Quantum Theory as operational probabilistic theory: what we have learnt

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Operational probabilistic theory (OPT)

\[ p(abc, \ldots, o|\gamma_1 \cup \gamma_2) = p(abc, \ldots, l|\gamma_1)p(n, \ldots, p|\gamma_2) \]

NOTICE: marginals depend on the marginalised part of the graph!
Goal of Science

1. To connect “objective things happening” (events)
2. To devise a theory of such “connections” (systems)
3. To make predictions for future occurrence (predict joint probabilities of events depending on their connections).

Which **events** happen is **objective**

**Systems** are **theoretical**

OPT: methodologically fit, falsification-ready
Goal of an OPT

To provide a mathematical description of systems and events consistent with their composition rules, allowing to evaluate their joint probability distribution depending on the graph of connections.
An OPT is an Information Theory

preparations

systems

events

observations

finite DAG
An OPT is an Information Theory

START

Decode 1

Stall detected

Average

Update pipeline and writeback

Execute

END

Decode Address, bla bla

YES

Decode Address, bla bla

Update pipeline

Execute

END
OPT framework

joint probabilities + connectivity

Marginal probability

\[ \sum_{i, j, k, \ldots} p(i, j, k, \ldots | \text{circuit}) = p(j | \text{circuit}) \]
OPT framework

joint probabilities + connectivity

Marginal probability

\[ \sum_{ik\ldots} p(i, j, k, \ldots | \text{circuit}) = p(j|\text{circuit}) \]
OPT framework

joint probabilities + connectivity

Probabilistic

equivalence classes
OPT framework

Joint probabilities + connectivity

Category theory:
- Transformations \( \Rightarrow \) morphisms
- Systems \( \Rightarrow \) objects

OPT: strict monoidal braided category
OPT framework

Sequential composition (associative)

\[
\begin{array}{ccc}
A & \{\mathcal{A}_x\}_{x \in X} & B \{\mathcal{B}_y\}_{y \in Y} & C &=& A \{\mathcal{B}_x \circ \mathcal{A}_y\}_{(x,y) \in X \times Y} & C
\end{array}
\]

Identity test

\[
\begin{array}{ccc}
\end{array}
\]
**OPT framework**

Parallel composition (associative)

\[
\begin{align*}
A \{ \mathcal{A}_x \}_{x \in X} & \quad B \\
C \{ \mathcal{B}_y \}_{y \in Y} & \quad D \\
\end{align*}
\]

\[
= : \quad AC \{ \mathcal{A}_x \otimes \mathcal{B}_y \}_{(x,y) \in X \times Y} \quad BD
\]

\[
AB \simeq BA =: S_{A,B} AB \quad \text{(braided)}
\]

\[
AI = IA
\]

\[
A(BC) = A(BC) \quad \text{(strict monoidal)}
\]

\[
(AB)C \simeq A(BC) \quad \text{(monoidal)}
\]

**OPT: strict monoidal braided category**

Quantum Theory: symmetric OPT

\[
S_{A,B} AB = BA
\]

\[
S_{A,B}^{-1} = S_{B,A} \quad \text{(symmetric)}
\]
Sequential and parallel compositions commute

\[(A \otimes D) \circ (C \otimes B) = (A \circ C) \otimes (D \circ B)\]

OPT framework

wire-stretching
(foliations)
### Quantum Theory as OPT

<table>
<thead>
<tr>
<th>system</th>
<th>A</th>
<th>( \mathcal{H}_A )</th>
<th>(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>system composition</td>
<td>AB</td>
<td>( \mathcal{H}_{AB} = \mathcal{H}_A \otimes \mathcal{H}_B )</td>
<td></td>
</tr>
<tr>
<td>transformation</td>
<td>( \mathcal{T} \in \text{Transf}(A \to B) )</td>
<td>( \mathcal{T} \in \text{CP}_{\leq}(\mathcal{T}(\mathcal{H}_A) \to \mathcal{T}(\mathcal{H}_B)) )</td>
<td>(2)</td>
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</tbody>
</table>

### Theorems

<table>
<thead>
<tr>
<th>trivial system system</th>
<th>I</th>
<th>( \mathcal{H}_I = \mathbb{C} )</th>
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</thead>
<tbody>
<tr>
<td>deterministic transformation</td>
<td>( \mathcal{T} \in \text{Transf}_1(A \to B) )</td>
<td>( \mathcal{T} \in \text{CP}_{\leq}(\mathcal{T}(\mathcal{H}_A) \to \mathcal{T}(\mathcal{H}_B)) )</td>
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<tr>
<td>states</td>
<td>( \rho \in \text{St}(A) \equiv \text{Transf}(I \to A) )</td>
<td>( \rho \in \text{T}^+_\leq_1(\mathcal{H}_A) )</td>
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<td>( \rho \in \text{St}_1(A) \equiv \text{Transf}_1(I \to A) )</td>
<td>( \rho \in \text{T}^+_\leq_1(\mathcal{H}_A) )</td>
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<td>( \rho \in \text{St}(I) \equiv \text{Transf}(I \to I) )</td>
<td>( \rho \in [0, 1] )</td>
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<td>( \rho \in \text{St}_1(I) \equiv \text{Transf}_1(I \to I) )</td>
<td>( \rho = 1 )</td>
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<tr>
<td>effects</td>
<td>( \varepsilon \in \text{Eff}(A) \equiv \text{Transf}(A \to I) )</td>
<td>( \varepsilon(\cdot) = \text{Tr}_A[\cdot E], \ 0 \leq E \leq I_A )</td>
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<tr>
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<td>( \varepsilon \in \text{Eff}_1(A) \equiv \text{Transf}_1(A \to I) )</td>
<td>( \varepsilon = \text{Tr}_A )</td>
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</tbody>
</table>
Principles for Quantum Theory

P1. Causality
P2. Local discriminability
P3. Purification
P4. Atomicity of composition
P5. Perfect distinguishability
P6. Lossless Compressibility

Principles for Quantum Theory

P1. **Causality**

P2. Local discriminability

P3. Purification

P4. Atomicity of composition

P5. Perfect distinguishability

P6. Lossless Compressibility

The probability of preparations is independent of the choice of observations.

$P(i, j|X', Y) := (a_j|\rho_i)$

$p(i|X, Y) = p(i|X, Y') = p(i|X)$

Iff conditions: a) the deterministic effect is unique; b) states are “normalizable”

$p(i, j, k, l, m, n, p, q|\text{circuit})$
Principles for Quantum Theory

P1. **Causality**
P2. Local discriminability
P3. Purification
P4. Atomicity of composition
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P6. Lossless Compressibility

The probability of preparations is independent of the choice of observations

no signaling without interaction

\[ p(i, j | X, Y) := (a_j | \rho_i) \]

\[ p(i | X, Y) = p(i | X, Y') = p(i | X) \]

Iff conditions: a) the deterministic effect is unique; b) states are “normalizable”

\[ \sigma_{AB \to e} =: \rho_{A}. \]

marginal state
Principles for Quantum Theory

P1. Causality

P2. Local discriminability

P3. Purification

P4. Atomicity of composition

P5. Perfect distinguishability

P6. Lossless Compressibility

It is possible to discriminate any pair of states of composite systems using only local measurements.
Principles for Quantum Theory

P1. Causality
P2. **Local discriminability**
P3. Purification
P4. Atomicity of composition
P5. Perfect distinguishability
P6. Lossless Compressibility

It is possible to discriminate any pair of states of composite systems using only local measurements.

Origin of the complex tensor product

Local characterization of transformations
Principles for Quantum Theory

P1. Causality
P2. Local discriminability
P3. Purification
P4. Atomicity of composition
P5. Perfect distinguishability
P6. Lossless Compressibility

The composition of two atomic transformations is atomic

Complete information can be accessed on a step-by-step basis
Principles for Quantum Theory

P1. Causality
P2. Local discriminability
P3. Purification
P4. Atomicity of composition
P5. Perfect distinguishability
P6. Lossless Compressibility

Every state that is not completely mixed (i.e. on the boundary of the convex) can be perfectly distinguished from some other state.

Falsifiability of the theory
Principles for Quantum Theory

P1. Causality
P2. Local discriminability
P3. Purification
P4. Atomicity of composition
P5. Perfect distinguishability
P6. **Lossless Compressibility**

For states that are not completely mixed there exists an ideal compression scheme.

Any face of the convex set of states is the convex set of states of some other system.
Principles for Quantum Theory

P1. Causality
P2. Local discriminability
P3. **Purification**
P4. Atomicity of composition
P5. Perfect distinguishability
P6. Lossless Compressibility

---

Every state has a purification. For fixed purifying system, every two purifications of the same state are connected by a reversible transformation on the purifying system.
Principles for Quantum Theory

P1. Causality
P2. Local discriminability
P3. **Purification**
Every state has a purification.
For fixed purifying system, every two purifications of the same state are connected by a reversible transformation on the purifying system.

P4. Atomicity of composition
P5. Perfect distinguishability
P6. Lossless Compressibility

Consequences

1. Existence of entangled states: the purification of a mixed state is an entangled state; the marginal of a pure entangled state is a mixed state;

2. Every two normalized pure states of the same system are connected by a reversible transformation

\[ \psi' \begin{array}{c} B \\ \hline \end{array} = \begin{array}{c} \psi \\ \begin{array}{c} B \\ \hline \end{array} \end{array} U_B \]

3. Steering: Let \( \psi \) purification of \( \rho \). Then for every ensemble decomposition \( \rho = \sum p_x \alpha_x \) there exists a measurement \( \{ b_x \} \), such that

\[ \begin{array}{c} \psi \\ \begin{array}{c} A \\ \hline \end{array} \end{array} \begin{array}{c} B \\ \hline \end{array} = \begin{array}{c} p_x \alpha_x \\ \begin{array}{c} A \\ \hline \end{array} \\ \begin{array}{c} b_x \\ \hline \end{array} \end{array} \quad \forall x \in X \]

4. Process tomography (faithful state):

\[ \begin{array}{c} \psi \\ \begin{array}{c} A \\ \hline \end{array} \begin{array}{c} A' \\ \hline \end{array} \end{array} \begin{array}{c} B \\ \hline \end{array} = \begin{array}{c} \psi \\ \begin{array}{c} A \\ \hline \end{array} \begin{array}{c} A' \\ \hline \end{array} \end{array} \]

\[ \mathcal{A} \rho = \mathcal{A}' \rho \quad \forall \rho \]

5. No information without disturbance
Principles for Quantum Theory

P1. Causality
P2. Local discriminability
P3. **Purification**
P4. Atomicity of composition
P5. Perfect distinguishability
P6. Lossless Compressibility

Every state has a purification. For fixed purifying system, every two purifications of the same state are connected by a reversible transformation on the purifying system.

Consequences

6. Teleportation

\[
\Phi_{AB} = p_x \ x \ x_{\Phi_x} \ \forall x \in X
\]

7. Reversible dilation of "channels"

\[
A \ E = \eta \ U \ E \ x \ e
\]

8. Reversible dilation of "instruments"

\[
A \ A = \eta \ U \ B \ b_x \ \forall x \in X
\]

9. State-transformation cone isomorphism

10. Reversible transform. for a system make a compact Lie group
## Other OPTs

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**Legend:**
- **QT:** Quantum theory
- **CT:** Classical theory
- **QBIT:** Qubit theory
- **FQT:** Fermionic quantum theory
- **RQT:** Real quantum theory
- **NSQT:** Number superselected quantum theory
- **PR:** PR-boxes theory
- **DPR:** Dual PR-boxes theory
- **HPR:** Hybrid PR-boxes theory
- **FOCT:** First order classical theory
- **FOQT:** First order quantum theory
- **NLCT:** Non-local classical theory
- **NLQT:** Non-local quantum theory
- **Compr.** indicates whether the theory is compressible.
- **Purification** indicates whether the theory allows purification.
- **NIWD** indicates whether the theory violates no的信息-without-disturbance.

**Notes:**
- A theory is no-cloning if for some state it is no-cloning for such that.
- At the theory is no-information without disturbance upon input of.
“HOW TO GET THE “MECHANICS?”

QUANTUM FIELD THEORY: an ultra-short account
Info-theoretical principles for Quantum Field Theory

- Causality
- Local discriminability
- Purification
- Atomicity of composition
- Perfect discriminability
- Ideal compressibility

Quantum Theory
- Locality
- Homogeneity
- Isotropy
- Unitarity

Quantum Cellular Automata on a Cayley graph of G
- Cayley graph quasi-isometrically embeddable in Euclidean space
- G virtually Abelian

Quantum Walk on Cayley graph of G
- Quantum Walk on Cayley graph of Abelian G

Relativity Principle without space-time
- Relativistic limit
- Free Quantum Field Theory

GR heuristics
- $m>0$: deformed De Sitter
- $m=0$: deformed Lorentz
- $m \in \mathbb{S}^1$
- discrete proper time

LTM Standards

MATH. FRAMEWORK

PRINCIPLES

THEORY

RESTRICTIONS

INTERPRETATION
\[ \mathcal{H} = \bigoplus_{g \in G} \mathbb{C}^{s_g} \quad |G| \leq \mathbb{N}, \ s_g \in \mathbb{N} \]

Evolution

\[ \psi_g(t + 1) = \sum_{g' \in S_g} A_{gg'} \psi_{g'}(t) \]

\[ \sum_{g'} A_{gg'} A_{g''g'}^\dagger = \sum_{g'}ug'_{gg''} = \delta_{gg''} I_{s_g} \]

Build a directed graph with an arrow from \( g \) to \( g' \) wherever they are connected by \( A_{gg'} \neq 0 \)
Info-theoretical principles for Quantum Field Theory
“NO PURIFICATION ONTOLOGY”

NO PARADOXES!
Quantum Theory: no purification ontology

P3. **Purification**

1. Isolated systems don’t need to be in a pure state!
2. Isolated systems don’t need to undergo unitary transformations!

Purification of states

Unitary purification of channels

Unitary dilation of quantum instruments

Unfalsifiable ontologies!
Quantum Theory: no purification ontology

P3. **Purification**

1. Isolated systems don’t need to be in a pure state

2. Isolated systems don’t need to undergo unitary transformations

The necessity for Faddeev–Popov ghosts follows from the requirement that quantum field theories yield unambiguous, non-singular solutions. This is not possible in the path integral formulation when a gauge symmetry is present since there is no procedure for selecting among physically equivalent solutions related by gauge transformation. The path integrals overcount field configurations corresponding to the same physical state; the measure of the path integrals contains a factor which does not allow obtaining various results directly from the action.

It is possible, however, to modify the action, such that methods such as Feynman diagrams will be applicable by adding ghost fields which break the gauge symmetry. **The ghost fields do not correspond to any real particles in external states:** they appear as virtual particles in Feynman diagrams – or as the absence of gauge configurations. However, they are a necessary computational tool to preserve unitarity.
Quantum Theory: no purification ontology

P3. Purification

1. Isolated systems don’t need to be in a pure state
2. Isolated systems don’t need to undergo unitary transformations

\[ H(x) |\psi\rangle = 0 \]
This is more or less what I wanted to say

THANK YOU!
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Follow project on Researchgate: The algorithmic paradigm: deriving the whole physics from information-theoretical principles.

