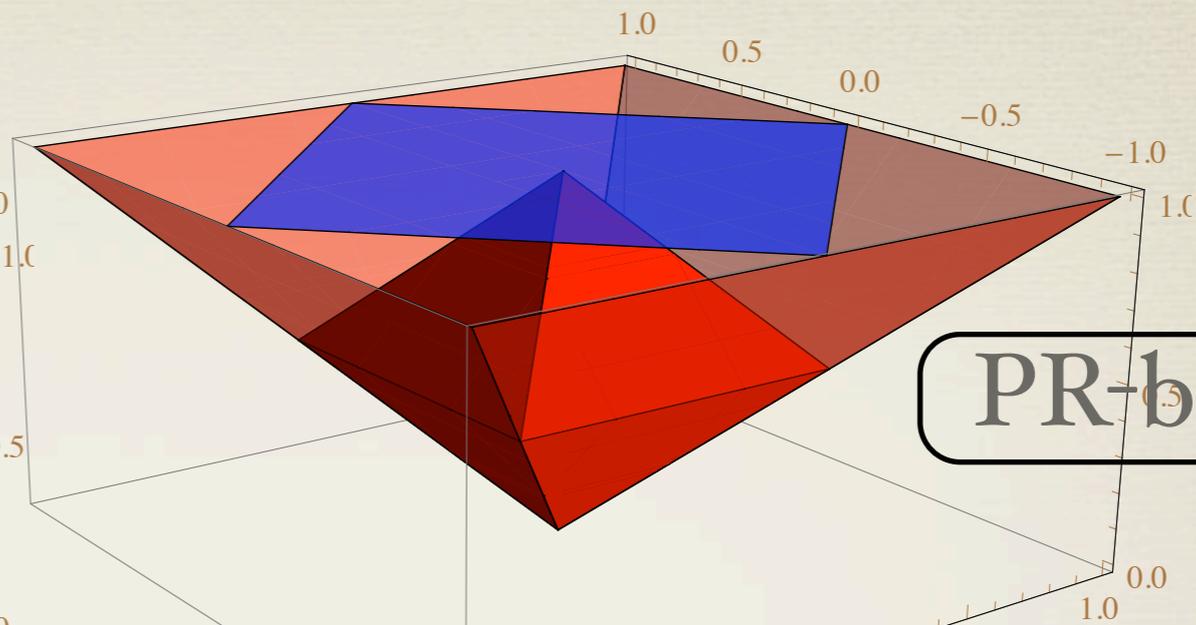
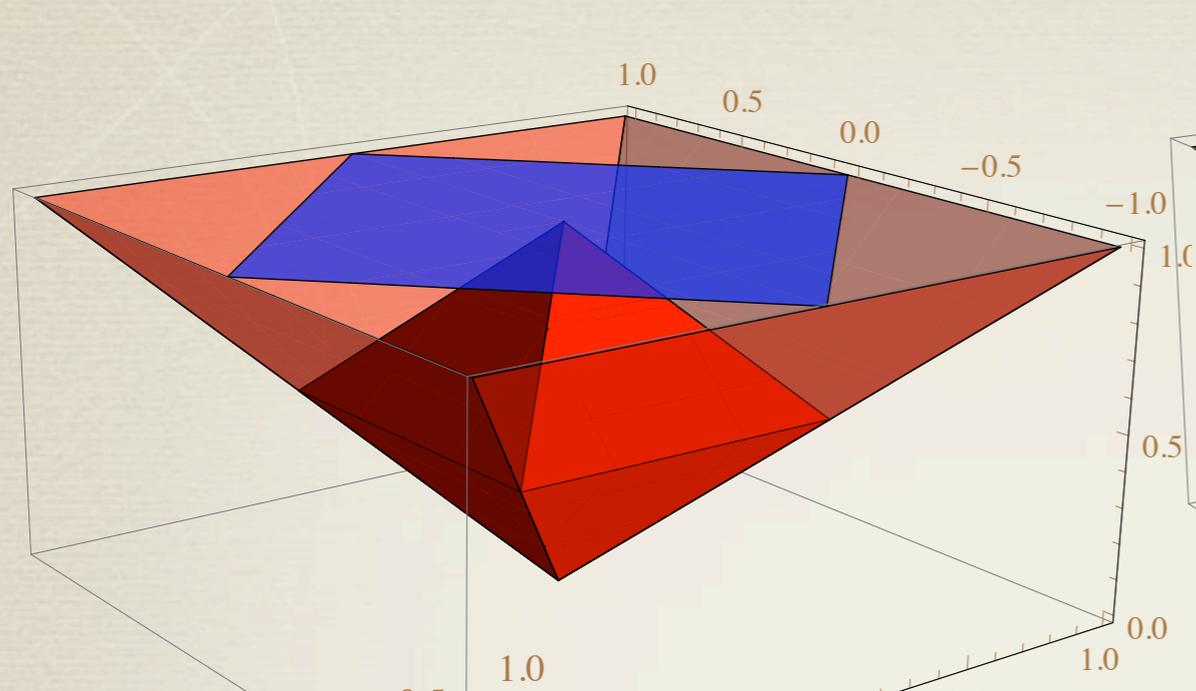
# Toy theories



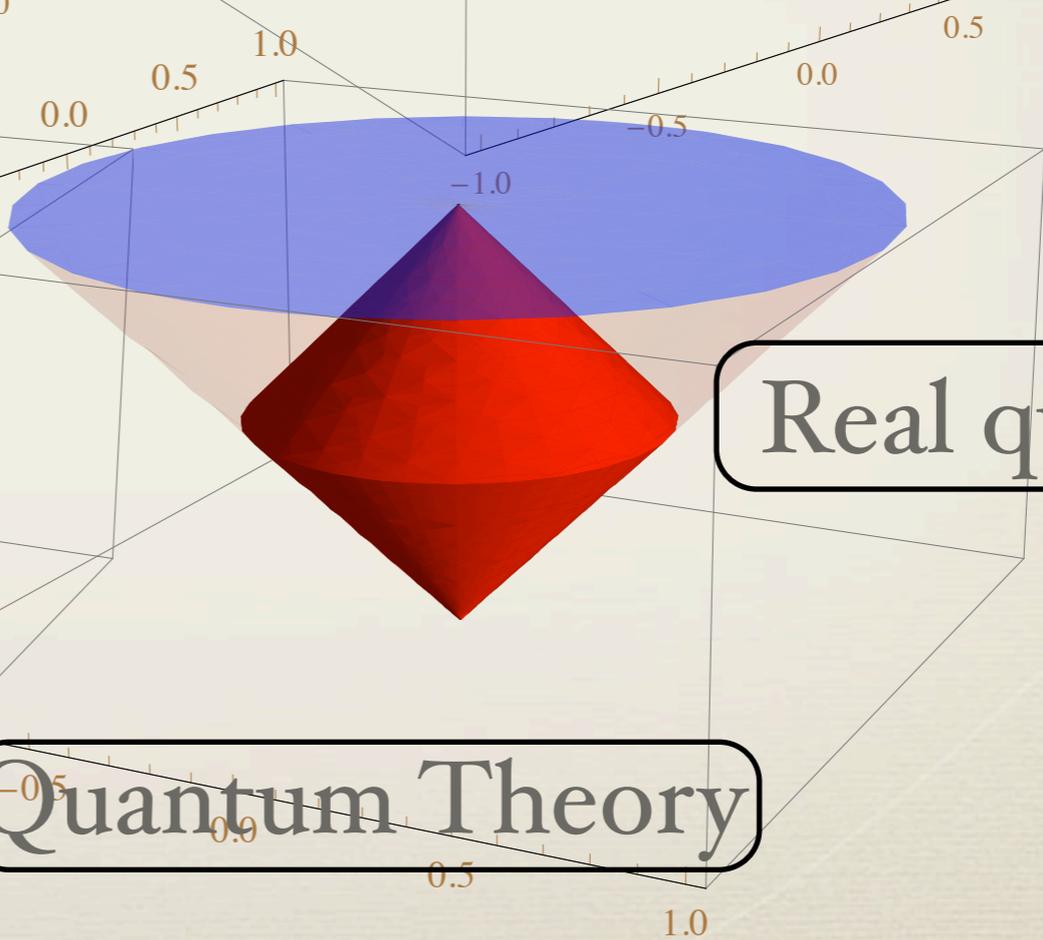
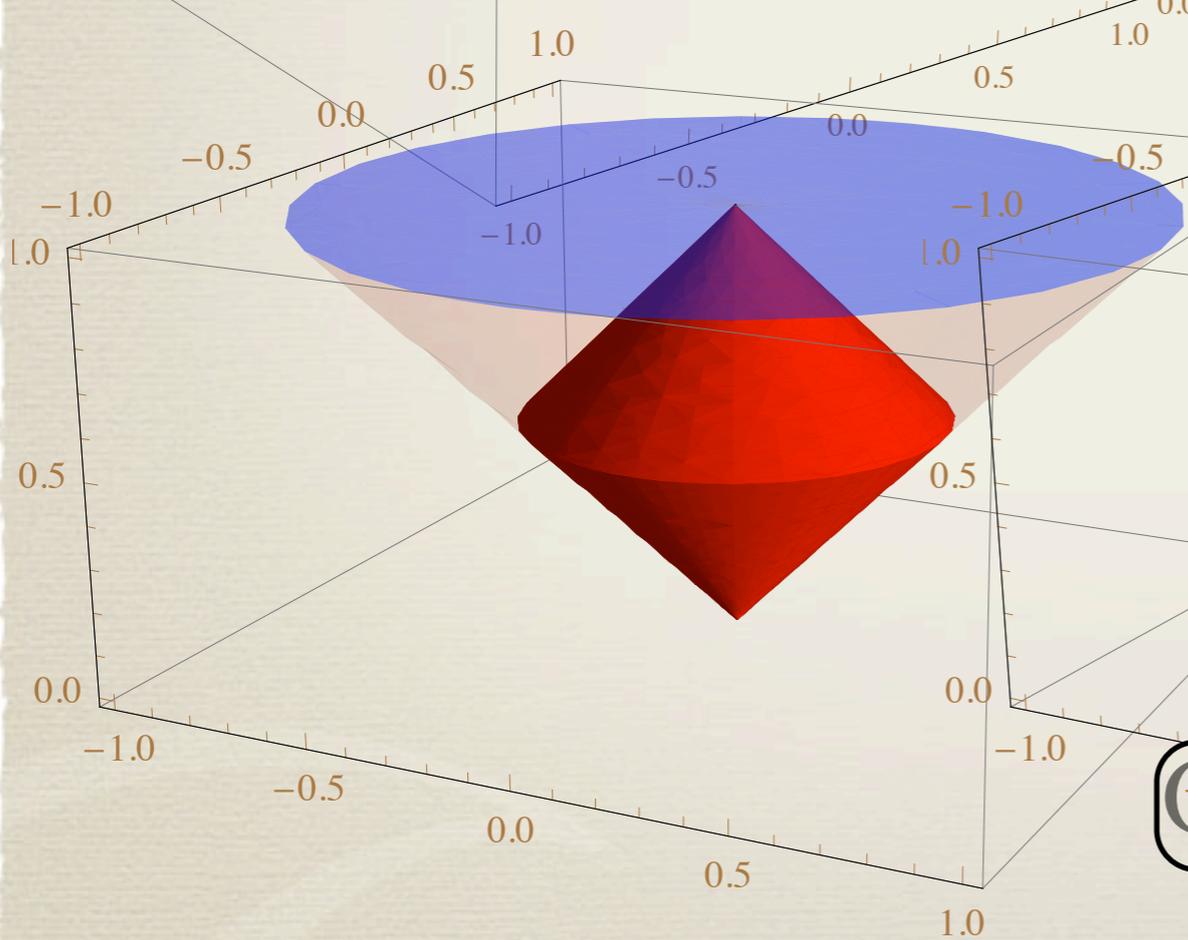
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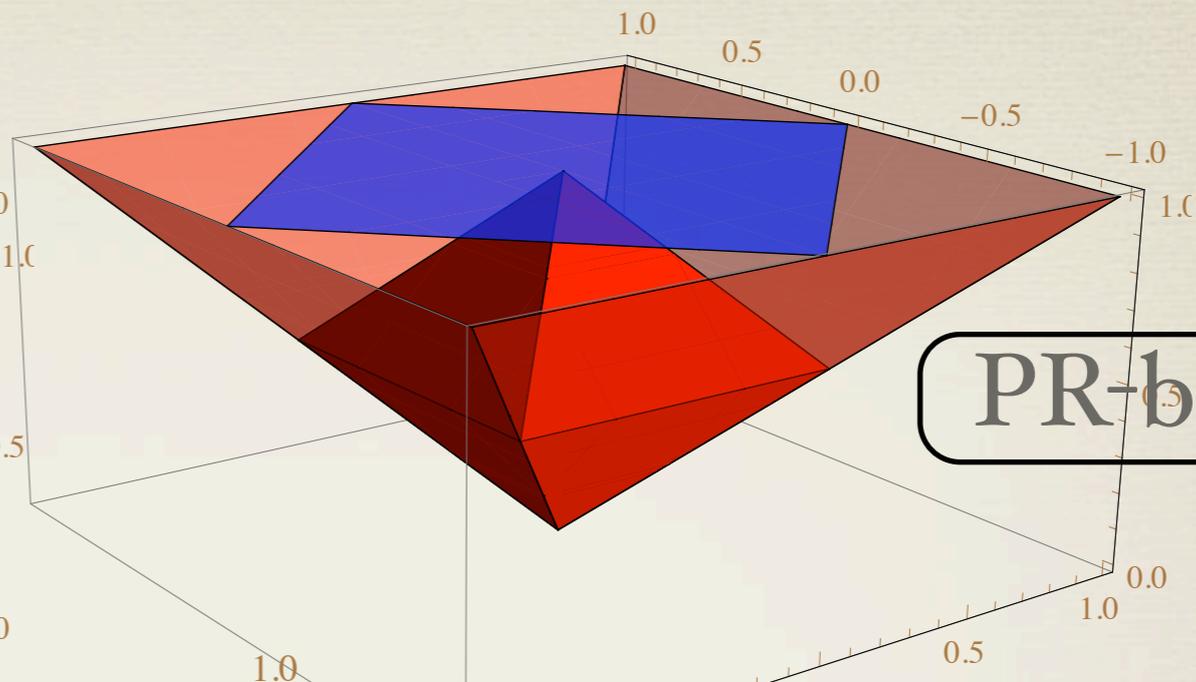
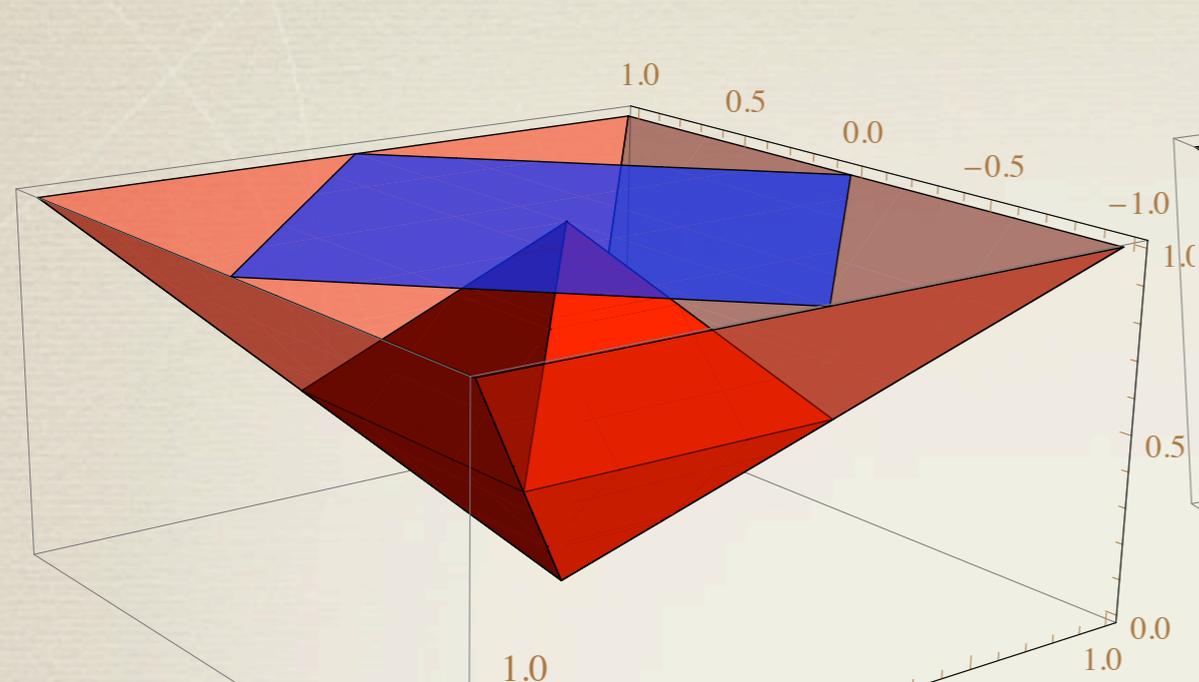
PR-boxes



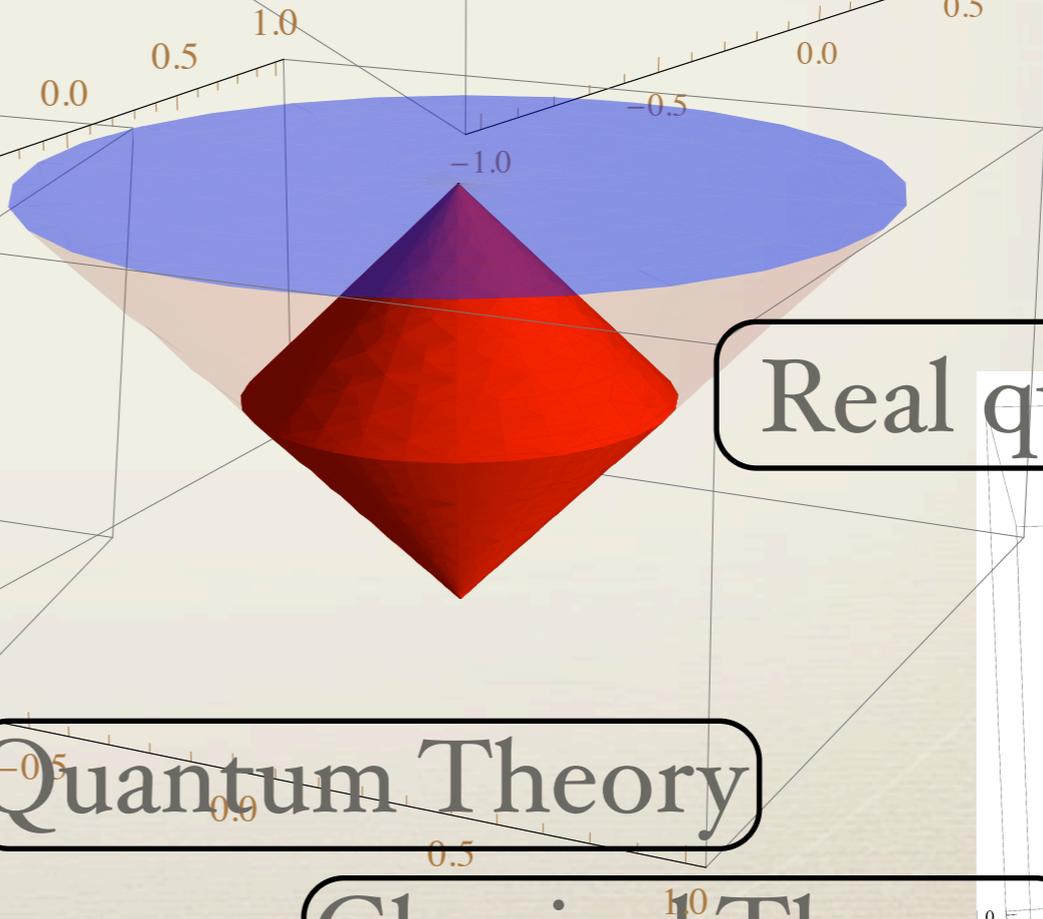
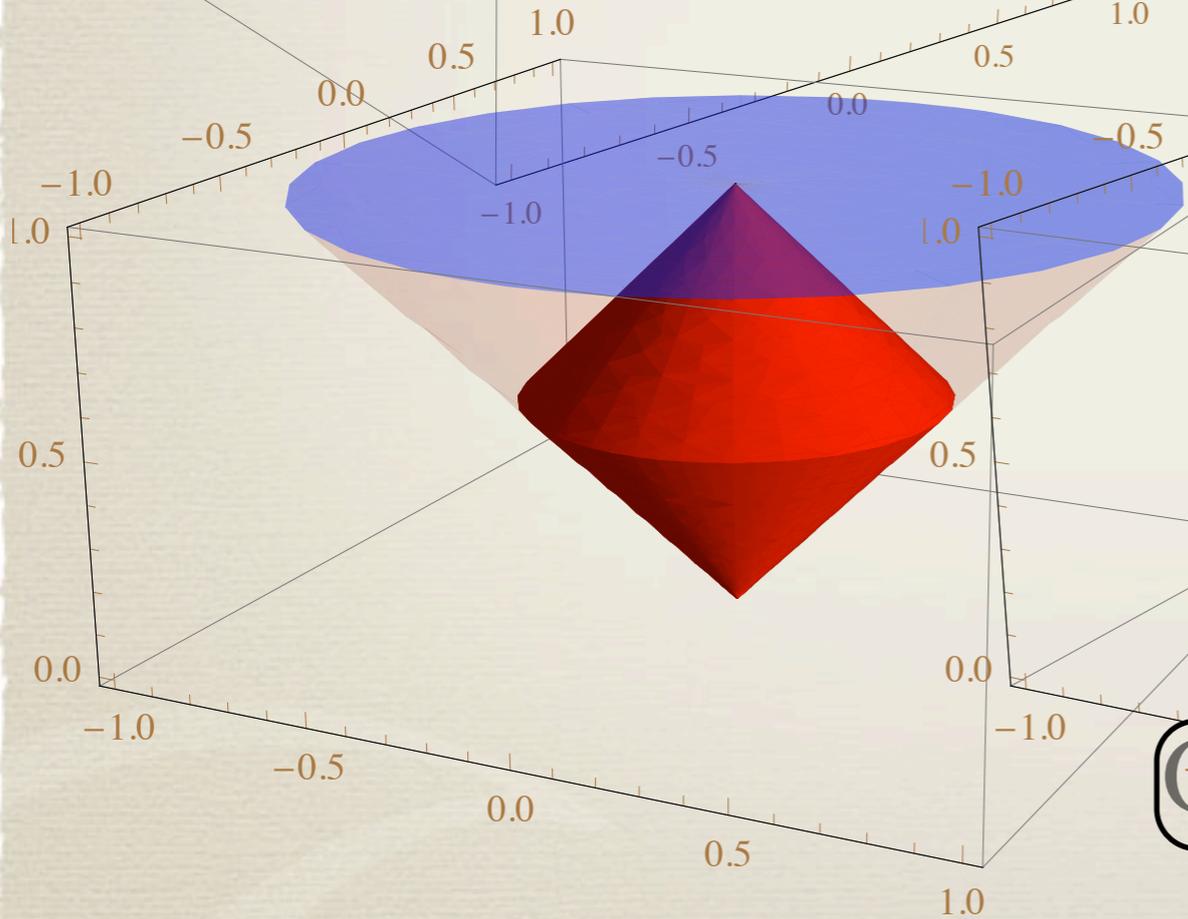
Real qubits

Quantum Theory

# Toy theories



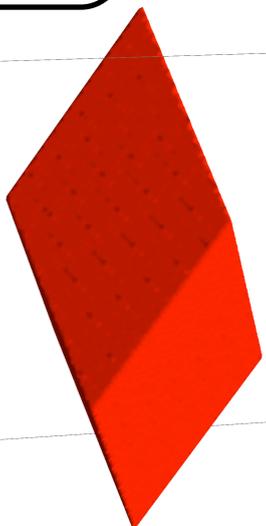
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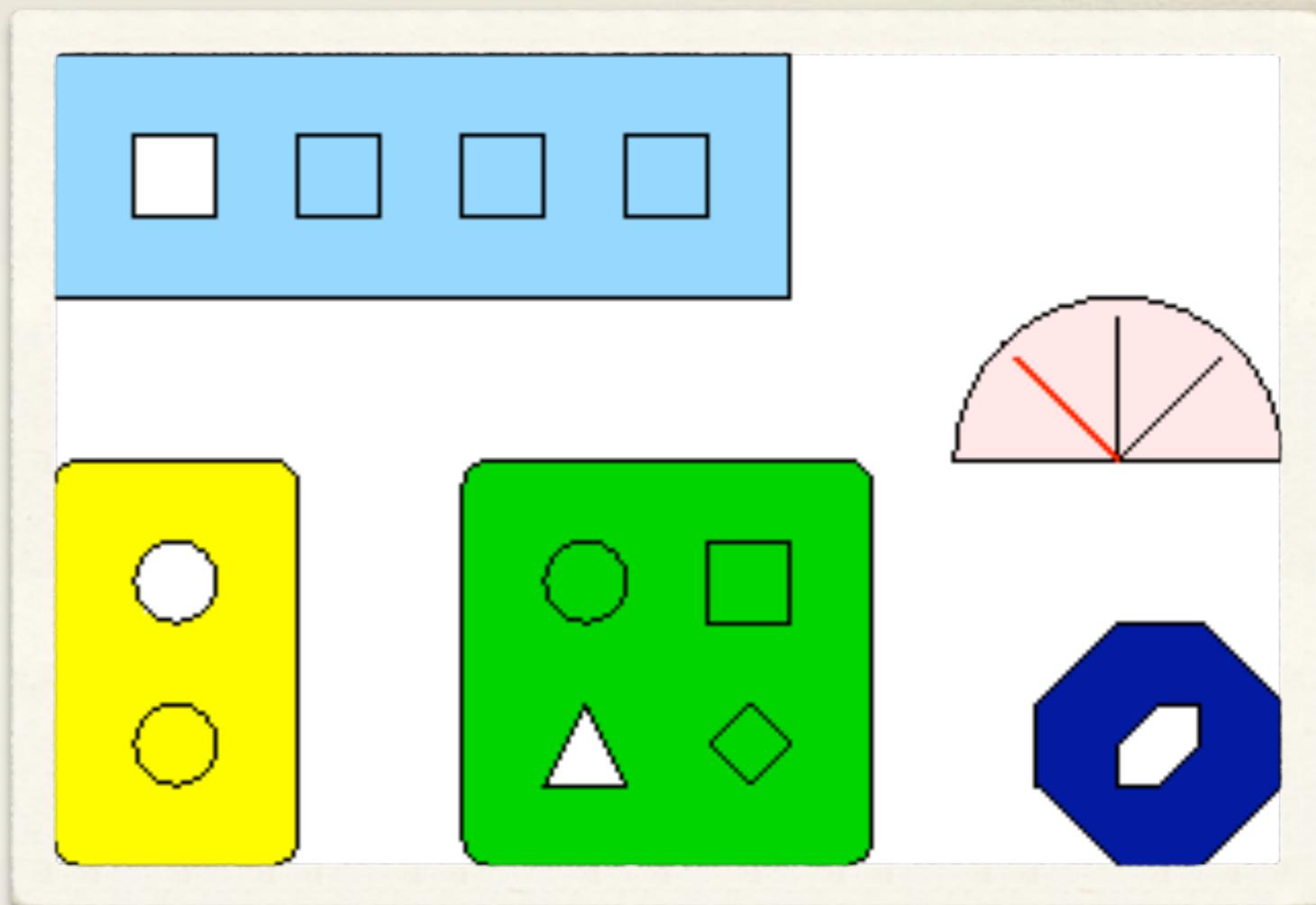
Quantum Theory

Classical Theory



# TESTS

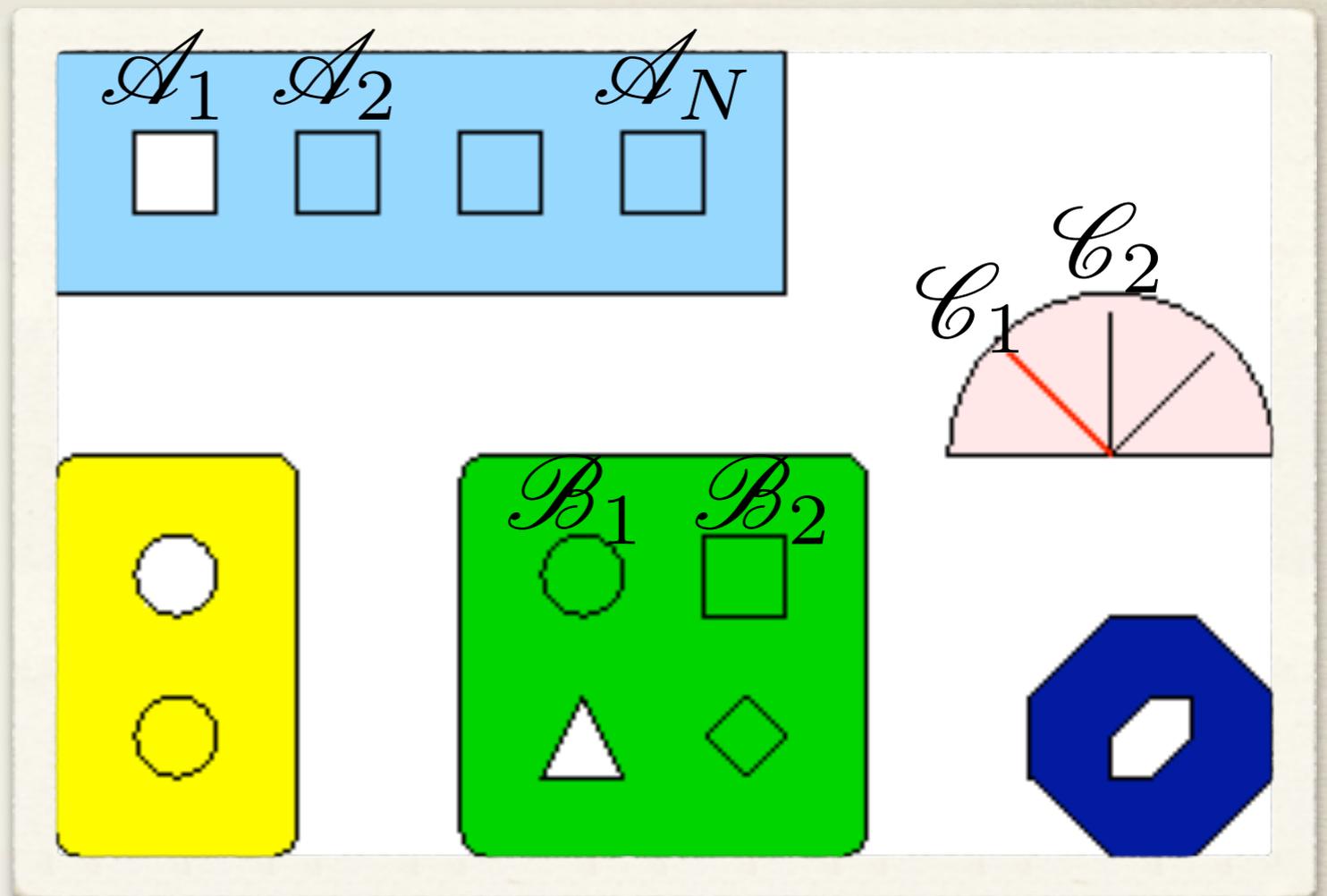
Test:  $\mathbb{A} \equiv \{\mathcal{A}_j\}$  set of possible events  $\mathcal{A}_j$



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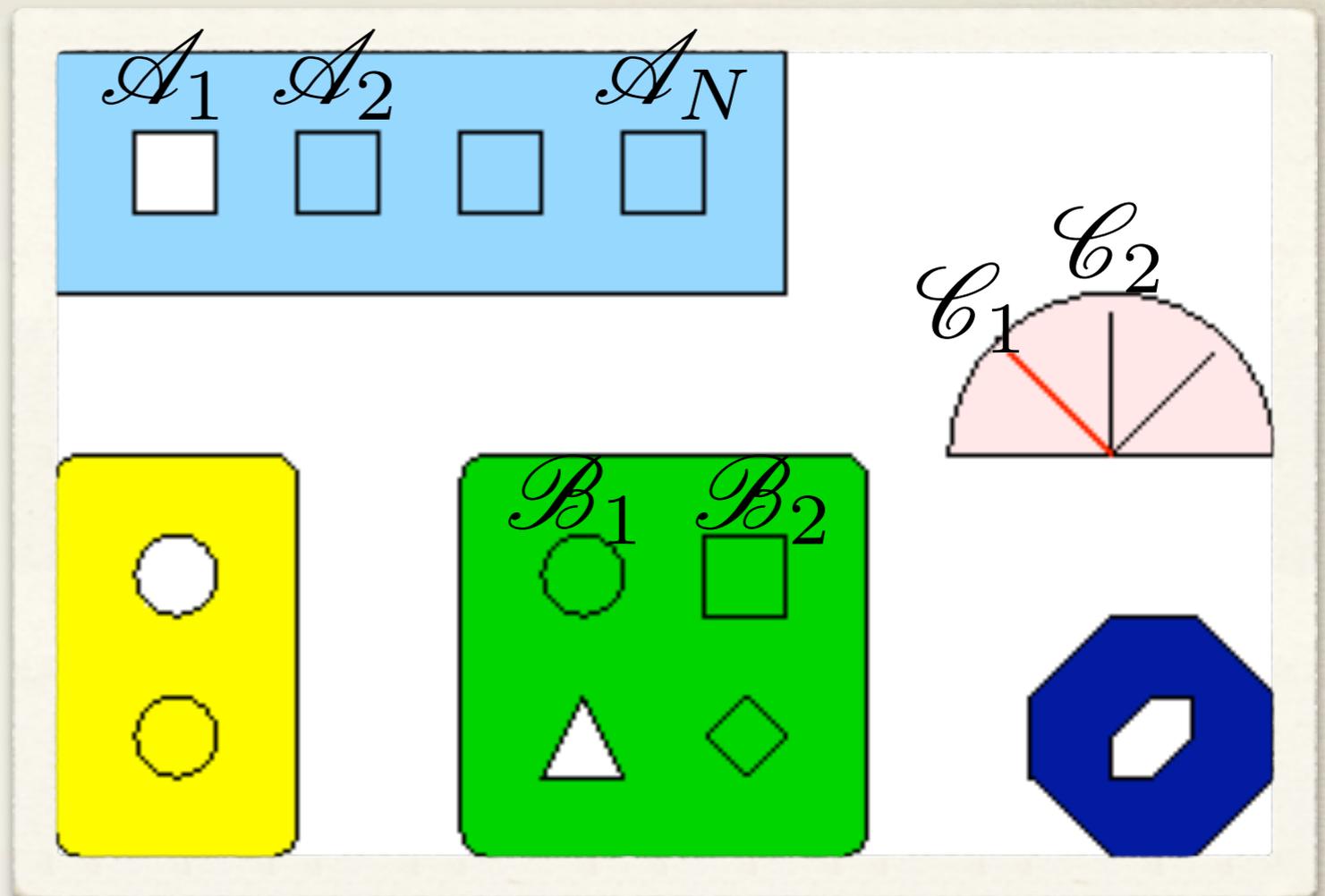
- \* The same event can occur in different tests
- \* Deterministic test = singleton



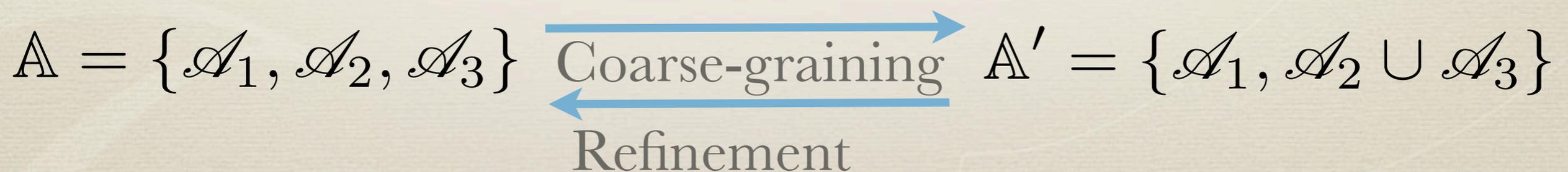
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Coarse-graining of events:  $\mathcal{A} \cup \mathcal{B}$



# STATES

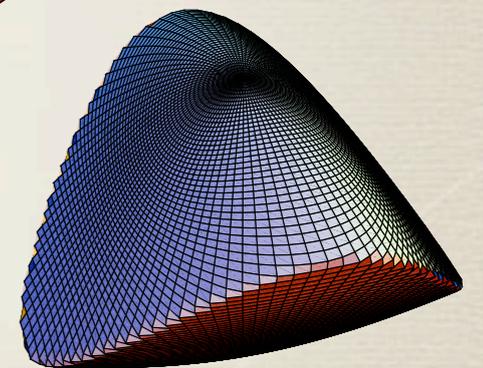
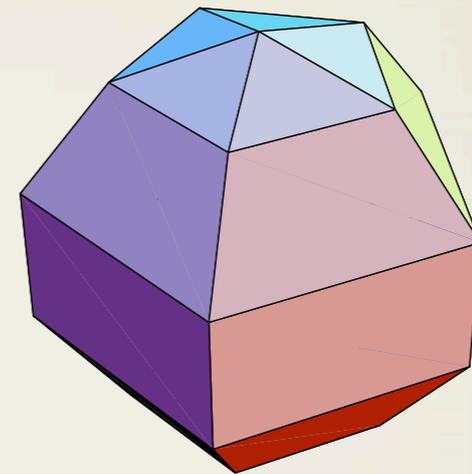
**State**  $\omega$ : probability rule  $\omega(\mathcal{A})$  for any possible event  $\mathcal{A}$  in any test

Normalization: 
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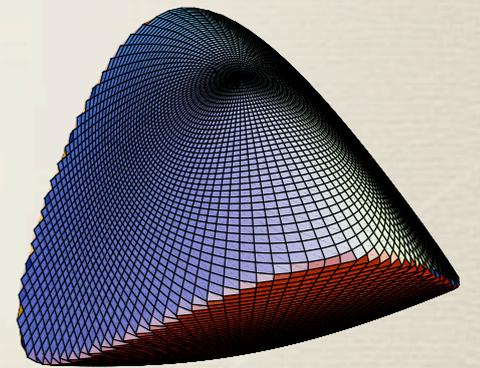
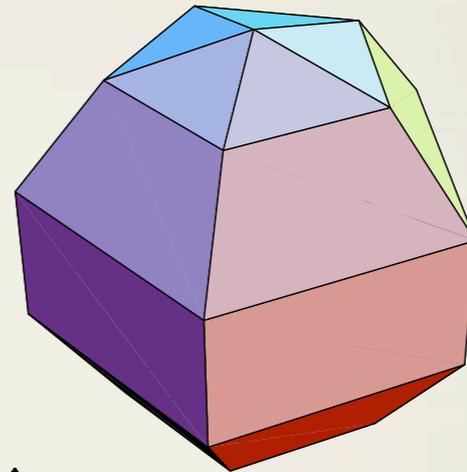


Convex set of states:  $\mathcal{S}$

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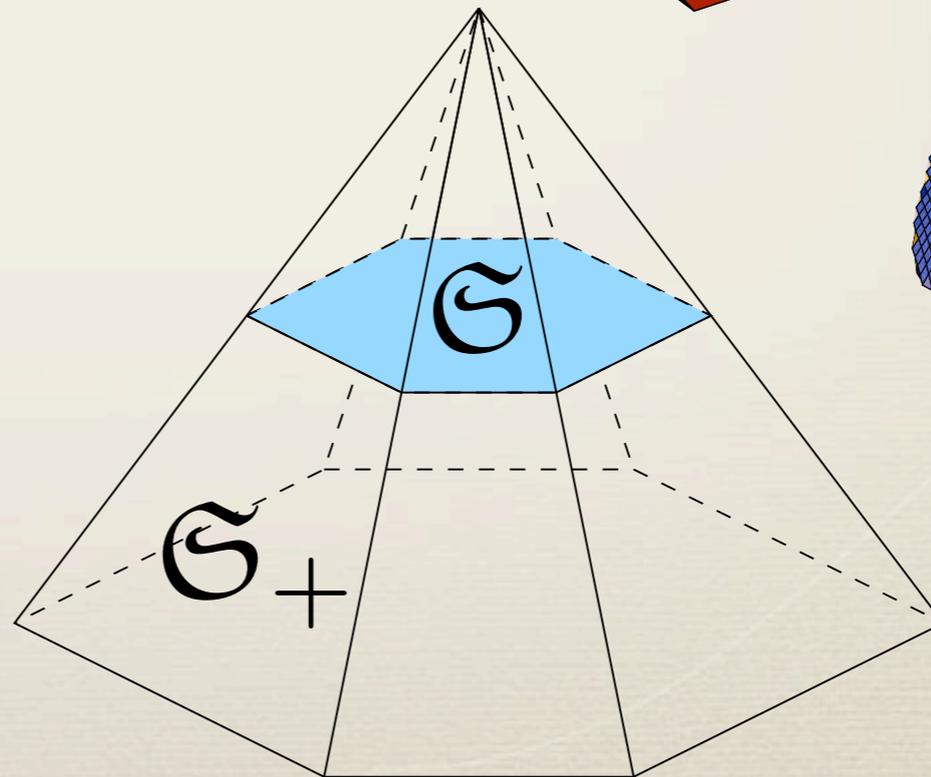
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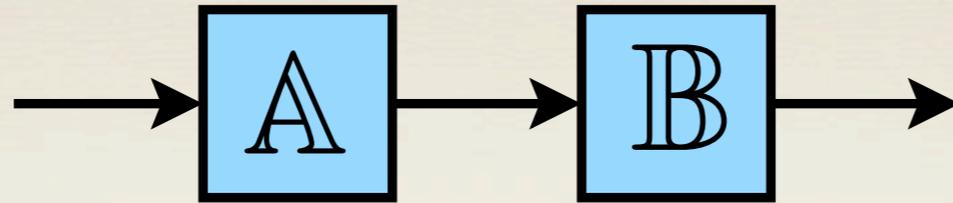
Convex set of states:  $\mathcal{S}$

Convex cone of unnormalized states:  $\mathcal{S}_+$



# CASCADES OF TESTS

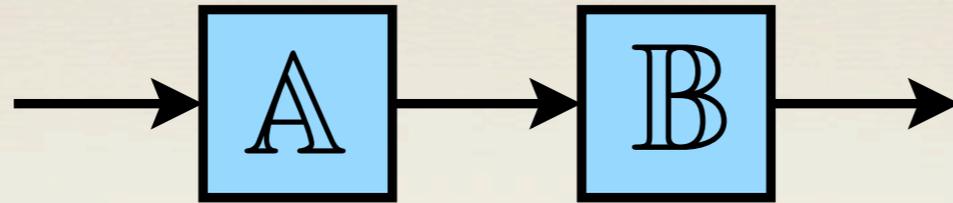
Time-cascade:



$\mathbb{B} \circ \mathbb{A} = \{\mathcal{B}_j \circ \mathcal{A}_i\}$  cascade of tests  $\mathbb{A} = \{\mathcal{A}_i\}$ ,  $\mathbb{B} = \{\mathcal{B}_j\}$ .

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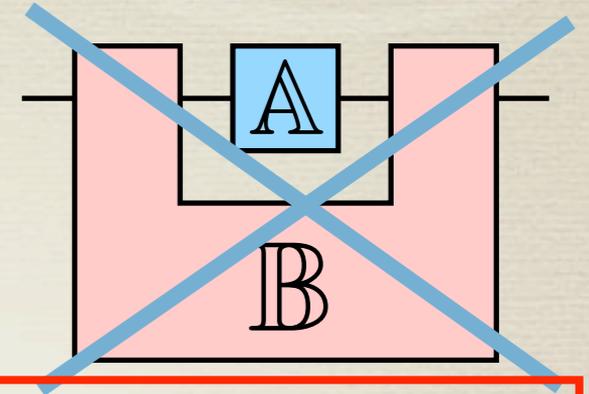
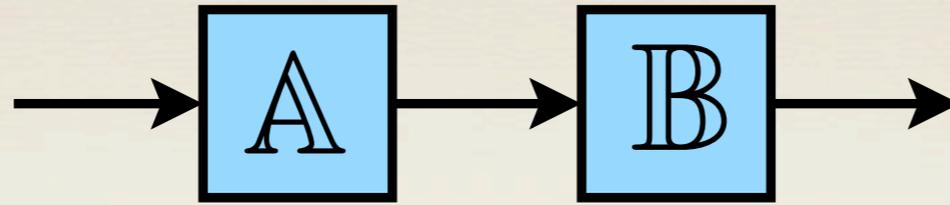
collection of joined events with the following rule for marginals:

$$\sum_{\mathcal{B}_j \in \mathbb{B}} \omega(\mathcal{B}_j \circ \mathcal{A}) =: f(\mathbb{B}, \mathcal{A}) \equiv \omega(\mathcal{A}), \quad \forall \mathbb{B}, \mathcal{A}, \omega$$

NSF (No signaling from the future)

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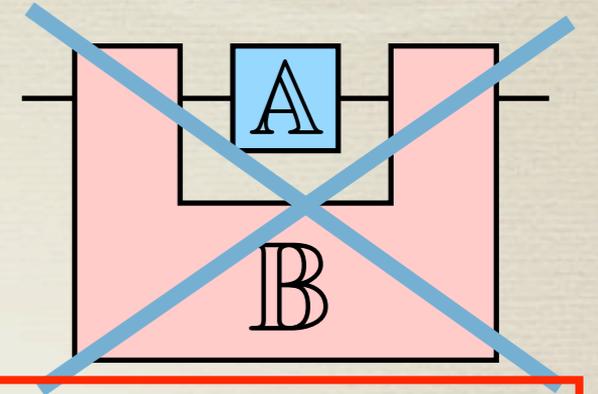
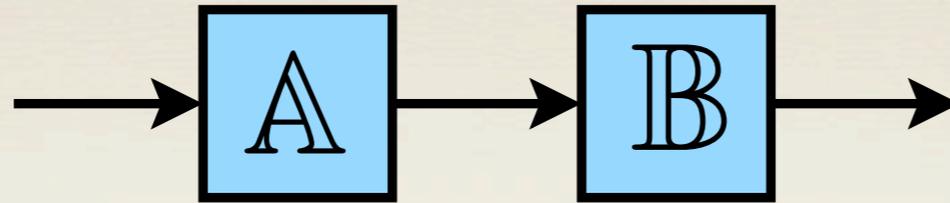
composition of events

$$\mathcal{B} \circ \mathcal{A} +$$

notion of conditional state

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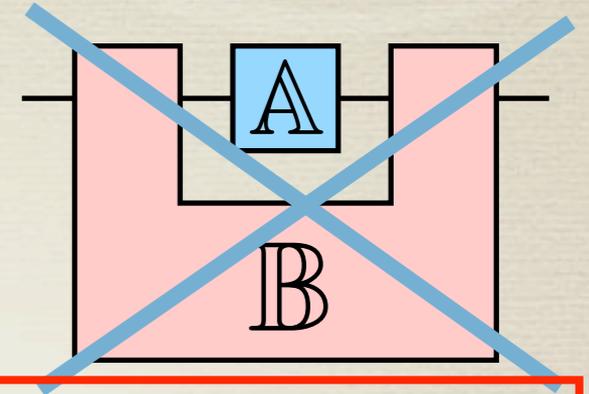
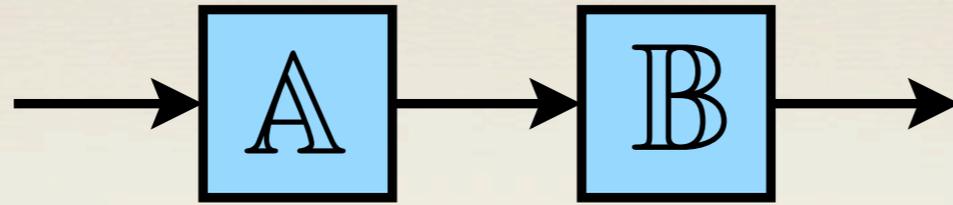
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→ composition of events  $\mathcal{B} \circ \mathcal{A}$  + notion of conditional state

$$\omega_{\mathcal{A}}(\mathcal{B}) = \frac{\omega(\mathcal{B} \circ \mathcal{A})}{\omega(\mathcal{A})}$$

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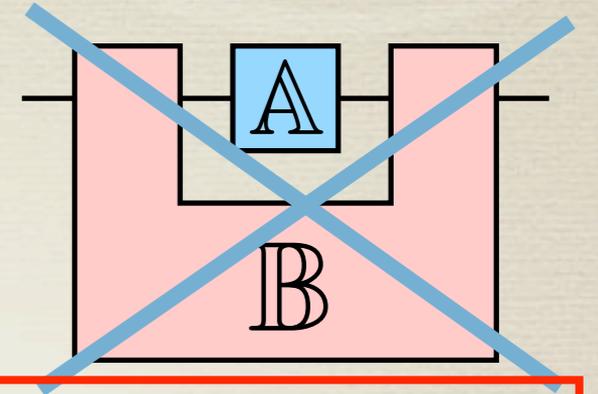
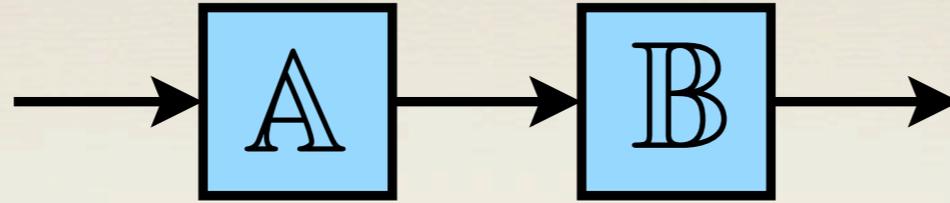
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→ composition of events  $\mathcal{B} \circ \mathcal{A}$  + notion of conditional state

→ events  $\equiv$  transformations + linearity of evolution

$$\mathcal{A}\omega := \omega(\cdot \circ \mathcal{A})$$

variable

# Equivalence classes for transformations

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Probabilistic-equivalence class

A transformation is completely specified by the two classes:

$$\mathcal{A}\omega = \omega(\mathcal{A})\omega_{\mathcal{A}}$$

# EFFECTS

Effect  $a$ : equivalence class of transformations occurring with the same probability as  $\mathcal{A}$  for all states.

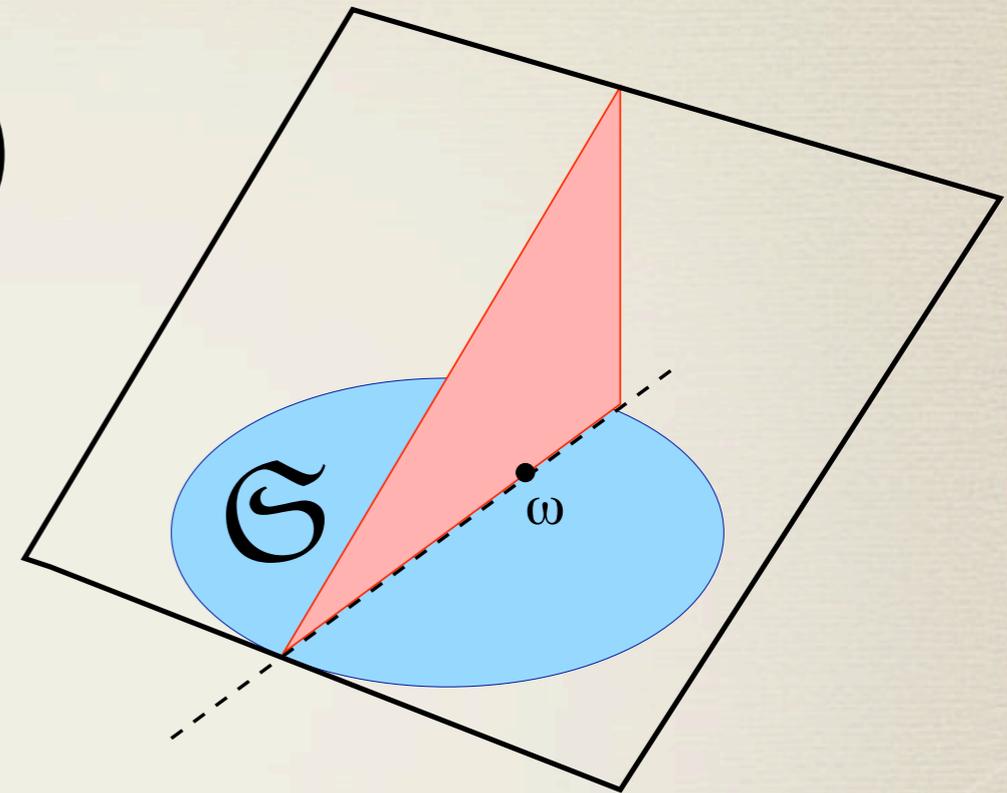
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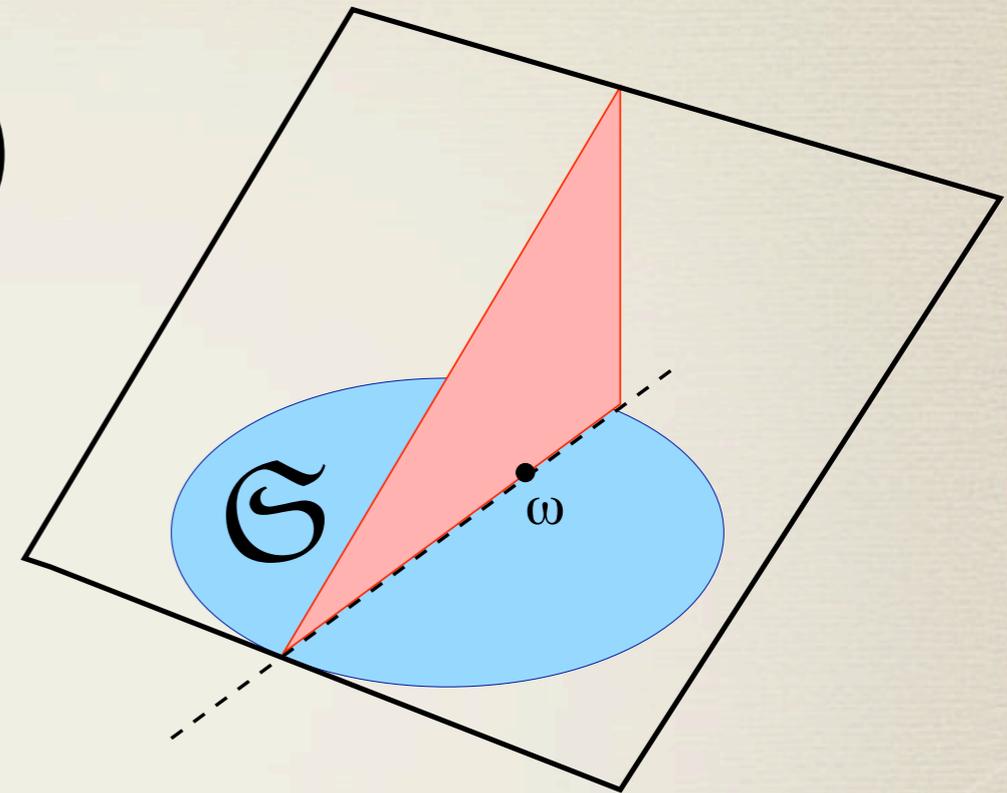
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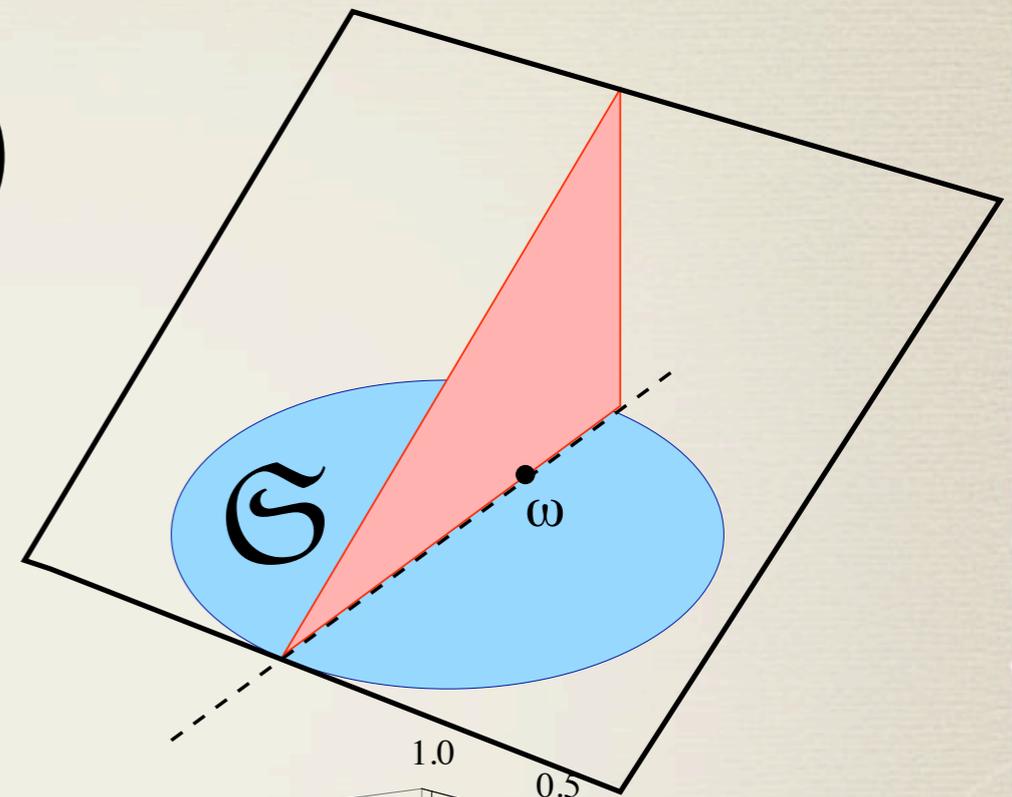
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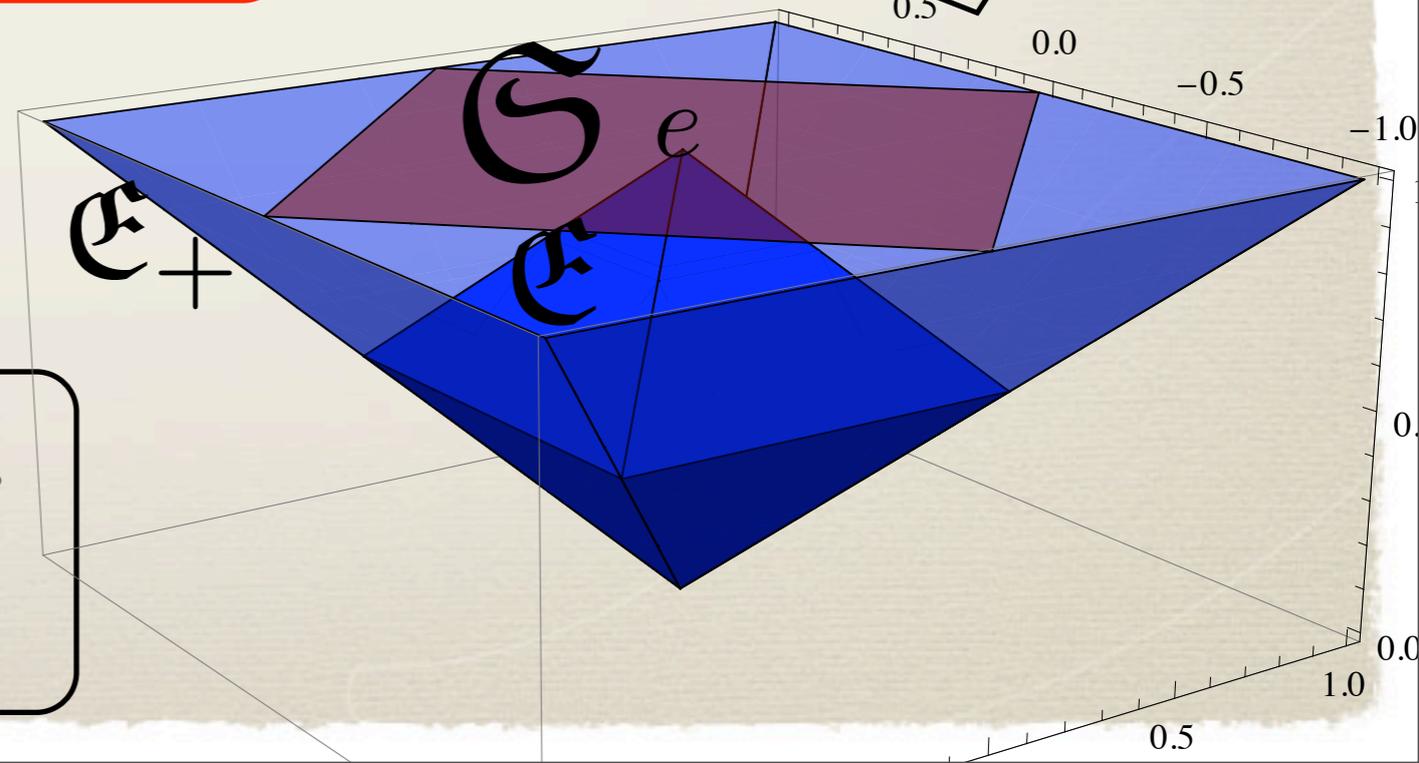
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Convex set of effects  $\mathcal{E}$

Convex cone  $\mathcal{E}_+$



$e$  deterministic effect i.e.

$$\omega(e) = 1 \quad \forall \omega \in \mathcal{S}$$

# Preparation-test and observation-tests

Preparation-test  $\{\omega_i\}, \sum_i \omega_i(e) = 1$

Observation-test  $\{a_i\}, \sum_i a_i = e$

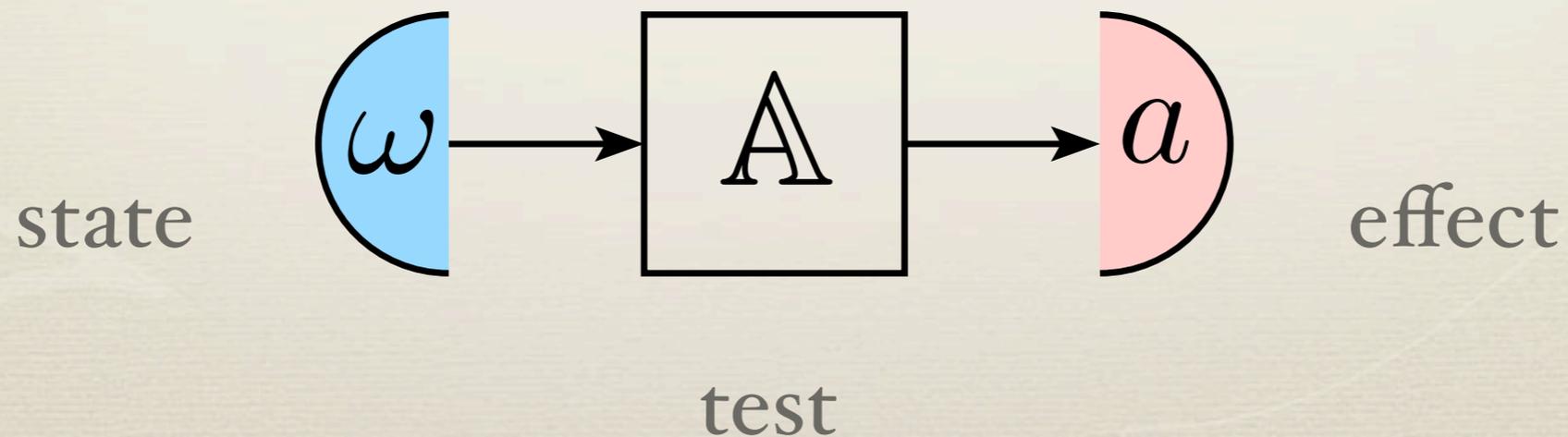
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## Dirac notation

$$(a|\omega) := \omega(a) \quad (a|\mathcal{B}|\omega) := \omega(a \circ \mathcal{B})$$



# Addition of transformations

Transformations  $\mathcal{A}, \mathcal{B}$  (generally belonging to different tests)

**Test-compatible** if:  $\omega(\mathcal{A}) + \omega(\mathcal{B}) \leq 1, \forall \omega \in \mathcal{G}$

For test-compatible transformations  $\mathcal{A}_1, \mathcal{A}_2$  define the transformation  $\mathcal{A}_1 + \mathcal{A}_2$  as the coarse-graining  $\mathcal{A}_1 \cup \mathcal{A}_2$  as if they belong to the same test

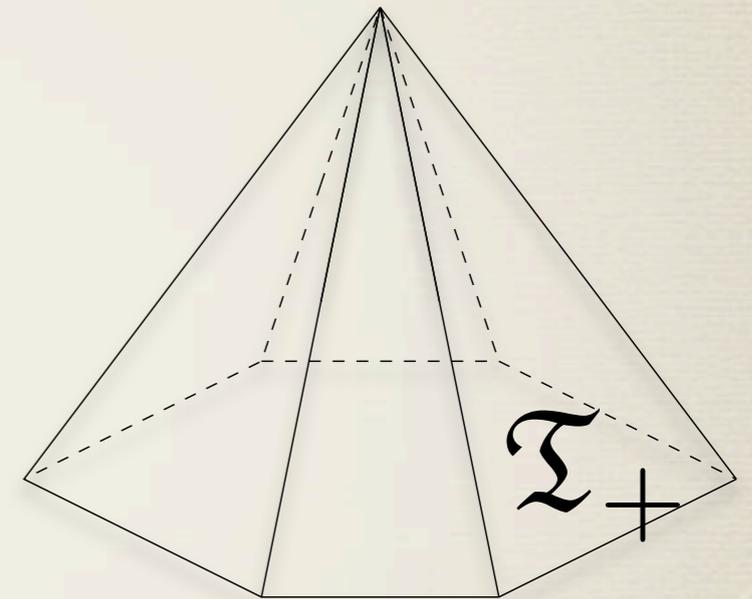
$$(\mathcal{A}_1 + \mathcal{A}_2)\omega = \mathcal{A}_1\omega + \mathcal{A}_2\omega$$

# Rescaling of transformations

The rescaled transformation  $\lambda\mathcal{A}$  of  $\mathcal{A}$ ,  $\lambda \in [0, 1]$  is the transformation giving the same conditioning but occurring with probability rescaled by  $\lambda$  for all states.

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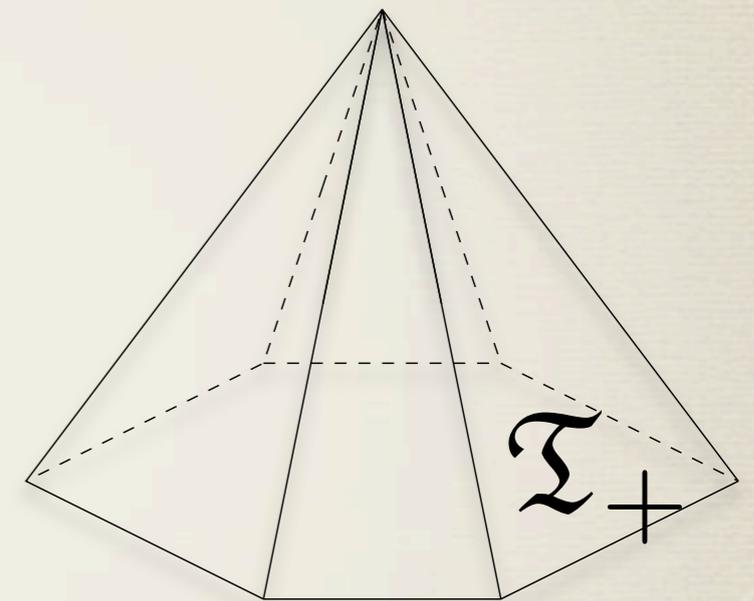


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**Atomic:** a transformation that cannot be refined in any test, i.e. it cannot be written as  $\mathcal{A} = \sum_i \mathcal{A}_i$  with  $\mathcal{A}_i \not\subseteq \mathcal{A} \forall i$



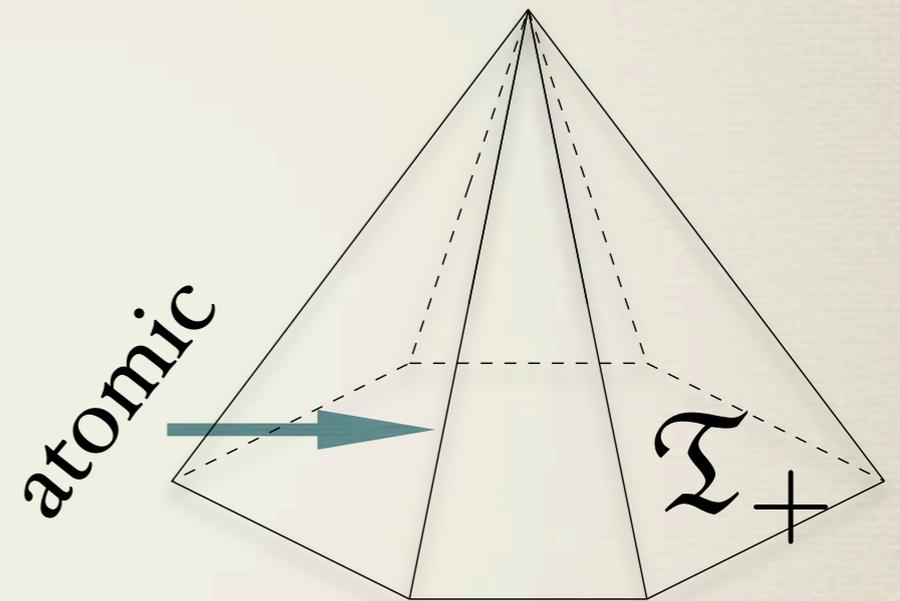
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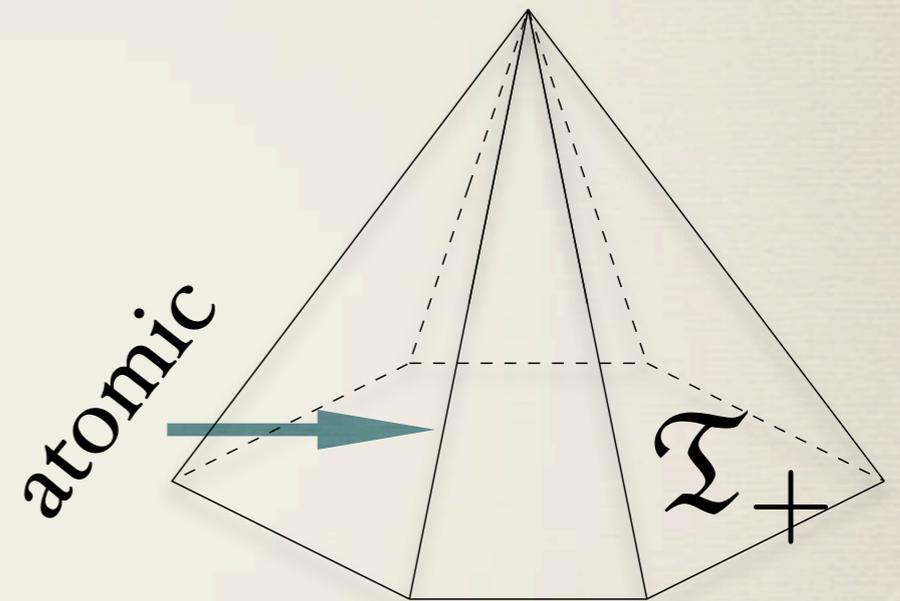
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Convex set of transformations  $\mathfrak{T}$   
Convex cone of transformations  $\mathfrak{T}_+$

Atomic transformations lie on extremal rays of  $\mathfrak{T}_+$

► The identity transformation  $\mathcal{I}$  is not necessarily atomic!

# STANDARD REFERENCE-TEST

$$\mathcal{S} = \{\mathcal{S}_i\}, \quad \mathcal{S}_i = |\lambda_i\rangle\langle l_i|$$

$\{\lambda_i\}$  minimal effect-separating set of states

$\{l_i\}$  minimal state-separating set of effects (info-complete)

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$$\langle l_i | \lambda_j \rangle = \delta_{ij}$$

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- take the last element  $l_N \equiv e$  and correspondingly  $\lambda_N$  giving the direction of the cone axis of  $\mathfrak{S}_+$

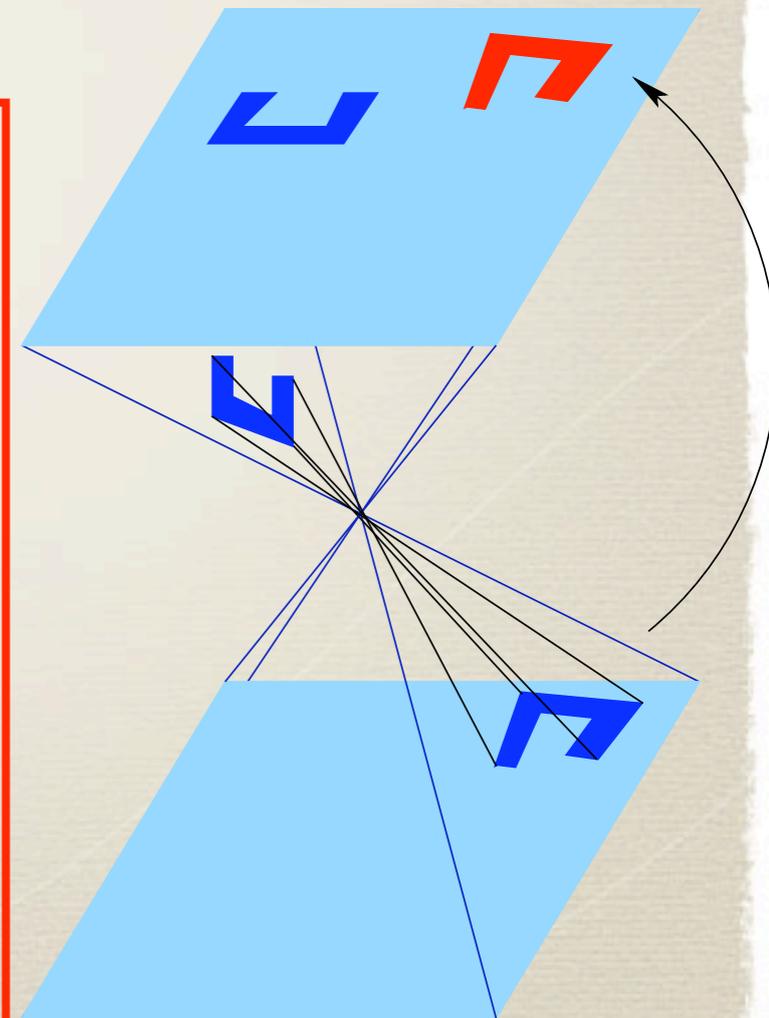
# Matrix representation of the algebra of transformations

$$\omega = \begin{bmatrix} (l_1|\omega) \\ (l_2|\omega) \\ \dots \\ (e|\omega) \end{bmatrix} = \begin{bmatrix} \hat{\omega} \\ \hat{\omega} \end{bmatrix}, \quad a = \begin{bmatrix} (a|\lambda_1) \\ (a|\lambda_2) \\ \dots \\ (a|\chi) \end{bmatrix} = \begin{bmatrix} \hat{a} \\ \hat{a} \end{bmatrix}$$

$$\mathcal{A} = \sum_{ij} A_{ij} |\lambda_i)(l_j|$$

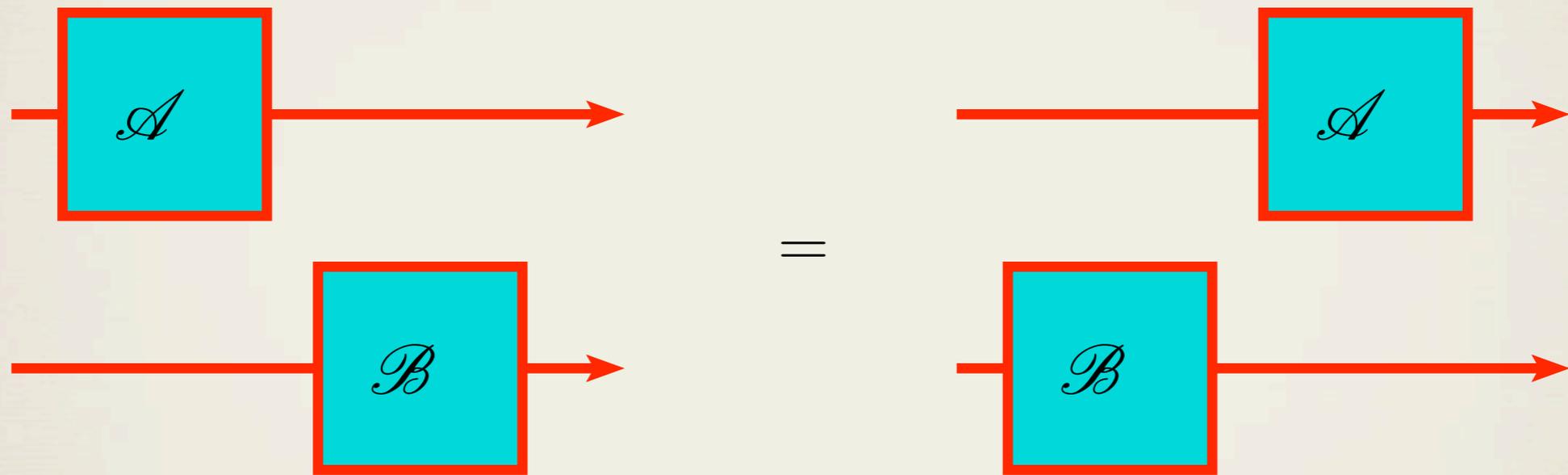
$$A = \begin{pmatrix} \hat{A} & \hat{\alpha} \\ \hat{a}^T & \hat{a} \end{pmatrix},$$

$$\begin{aligned} \widehat{A\omega} &= \hat{A}\hat{\omega} + \hat{\alpha}, \\ (a|\omega) &= \hat{a}^T \hat{\omega} + \hat{a}, \\ \hat{\omega} \rightarrow \hat{\omega}_{\mathcal{A}} &= \frac{\hat{A}\hat{\omega} + \hat{\alpha}}{\hat{a}^T \hat{\omega} + \hat{a}}. \end{aligned}$$



# INDEPENDENT SYSTEMS

Two systems are **independent** if on each system it is possible to perform all their tests as **local tests**, i.e. such that on every joint state one has the commutativity of the transformations from different systems



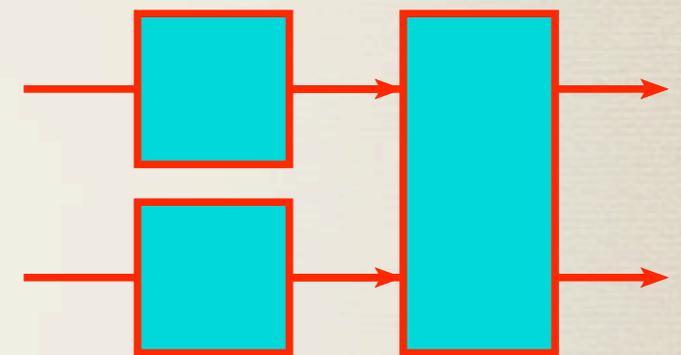
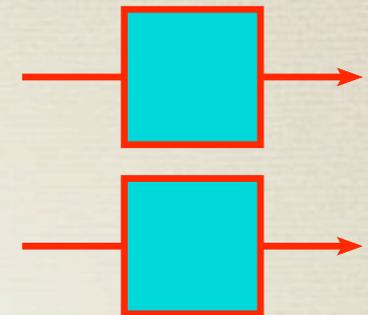
$$A^{(1)} \circ B^{(2)} = B^{(2)} \circ A^{(1)}$$

# MULTIPARTITE SYSTEMS

We compose the two systems  $A$  and  $B$  into the bipartite system  $AB$  considered as a new system containing all **local tests**  $A \times B$  plus other tests, and closing w.r.t. coarse graining, convex combination and cascading:

$$AB \supseteq A \times B$$

**Nonlocal tests:**  $AB \setminus A \times B$



# MARGINAL STATE

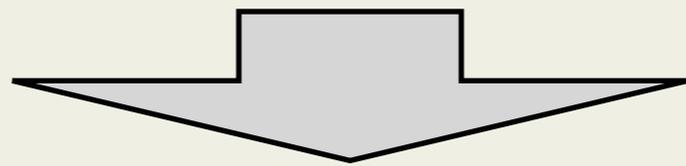
For a multipartite system we define the marginal state  $\Omega|_n$  of the  $n$ -th system the state that gives the probability of any local transformation  $\mathcal{A}$  on the  $n$ -th system with all other systems untouched, namely

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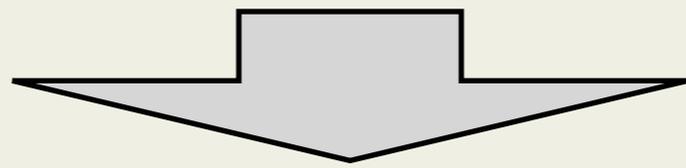


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**NS: (no-signaling)** any local test on a system is equivalent to no-test on another independent system.

# Bipartite states effects

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**$\supseteq \Rightarrow$  NO local discriminability:**

there are local effects are not separated by local states and/or viceversa.

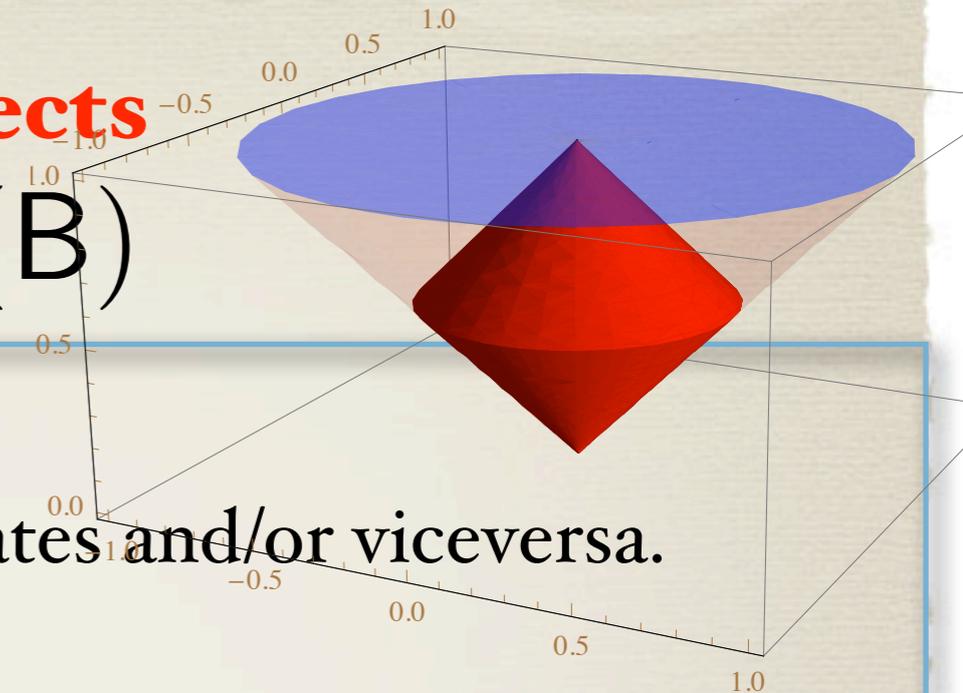
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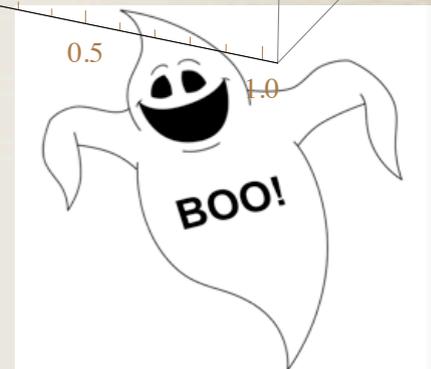
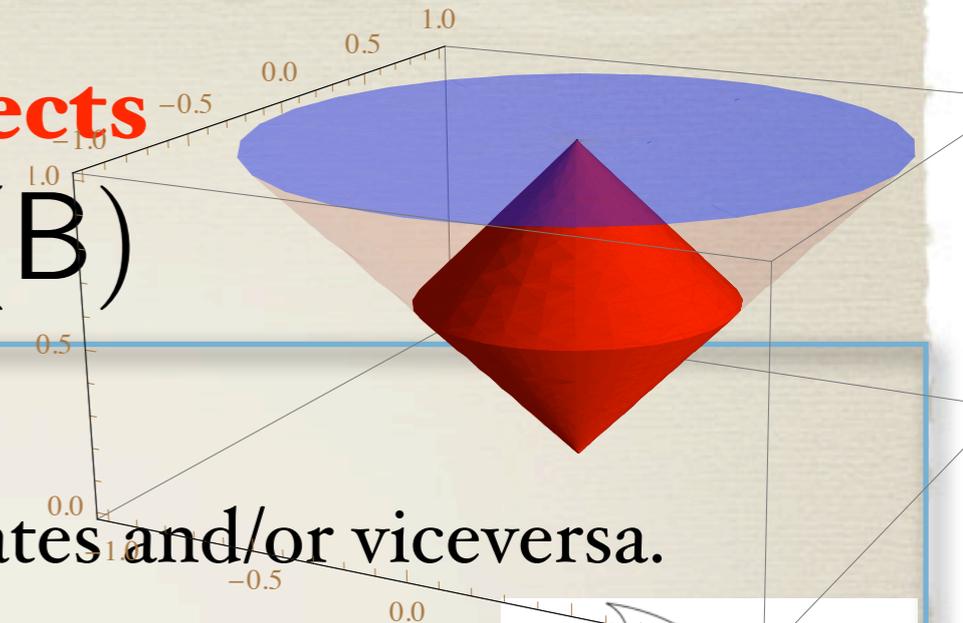
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Effects/states that are locally indistinguishable becomes distinguishable using joint tests. **Recipe:** add local “ghost” states/effects to the reference-test to represent everything within the tensor product.



# Bipartite states effects

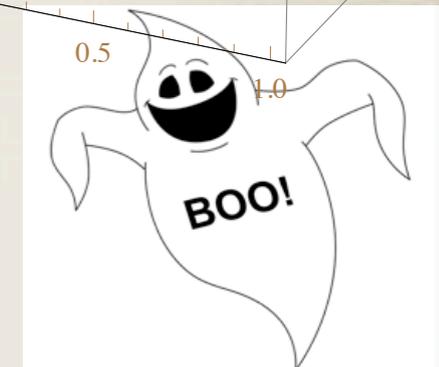
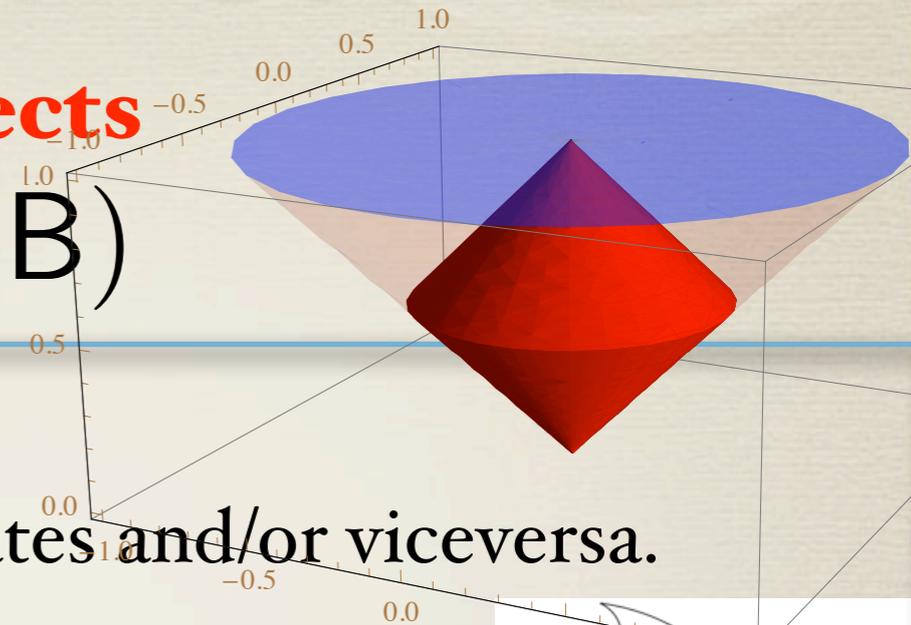
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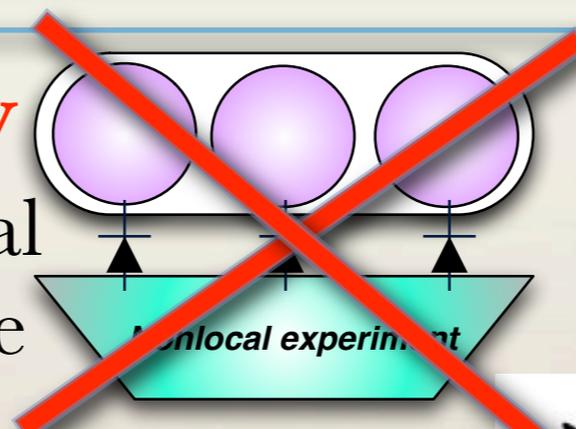
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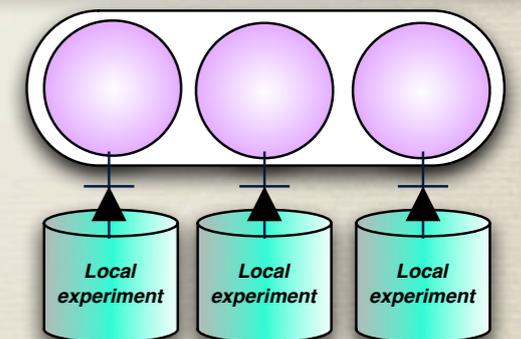


$= \Rightarrow$  **Local discriminability**

+ **local observability:** global info-complete observables made of local info-complete



**Holism**



**Reductionism**

# Matrix representation of bipartite states/effects

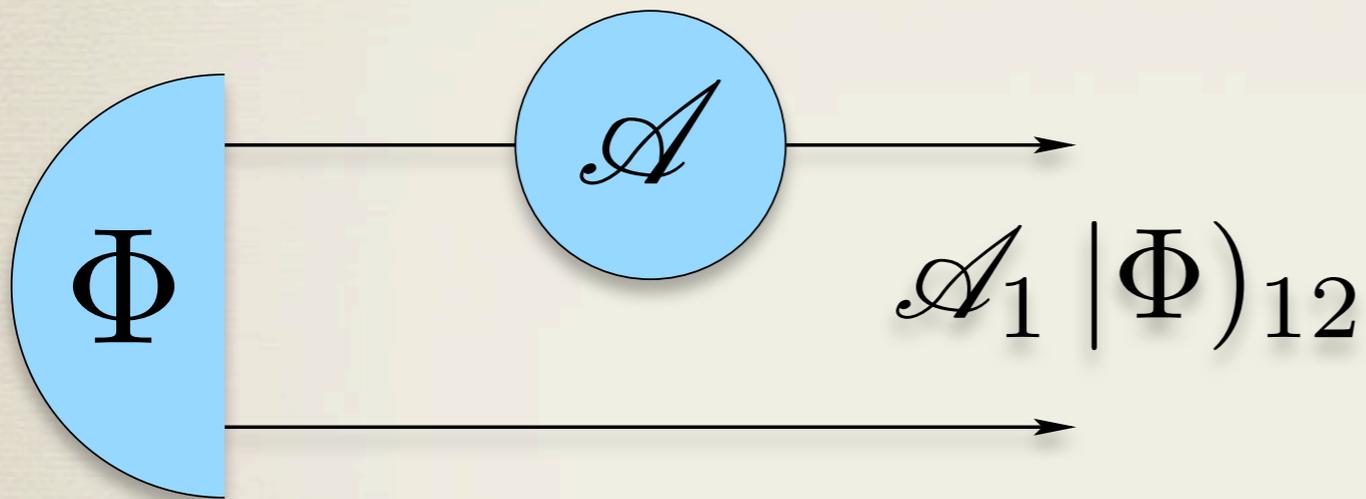
With respect to the standard test we can represent bipartite states and effects as follows

$$|\Psi\rangle = \sum_{ij} \Psi_{ij} |\lambda_i\rangle \otimes |\lambda_j\rangle, \quad (E| = \sum_{ij} E_{ij} (l_i| \otimes (l_j|,$$

# FAITHFUL STATES

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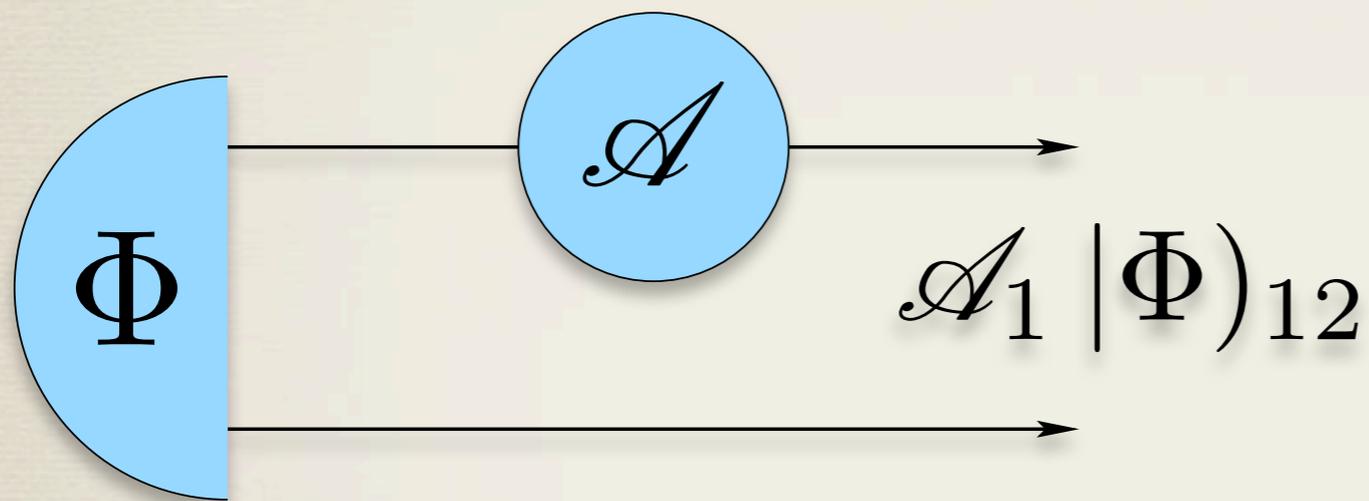
A state  $\Phi$  of a bipartite system is **dynamically faithful** when the output state  $\mathcal{A}_1 |\Phi\rangle_{12}$  from a local transformation  $\mathcal{A}$  on one system is in 1-to-1 correspondence with the transformation  $\mathcal{A}$



calibrability of tests

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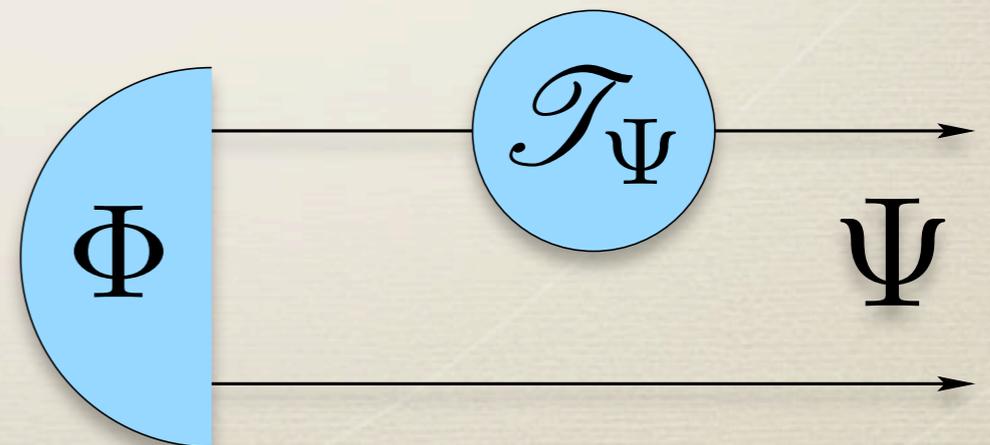
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calibrability of tests

local state-preparability

A state  $\Phi$  of a bipartite system is **preparationally faithful** if every joint state  $\Psi$  can be achieved by a suitable local transformation  $\mathcal{T}_\Psi$  on one system occurring with nonzero probability



# FAITHFUL STATES

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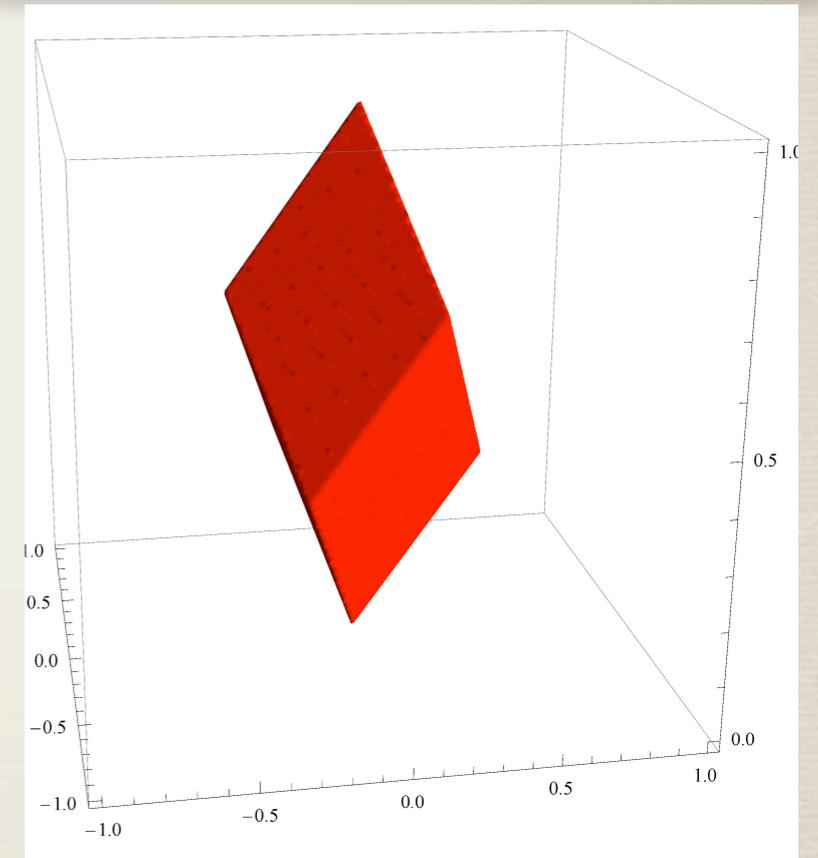
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- ▶ It is always possible to build up a symmetric preparationally faithful state over two identical systems.
- ▶ Faithful states are pure iff  $\mathcal{I}$  is atomic (joint property from local geometry!)



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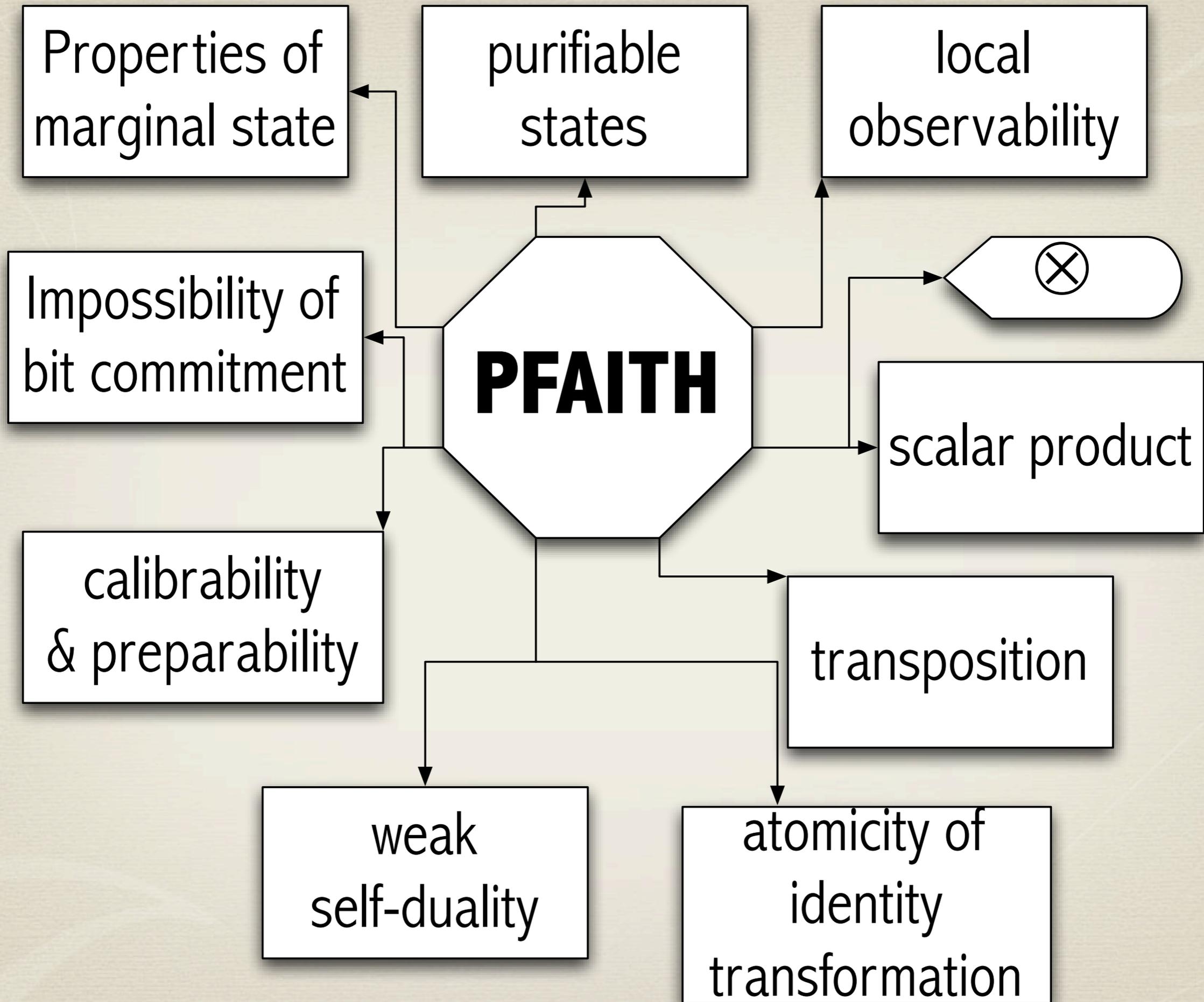
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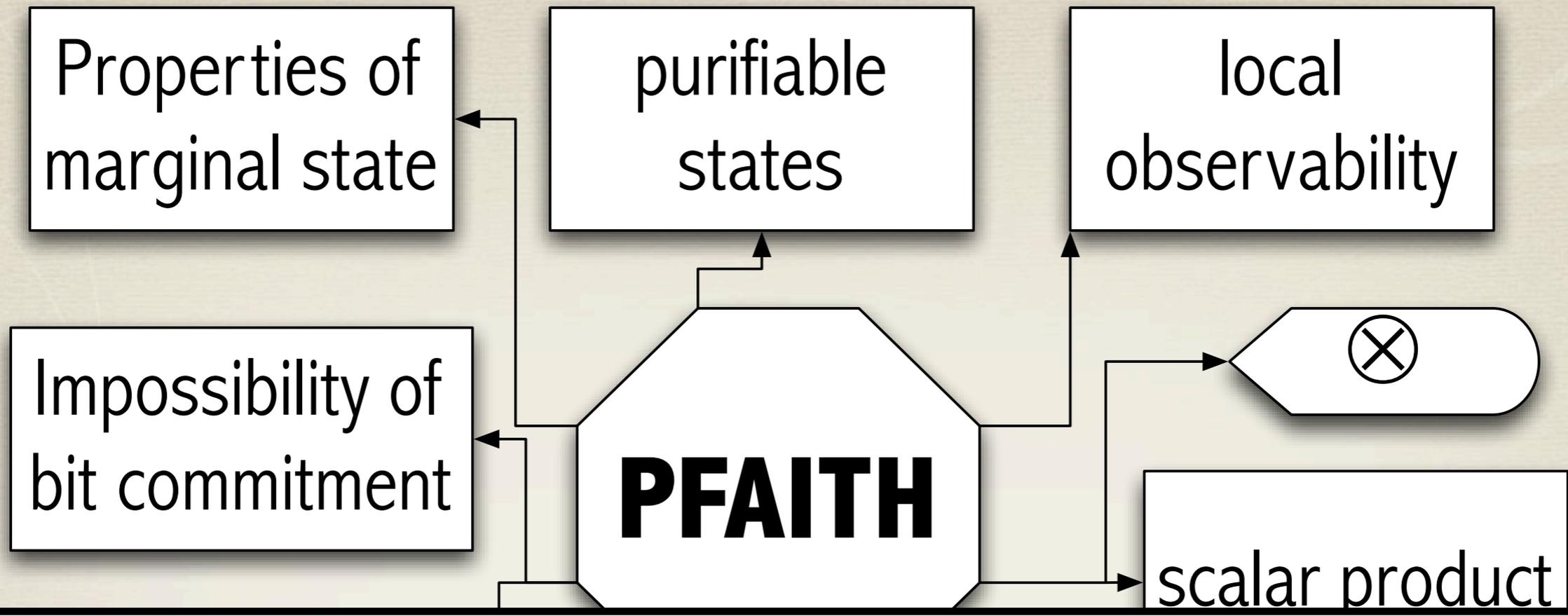
Lesson learnt:  
all test-theories have  
a nice matrix  
representation

EXPLORING POSSIBLE  
PRINCIPLES OF THE  
QUANTUMNESS

# Postulate PFAITH

**PFAITH:** For any couple of identical systems, there exist a symmetric pure state  $\Phi$  that is preparationally faithful.





**CLASSICAL TEST-THEORIES  
ARE EXCLUDED**

**PR-BOXES ARE INCLUDED**

self-duality

identity  
transformation

# Postulate: FAITHE

**Postulate FAITHE:** (faithful effect)

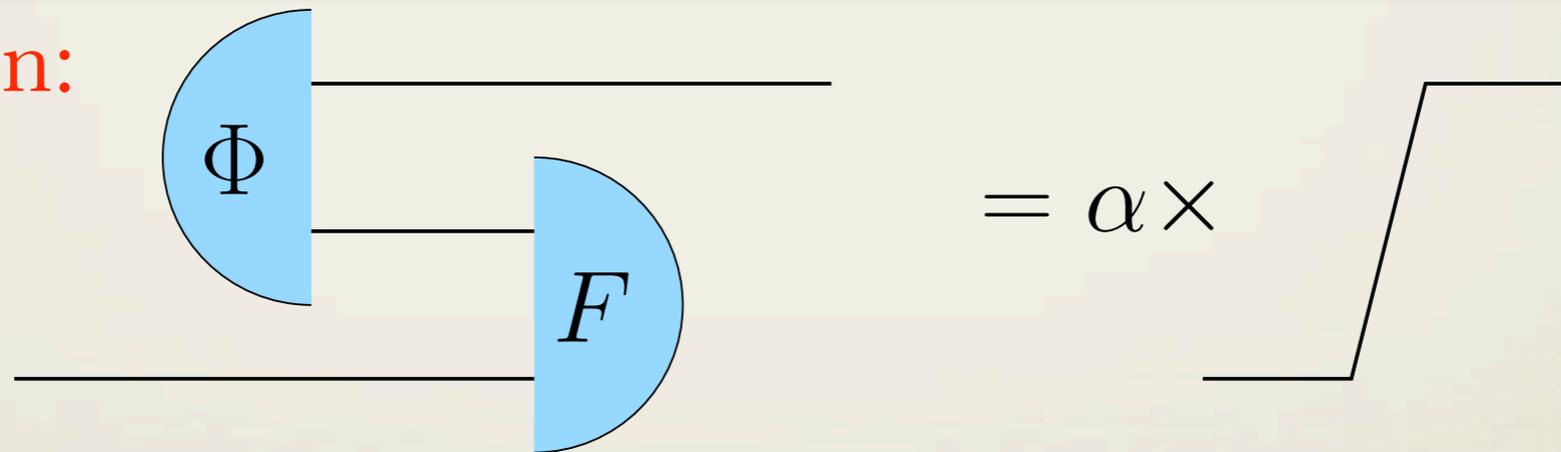
$F := \alpha (\Phi^{-1} | \in \mathfrak{E}(SS), 0 < \alpha \leq 1$   
proportional to a joint effect.

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► Teleportation:



# Postulate: FAITHE

**Postulate FAITHE:** (faithful effect)

BOTH CLASSICAL TEST-  
THEORIES AND P-BOXES ARE  
EXCLUDED

# Postulate: Purification

**Postulate PURIFY:** Every state has a purification on two identical systems.

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**Postulate PURIFY:** Every state has a purification on two identical systems.



▶ A symmetric preparationally faithful state is necessarily pure and  $\mathcal{I}$  is atomic.

▶ The sets of (bipartite) states/effects are strongly convex

▶ Each state can be obtained by applying an atomic transformation to the marginal state  $\chi = \Phi(e, \cdot)$

▶ Each effect contains an atomic transformation.

DO WE GET QUANTUM THEORY  
FROM OUR POSTULATES?

DO WE GET QUANTUM THEORY  
FROM OUR POSTULATES?

HOW TO PROVE THAT WE HAVE  
QUANTUM MECHANICS?

THANK YOU FOR  
YOUR ATTENTION