JOHN TEMPLETON FOUNDATION

SUPPORTING SCIENCE~INVESTING IN THE BIG QUESTIONS

Relativity principle without space-time

Giacomo Mauro D'Ariano
Università degli Studi di Pavia

Time in Physics

ETH Zurich, September 7-11 2015

Program

To derive the whole Physics from principles

as

an axiomatic theory with complete physical interpretation

in order to have a conceptual understanding in terms of the principles

Program

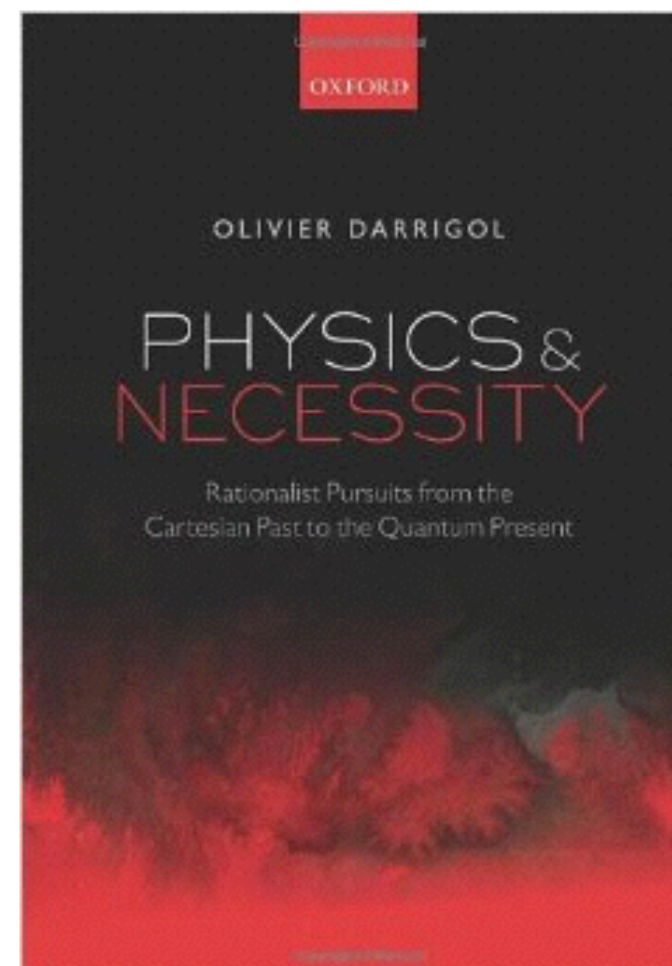
To understand Physics in terms of "principles",

namely:

to derive the whole Physics from principles

as

an axiomatic theory with complete physical interpretation



Principles for Quantum Theory

Selected for a Viewpoint in *Physics*

PHYSICAL REVIEW A **84**, 012311 (2011)

Informational derivation of quantum theory

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We derive quantum theory from purely informational principles. Five elementary axioms—causality, perfect distinguishability, ideal compression, local distinguishability, and pure conditioning—define a broad class of theories of information processing that can be regarded as standard. One postulate—purification—singles out quantum theory within this class.

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Principles for Quantum Theory

P1. Causality

P2. Local discriminability

P3. Purification

P4. Atomicity of composition

P5. Perfect distinguishability

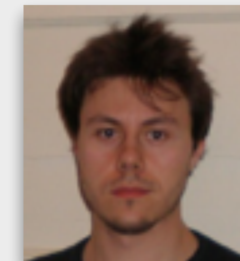
P6. Lossless Compressibility

Book from CUP soon!

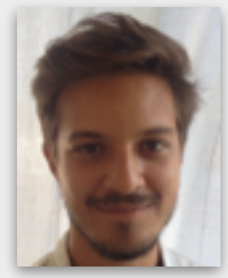
Principles for Mechanics

Paolo Perinotti

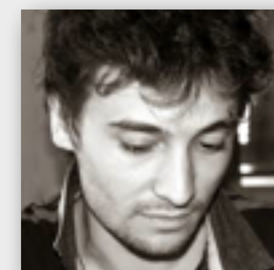
Alessandro Bisio



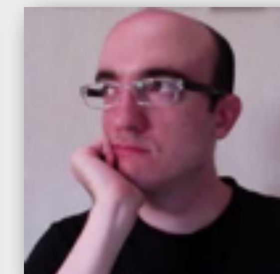
Alessandro Tosini



Marco Erba



Franco Manessi



Nicola Mosco

- *Mechanics (QFT) derived in terms of countably many quantum systems in interaction*

add principles

Min algorithmic complexity principle

- homogeneity
- locality
- reversibility

Principles for Quantum Field Theory

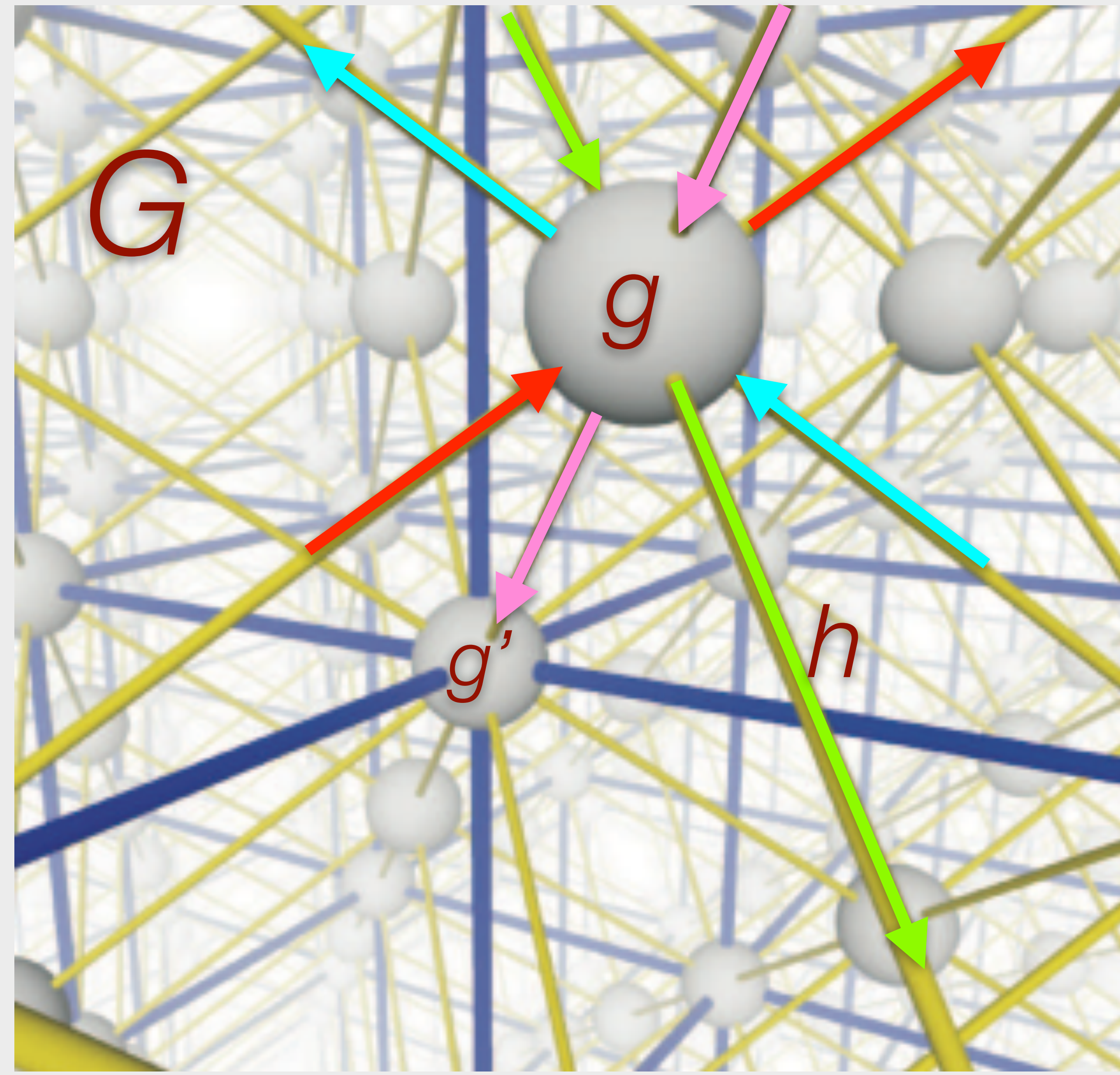
- *QFT derived in terms of countably many quantum systems in interaction*

add principles

Min algorithmic complexity principle

- homogeneity
- locality
- reversibility

} Quantum Cellular Automata on the Cayley graph of a group *G*



$G = \langle h_1, h_2, \ldots | r_1, r_2, \ldots \rangle =: \langle S_+ | R \rangle$

Principles for Quantum Field Theory

- *QFT derived in terms of countably many quantum systems in interaction*

add principles

Min algorithmic complexity principle

- homogeneity
- locality
- reversibility

} Quantum Cellular Automata on the Cayley graph of a group G

- linearity
- isotropy
- minimal-dimension
- Cayley qi-embedded in R^d

} Restrictions

Cells labeled by $g \in G$, $|G| \leq \aleph$; $\psi_g \in \mathbb{C}^{s_g}$, $0 < s_g < \infty$

linearity	The interaction between systems is described by $s_{g'} \times s_g$ *transition matrices* $A_{gg'}$ with evolution from step t to step $t+1$ given by $\psi_g(t+1) = \sum_{g' \in G} A_{gg'} \psi_{g'}(t)$
unitarity	$\sum_{g'} A_{gg'} A^\dagger_{g''g'} = \sum_{g'} A^\dagger_{gg'} A_{g''g'} = \delta_{gg''} I_{s_g}$
locality	$A_{gg'} \neq 0 \iff A_{g'g} \neq 0$: g' and g are *interacting* $\lvert S_g \rvert \leq k < \infty$ for every $g \in G$, where $S_g \subseteq G$ set of cells g' interacting with g
homogeneity	All cells $g \in G$ are equivalent $\Longrightarrow \lvert S_g \rvert$, s_g, $\{A_{gg'}\}_{g' \in S_g}$ independent of g Identify the matrices $A_{gg'} = A_h$ for some $h \in S$ with $\lvert S \rvert = \lvert S_g \rvert$ Define $gh := g'$ if $A_{gg'} = A_h$ and define $A_{g'g} := A_{h^{-1}}$

Linearity ⇒ Quantum Walk (free QFT)

Quantum Cellular Automaton

$$U \psi U^\dagger = A \psi$$

Fock space ⇒ von Neumann algebra

Principles for Quantum Field Theory

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} Quantum Cellular Automata on the Cayley graph of a group *G*

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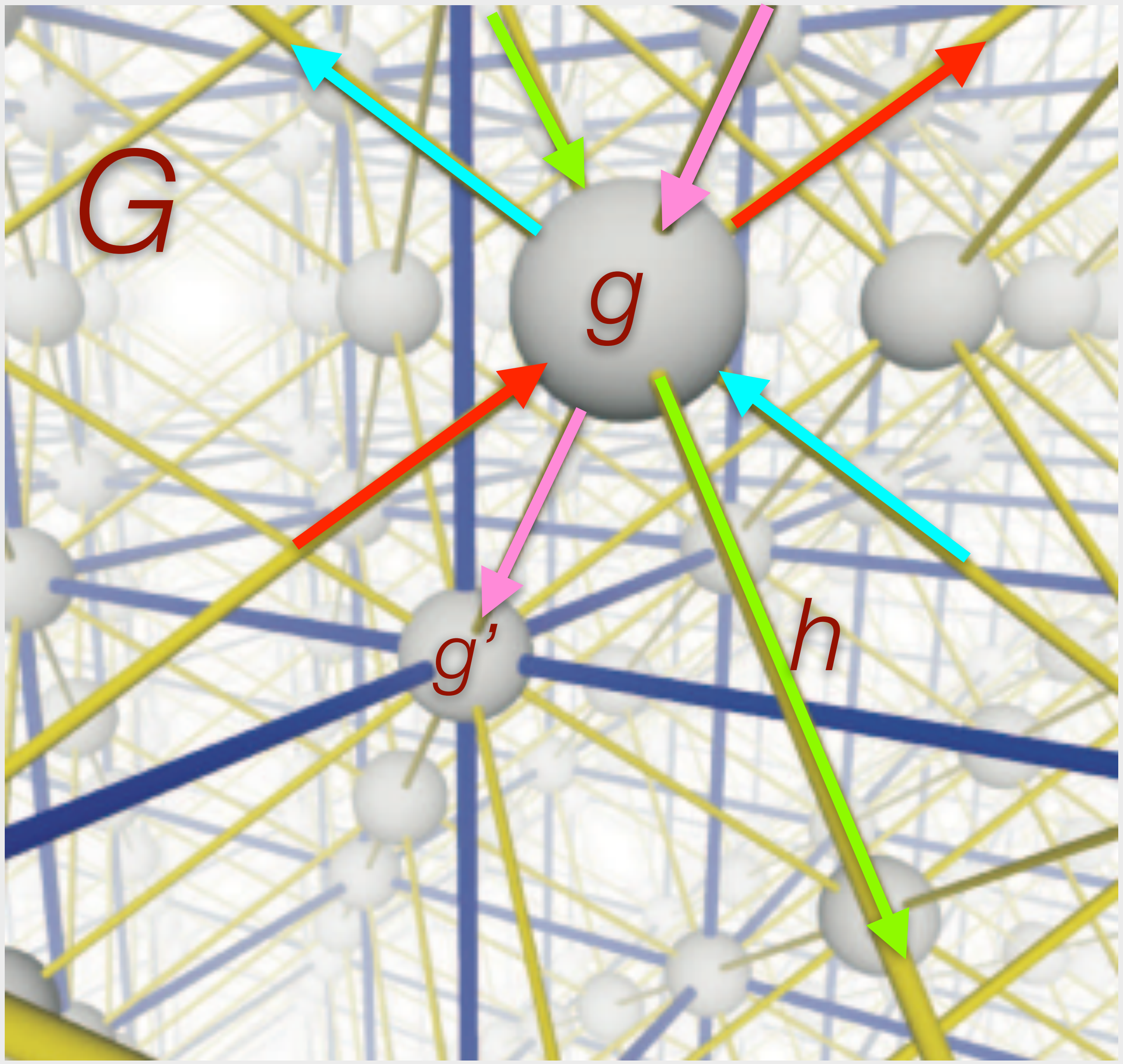
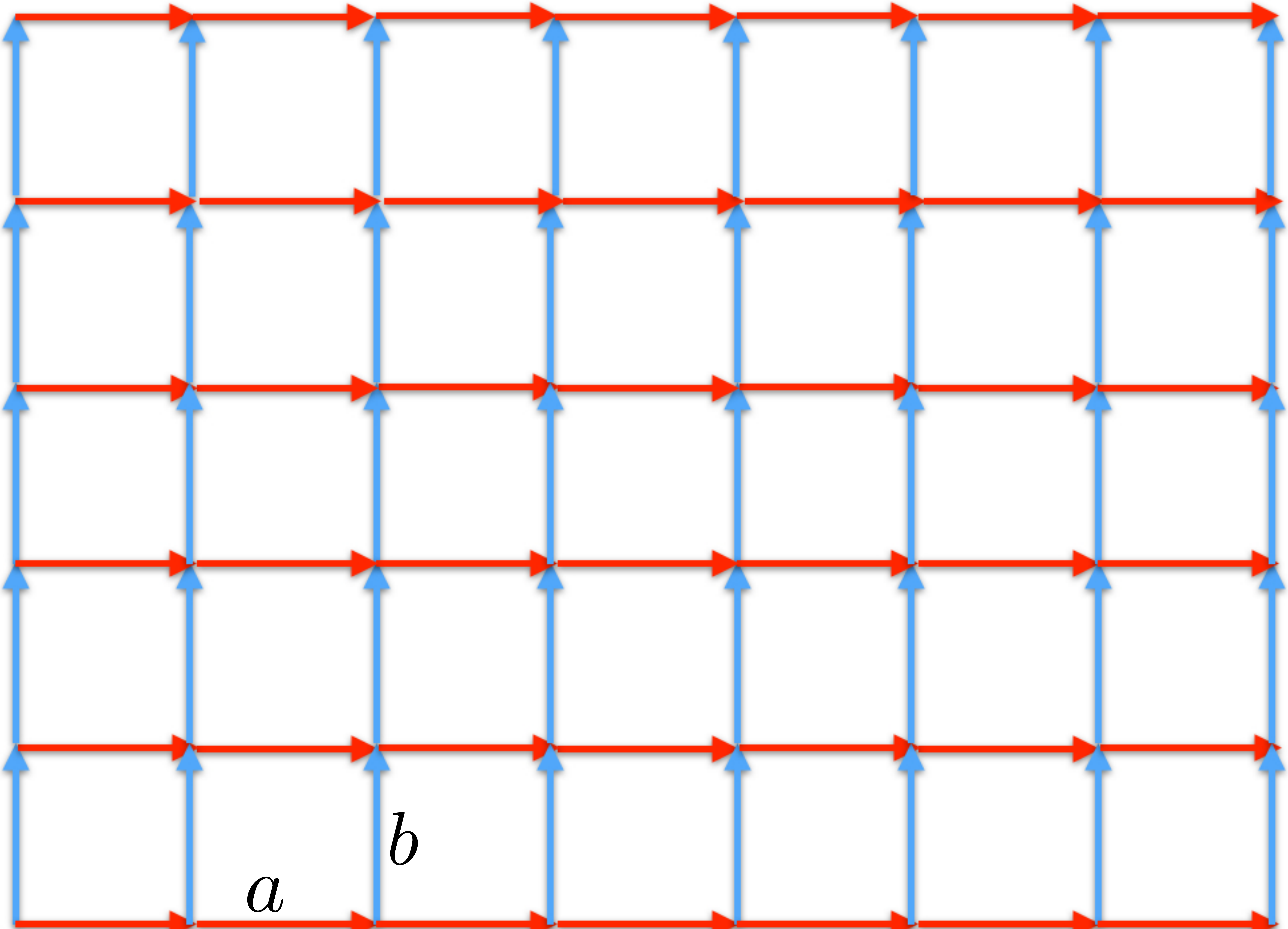
} Restrictions

→ *G virtually Abelian*

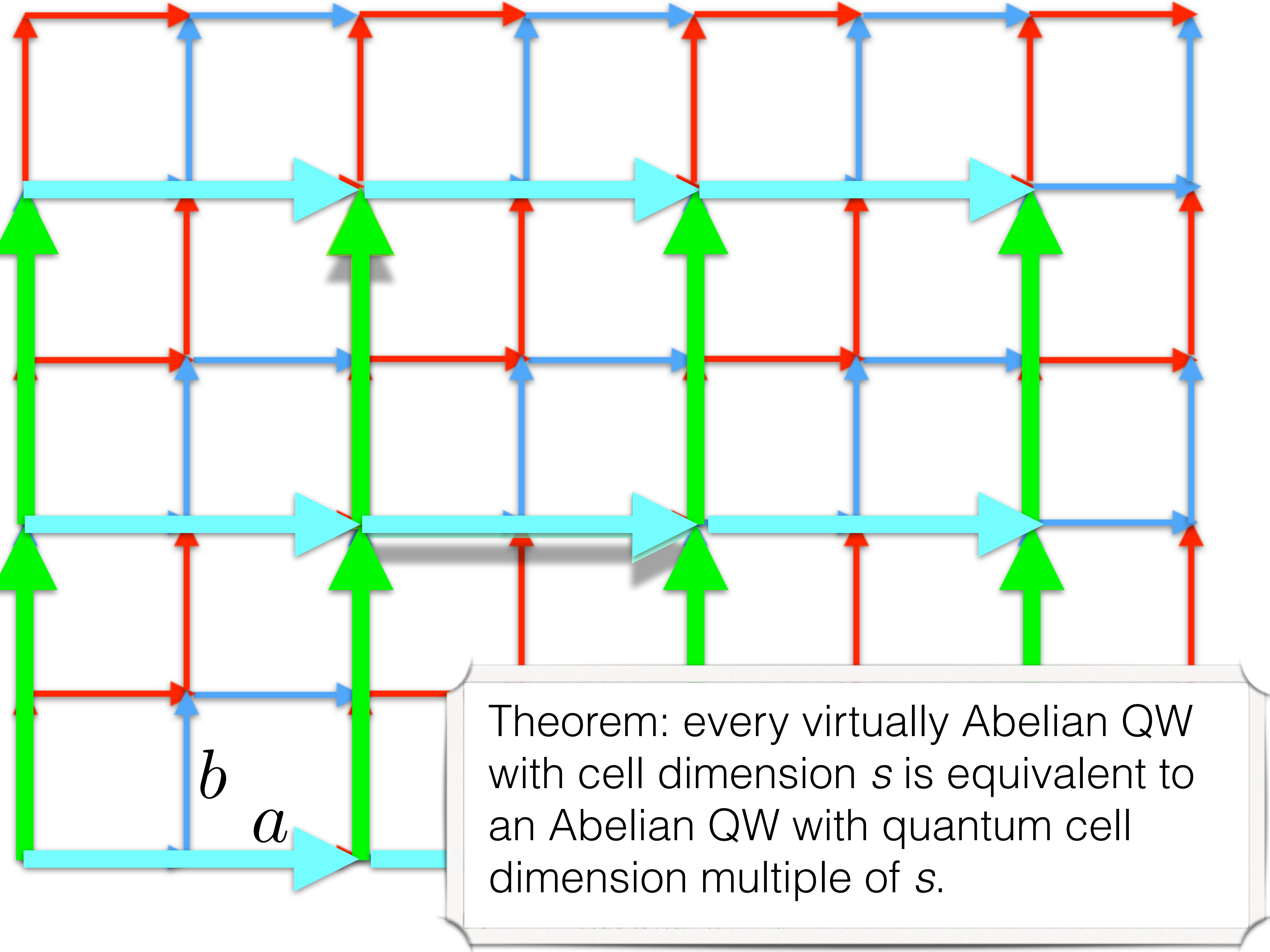
(geometric group theory)



$G = \langle h_1, h_2, \ldots | r_1, r_2, \ldots \rangle =: \langle S_+ | R \rangle$



$$G = \langle a, b | aba^{-1}b^{-1} \rangle$$



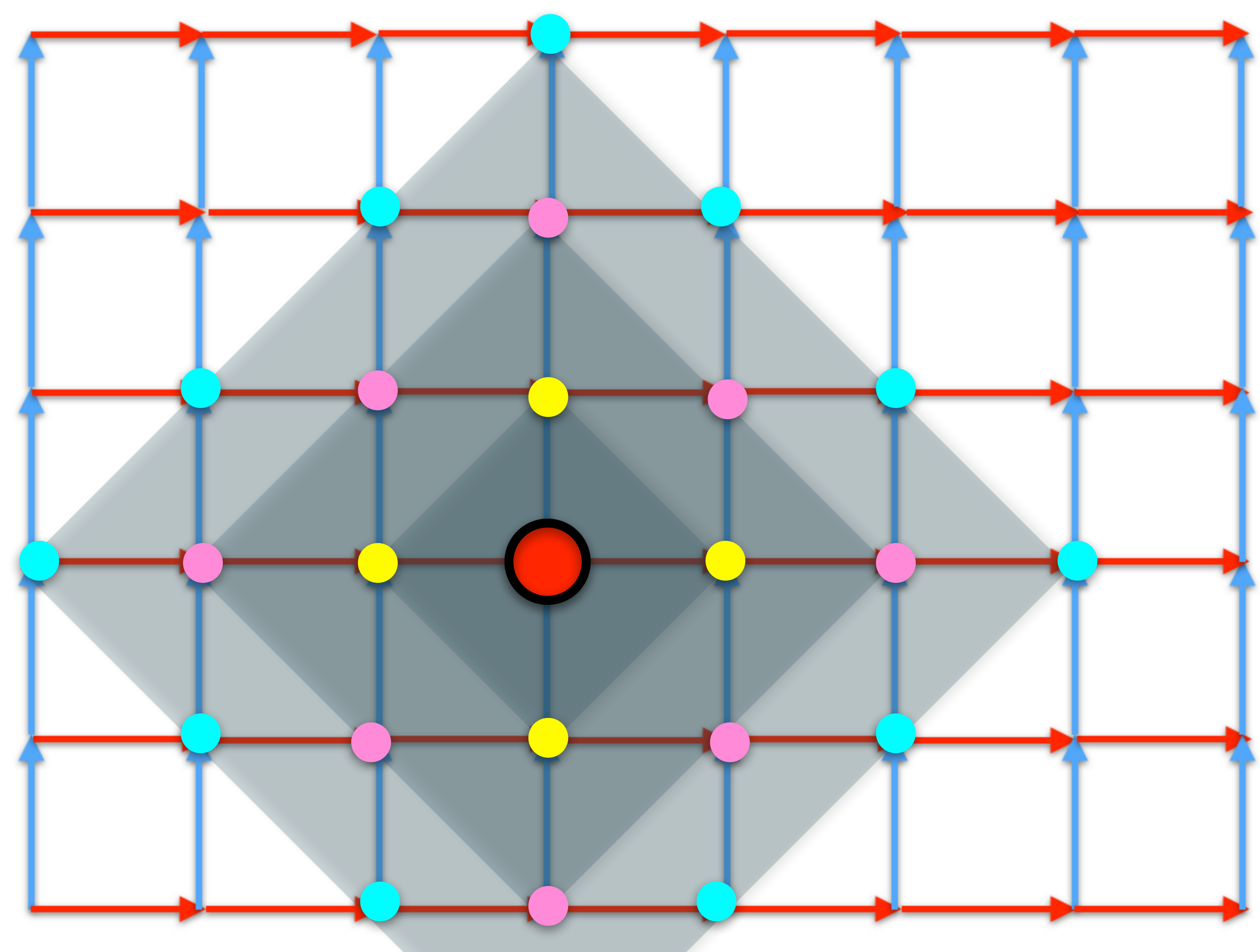
Theorem: every virtually Abelian QW with cell dimension *s* is equivalent to an Abelian QW with quantum cell dimension multiple of *s*.

Quantum walk on Cayley graph

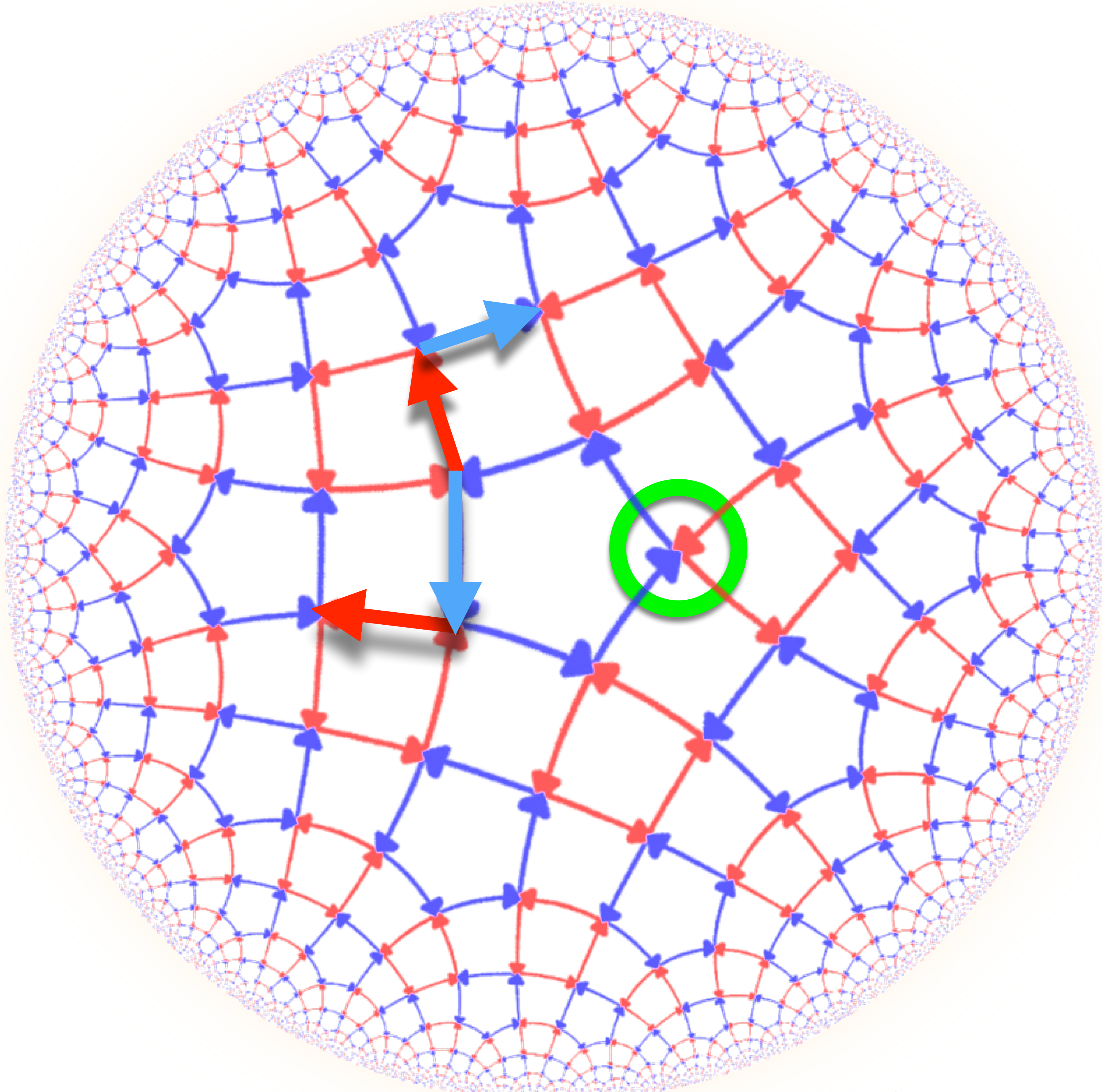
Theorem: A group is quasi-isometrically embeddable in R^d iff it is virtually Abelian

Virtually Abelian groups have polynomial growth (Gromov)

points $\sim r^d$

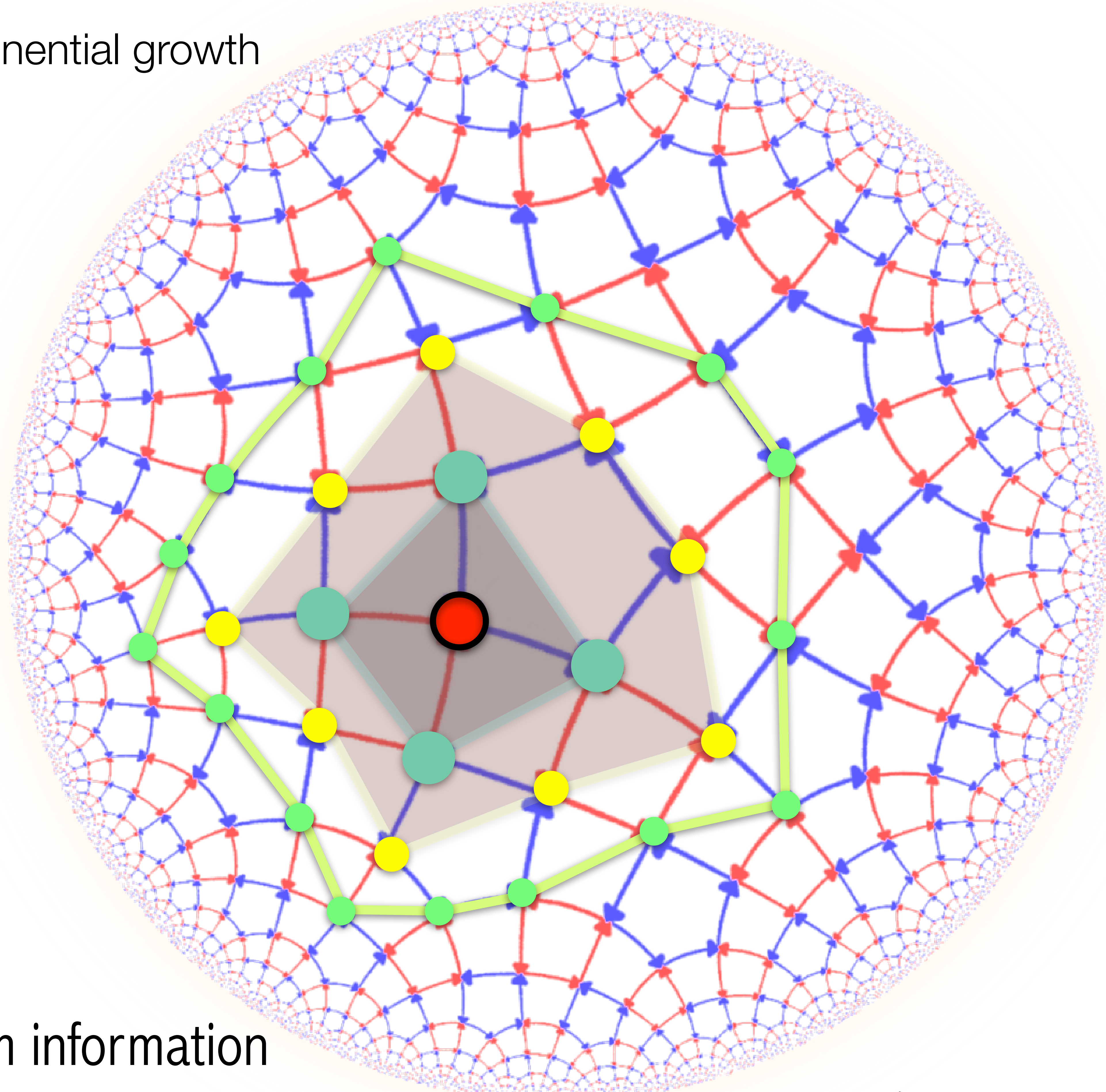


- *G* hyperbolic



$$G = \langle a, b | a^5, b^4, (ab)^2 \rangle$$

- G hyperbolic → exponential growth



points ~exp(r)

transmitted quantum information decrease as exp(-r)

$$G = \langle a, b | a^5, b^4, (ab)^2 \rangle$$

Informationalism: Principles for QFT

- *QFT derived in terms of countably many quantum systems in interaction*

add principles

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} Quantum Cellular Automata on the Cayley graph of a group *G*

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} Restrictions

G virtually Abelian

Isotropy

- There exists a group *L* of permutations of S_+, transitive over S_+ that leaves the Cayley graph invariant
- a nontrivial unitary *s*-dimensional (projective) representation $\{L_l\}$ of *L* such that:

$$A = \sum_{h\in S} T_h \otimes A_h = \sum_{h\in S} T_{lh} \otimes L_l A_h L_l^\dagger$$

Informationalism: Principles for QFT

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} Quantum Cellular Automata on the Cayley graph of a group *G*

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} Restrictions

G virtually Abelian

- *Relativistic regime (k≪1):* free QFT (Weyl, Dirac, and Maxwell)
- *Ultra-relativistic regime (k~1)* [Planck scale]: nonlinear Lorentz

- QFT derived:
  - without assuming Special Relativity
  - without assuming mechanics (quantum ab-initio)

- QCA is a *discrete* theory

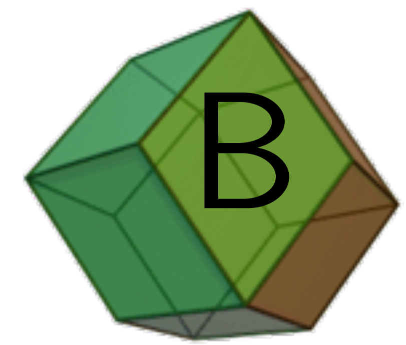
Motivations to keep it discrete:

1. Discrete contains continuum as special regime
2. Testing mechanisms in quantum simulations
3. Falsifiable discrete-scale hypothesis
4. Natural scenario for holographic principle
5. Solves all issues in QFT originating from continuum:
   i) uv divergencies
   ii) localization issue
   iii) Path-integral
6. Fully-fledged theory to evaluate cutoffs

D'Ariano, Perinotti, PRA **90** 062106 (2014)

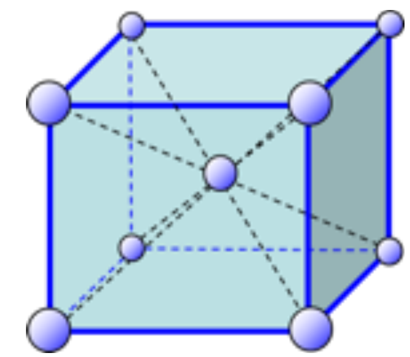
The Weyl QCA

☞ Minimal dimension for nontrivial unitary *A:* s=2



- Unitarity ⇒ for d=3 the only possible *G* is the BCC!!

- Isotropy ⇒ Fermionic ψ (d=3)



Unitary operator: $$A = \int_B^{\oplus} d\mathbf{k}\, A_{\mathbf{k}}$$

Two QCAs connected by P

$$\begin{aligned} A^{\pm}_{\mathbf{k}} = &- i\sigma_x\left(s_x c_y c_z \pm c_x s_y s_z\right) \\ &\mp i\sigma_y\left(c_x s_y c_z \mp s_x c_y s_z\right) \\ &- i\sigma_z\left(c_x c_y s_z \pm s_x s_y c_z\right) \\ &+ I\left(c_x c_y c_z \mp s_x s_y s_z\right) \end{aligned}$$

$$s_\alpha = \sin\frac{k_\alpha}{\sqrt{3}}$$

$$c_\alpha = \cos\frac{k_\alpha}{\sqrt{3}}$$

D'Ariano, Perinotti, PRA **90** 062106 (2014)

The Weyl QCA

$$i\partial_t\psi(t) \simeq \frac{i}{2}[\psi(t+1)-\psi(t-1)] = \frac{i}{2}(A-A^\dagger)\psi(t)$$

$$\begin{aligned}\frac{i}{2}(A^\pm_{\mathbf{k}} - A^{\pm\dagger}_{\mathbf{k}}) = &+\sigma_x(s_xc_yc_z \pm c_xs_ys_z) \\ &\pm\sigma_y(c_xs_yc_z \mp s_xc_ys_z) \\ &+\sigma_z(c_xc_ys_z \pm s_xs_yc_z)\end{aligned}$$

"Hamiltonian"

$k \ll 1$ ⇒ $i\partial_t\psi = \frac{1}{\sqrt{3}}\boldsymbol{\sigma}^\pm \cdot \mathbf{k}\,\psi$ ☜ Weyl equation! $\boldsymbol{\sigma}^\pm := (\sigma_x, \pm\sigma_y, \sigma_z)$

⇓ Two QCAs connected by P

$$\begin{aligned}A^\pm_{\mathbf{k}} = &-i\sigma_x(s_xc_yc_z \pm c_xs_ys_z) \\ &\mp i\sigma_y(c_xs_yc_z \mp s_xc_ys_z) \\ &-i\sigma_z(c_xc_ys_z \pm s_xs_yc_z) \\ &+I(c_xc_yc_z \mp s_xs_ys_z)\end{aligned}$$

$$s_\alpha = \sin\frac{k_\alpha}{\sqrt{3}} \qquad c_\alpha = \cos\frac{k_\alpha}{\sqrt{3}}$$

D'Ariano, Perinotti, PRA **90** 062106 (2014)

Dirac QCA ⊕

Local coupling: $A_{\mathbf{k}}$ coupled with its inverse with off-diagonal identity block matrix

$$E^{\pm}_{\mathbf{k}} = \begin{pmatrix} nA^{\pm}_{\mathbf{k}} & imI \\ imI & nA^{\pm\dagger}_{\mathbf{k}} \end{pmatrix}$$
$$n^2 + m^2 = 1$$

$E^{\pm}_{\mathbf{k}}$ CPT-connected!

$$\omega^{E}_{\pm}(\mathbf{k}) = \cos^{-1}[n(c_x c_y c_z \mp s_x s_y s_z)]$$

Dirac in relativistic limit $k \ll 1$

m≤1: mass

n^{-1}: refraction index

Bisio, D'Ariano, Perinotti, arXiv:1407.6928

Maxwell QCA ⊗

$$M_{\mathbf{k}} = A_{\mathbf{k}} \otimes A^{*}_{\mathbf{k}}$$

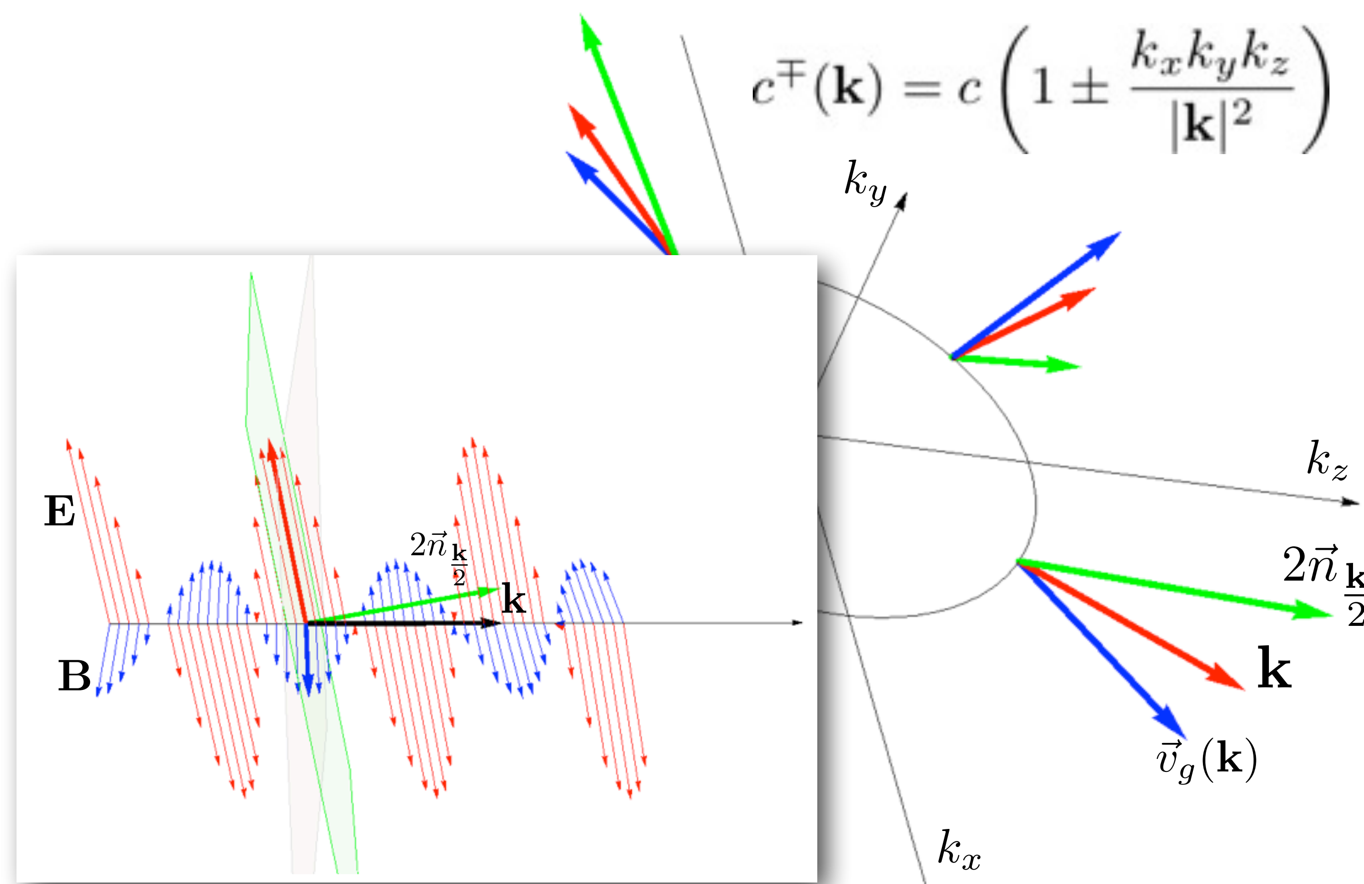
$$F^{\mu}(\mathbf{k}) = \int \frac{\mathrm{d}\,\mathbf{q}}{2\pi} f(\mathbf{q}) \tilde{\psi}(\tfrac{\mathbf{k}}{2} - \mathbf{q}) \sigma^{\mu} \varphi(\tfrac{\mathbf{k}}{2} + \mathbf{q})$$

Maxwell in relativistic limit $k \ll 1$

Boson: emergent from convolution of fermions

(De Broglie neutrino-theory of photon)

$$c^{\mp}(\mathbf{k}) = c\left(1 \pm \frac{k_x k_y k_z}{|\mathbf{k}|^2}\right)$$



G. M. D'Ariano, N. Mosco, P. Perinotti, A. Tosini,
PLA **378** 3165 (2014); EPL **109** 40012 (2015)

Exact solution of Dirac (d=1) and Weyl (d=1,2,3)

The analytical solution of the Dirac automaton can also be expressed in terms of Jacobi polynomials $P_k^{(\zeta,\rho)}$ performing the sum over f in Eq. (16) which finally gives

$$\psi(x,t) = \sum_y \sum_{a,b\in\{0,1\}} \gamma_{a,b} P_k^{(1,-t)}\left(1+2\left(\frac{m}{n}\right)^2\right) A_{ab}\psi(y,0),$$

$$k = \mu_+ - \frac{a\oplus b+1}{2},$$

$$\gamma_{a,b} = -(\mathrm{i}^{a\oplus b}) n^t \left(\frac{m}{n}\right)^{2+a\oplus b} \frac{k!\left(\mu_{(-)^{ab}} + \frac{\overline{a\oplus b}}{2}\right)}{(2)_k}, \qquad (18)$$

where $\gamma_{00} = \gamma_{11} = 0$ ($\gamma_{10} = \gamma_{01} = 0$) for $t + x - y$ odd (even) and $(x)_k = x(x+1)\cdots(x+k-1)$.

The LTM standards of the theory

Dimensionless variables

$$x = \frac{x_m}{\mathfrak{a}} \in \mathbb{Z}, \quad t = \frac{t_s}{\mathfrak{t}} \in \mathbb{N}, \quad m = \frac{m_g}{\mathfrak{m}} \in [0, 1]$$

Relativistic limit: → $c = \mathfrak{a}/\mathfrak{t}$ $\quad \hbar = \mathfrak{m}\mathfrak{a}c$

Measure $\mathfrak{m}$ from mass-refraction-index

→ $$n(m_g) = \sqrt{1 - \left(\frac{m_g}{\mathfrak{m}}\right)^2}$$

Measure $\mathfrak{a}$ from light-refraction-index

→ $$c^{\mp}(k) = c\left(1 \pm \frac{k}{\sqrt{3}k_{max}}\right)$$

Dirac emerging from the QCA

D'Ariano, Perinotti,
PRA **90** 062106 (2014)

fidelity with Dirac for a narrowband packets in the relativistic limit $k \simeq m \ll 1$

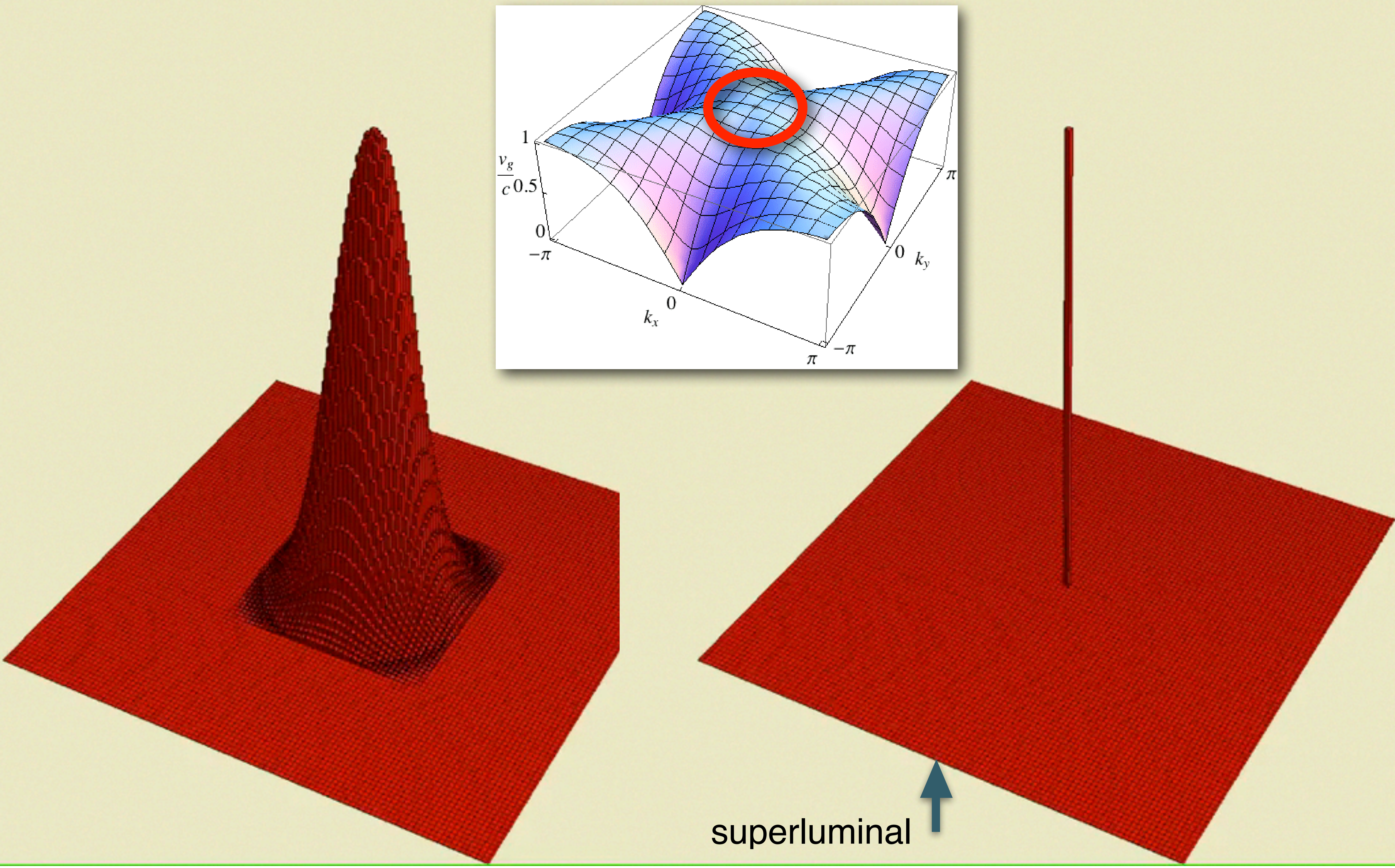
$$F = |\langle \exp\left[-iN\mathbf{\Delta}(\mathbf{k})\right]\rangle|$$

$$\Delta(\mathbf{k}) := (m^2 + \tfrac{k^2}{3})^{\frac{1}{2}} - \omega^E(\mathbf{k})$$

$$= \frac{\sqrt{3}k_x k_y k_z}{(m^2 + \frac{k^2}{3})^{\frac{1}{2}}} - \frac{3(k_x k_y k_z)^2}{(m^2 + \frac{k^2}{3})^{\frac{3}{2}}} + \tfrac{1}{24}(m^2 + \tfrac{k^2}{3})^{\frac{3}{2}} + \mathcal{O}(k^4 + N^{-1}k^2)$$

relativistic proton: $N \simeq m^{-3} = 2.2 * 10^{57} \;\Rightarrow\; t = 1.2 * 10^{14}\mathrm{s} = 3.7 * 10^6$ y

UHECRs: $k = 10^{-8} \gg m \;\Rightarrow N \simeq k^{-2} = 10^{16} \;\Rightarrow\; 5 * 10^{-28}$ s



2d automaton

- Evolution of a *narrow-band particle-state*
- Evolution of a *localized state*

The relativity principle

Virtually Abelian QW

$$A = \int_B^{\oplus} d\mathbf{k}\, A_{\mathbf{k}}$$

$$\mathbf{n}(\mathbf{k}) \cdot \mathbf{T} := \frac{i}{2}(A_{\mathbf{k}} - A_{\mathbf{k}}^{\dagger})$$

$\mathbf{n}(\mathbf{k})$ analytic in $\mathbf{k}$

$\mathbf{T}$ traceless $\quad T := (I, \mathbf{T}) = (T^{\mu})$ basis for $\mathrm{Lin}(\mathbb{C}^s)$

Dynamics: eigenvalue equation

$$A_{\mathbf{k}}\psi(\mathbf{k}, \omega) = e^{i\omega}\psi(\mathbf{k}, \omega)$$

$$(\sin\omega I - \mathbf{n}(\mathbf{k}) \cdot \mathbf{T})\psi(\mathbf{k}, \omega) = 0$$

For each value of $\mathbf{k}$ there are at most s eigenvalues $\{\omega_l(\mathbf{k})\}$

$\mathbf{n}(\mathbf{k})$ analytic in $\mathbf{k}$ + finite-dim irrep

$\Rightarrow$ $\omega_l(\mathbf{k})$ continuous

dispersion relations branches

Symmetries and Relativity Principle

Change of reference-frame: $(\omega, \mathbf{k}) \to (\omega', \mathbf{k}') = \mathcal{L}_\beta(\omega, \mathbf{k})$

$\mathcal{L}_\beta$ invertible (gen. non continuous) over $[-\pi, \pi] \times$ B

$\{\mathcal{L}_\beta\}_{\beta \in \mathbb{G}}$ Lie group (including also inversion, charge conjugation,...)

Covariance/symmetry of the dynamics:

there exists a pair of invertible matrices Γ_β and $\tilde{\Gamma}_\beta$ such that the following identity holds:

$$(\sin \omega I - \mathbf{n}(\mathbf{k}) \cdot \mathbf{T}) = \tilde{\Gamma}_\beta^{-1} (\sin \omega' I - \mathbf{n}(\mathbf{k}') \cdot \mathbf{T}) \Gamma_\beta$$

Γ_β and $\tilde{\Gamma}_\beta$ generally depending also on $(\omega, \mathbf{k})$ (continuously)

- $\mathbb{G}_0$ id-component of $\mathbb{G}$ preserve the branches
- change of reference-frame just a reshuffling of irreps: $\mathbf{k} \to \mathbf{k}'(\mathbf{k})$, $\mathcal{L}_\beta(\omega, \mathbf{k}) = (\omega(\mathbf{k}'), \mathbf{k}'(\mathbf{k}))$
- the definition of the change of reference-frame is the same for the whole class of virtually Abelian QW

$\mathbb{G}_0, \mathbb{G}$ depend on the QW!

Symmetries and Relativity Principle

$$(\sin \omega I - \mathbf{n}(\mathbf{k}) \cdot \mathbf{T}) = \tilde{\Gamma}^{-1}_{\lambda,k}(\sin \omega' I - \mathbf{n}(\mathbf{k'}) \cdot \mathbf{T})\Gamma_{\lambda,k}$$

Simplest symmetry: “gauge” transformation

$$\omega' = \omega, \; \mathbf{k}' = \mathbf{k}, \quad \Gamma_{\lambda,k} = \tilde{\Gamma}_{\lambda,k} = e^{i\lambda(\mathbf{k})}$$

(includes the group of “translations” of the Cayley graph)

Relativity Principle for Weyl QW

Weyl QW

eigenvalue equation $(\sin\omega I - \mathbf{n}(\mathbf{k})\cdot\boldsymbol{\sigma})\psi(\mathbf{k},\omega) = 0$

⇒ $\sin^2\omega - |\mathbf{n}(\mathbf{k})|^2 = 0$ ⇒ $(\omega,\mathbf{k}) \in \mathrm{Disp}(A) \subset [-\pi,\pi]\times \mathsf{B}$
$(\sin\omega, \mathbf{n}(\mathbf{k})) \in \mathbb{M}^4$ light-like

$\mathcal{D}^{(f)}$: $\mathcal{D}^{(f)}(\omega,\mathbf{k}) := f(\omega,\mathbf{k})(\sin\omega,\mathbf{n}(\mathbf{k})) =: p^{(f)}$

⇒ $p^{(f)}_\mu \sigma^\mu \psi(\mathbf{k},\omega) = 0$

for suitable choice of $f(\omega,\mathbf{k})$ and L_β matrix of the Lorentz group one has:

$\mathcal{L}^{(f)}_\beta := \mathcal{D}^{(f)-1} L_\beta \mathcal{D}^{(f)}$ is well defined on $\mathrm{Disp}(A)$

Bisio, D'Ariano, Perinotti, arXiv:1503.01017

Relativity Principle for Weyl QW

Non-linear Lorentz group

$$\mathcal{L}_\beta^{(f)} := \mathcal{D}^{(f)-1} L_\beta \mathcal{D}^{(f)}$$

acting on $[-\pi, \pi] \times$ B

leaving $\mathrm{Disp}(A)$ invariant

L_β linear Lorentz

$$\mathcal{D}^{(f)}(k_\mu) = p_\mu^{(f)}$$

Relativistic covariance of dynamics

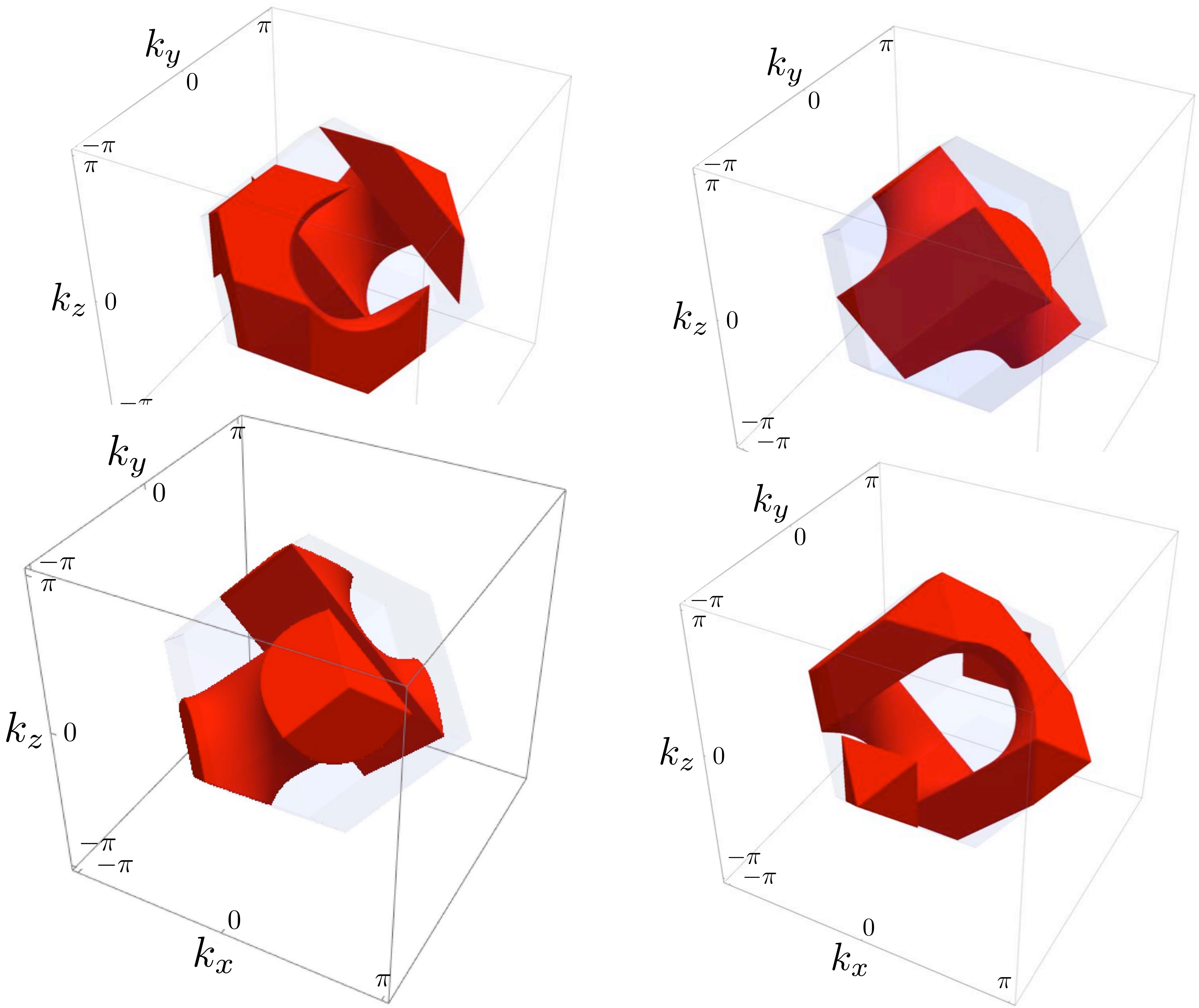
$$(\sin \omega I - \mathbf{n}(\mathbf{k}) \cdot \boldsymbol{\sigma}) = \tilde{\Lambda}_\beta^\dagger (\sin \omega' I - \mathbf{n}(\mathbf{k'}) \cdot \boldsymbol{\sigma}) \Lambda_\beta$$

$\Lambda_\beta \in \mathrm{SL}_2(\mathbb{C})$ independent of (k_μ)

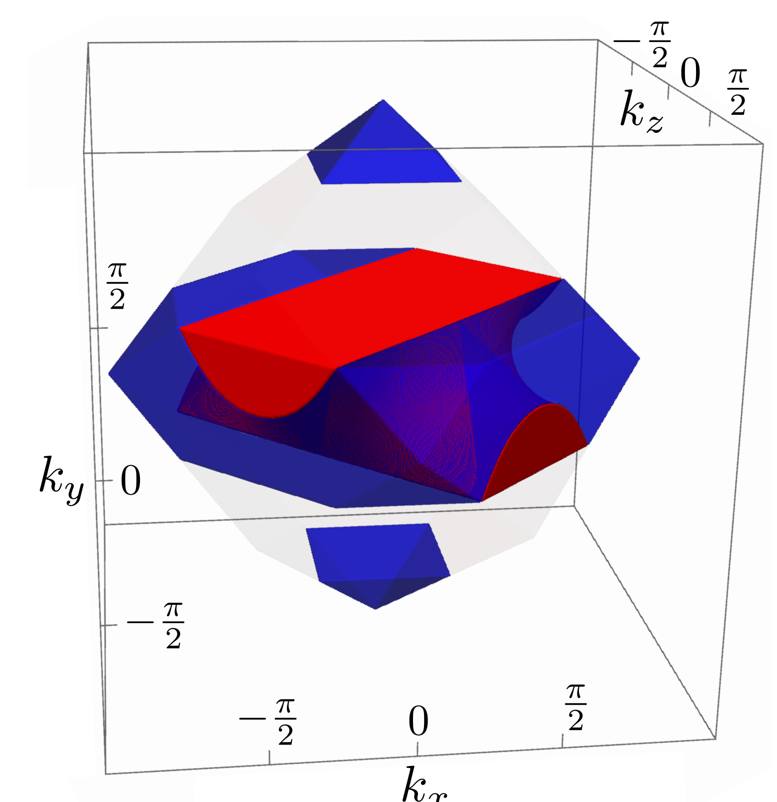
Bisio, D'Ariano, Perinotti, arXiv:1503.01017

Relativity Principle for Weyl QW

Includes the group of “translations” of the Cayley graph: $\mathbb{G}_0$ is the Poincaré group



The Brillouin zone separates into ***four invariant regions*** diffeomorphic to balls, corresponding to four different ***particles***.



Relativity Principle for Dirac QW

Dirac automaton: De Sitter covariance (non linear)

Covariance for Dirac QCA cannot leave m invariant

invariance of de Sitter norm:

$$\mathrm{Disp}(A): \quad \sin^2\omega - (1-m^2)|\mathbf{n}(\mathbf{k})|^2 - m^2 = 0$$

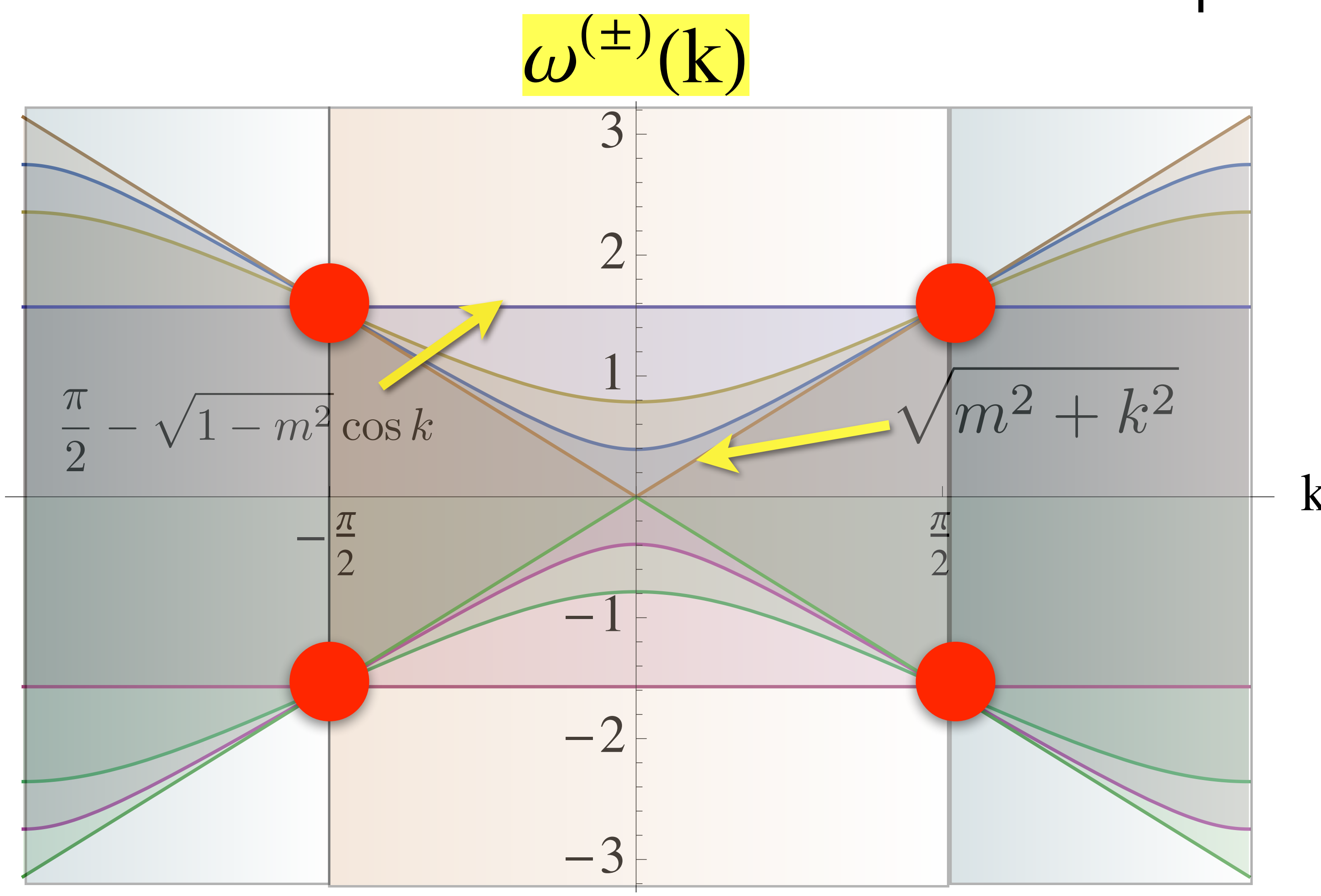
➡ $SO(1,4)$ invariance

$SO(1,4) \longrightarrow SO(1,3)$ for $m \to 0$ $\qquad \mathcal{O}(m^2)$

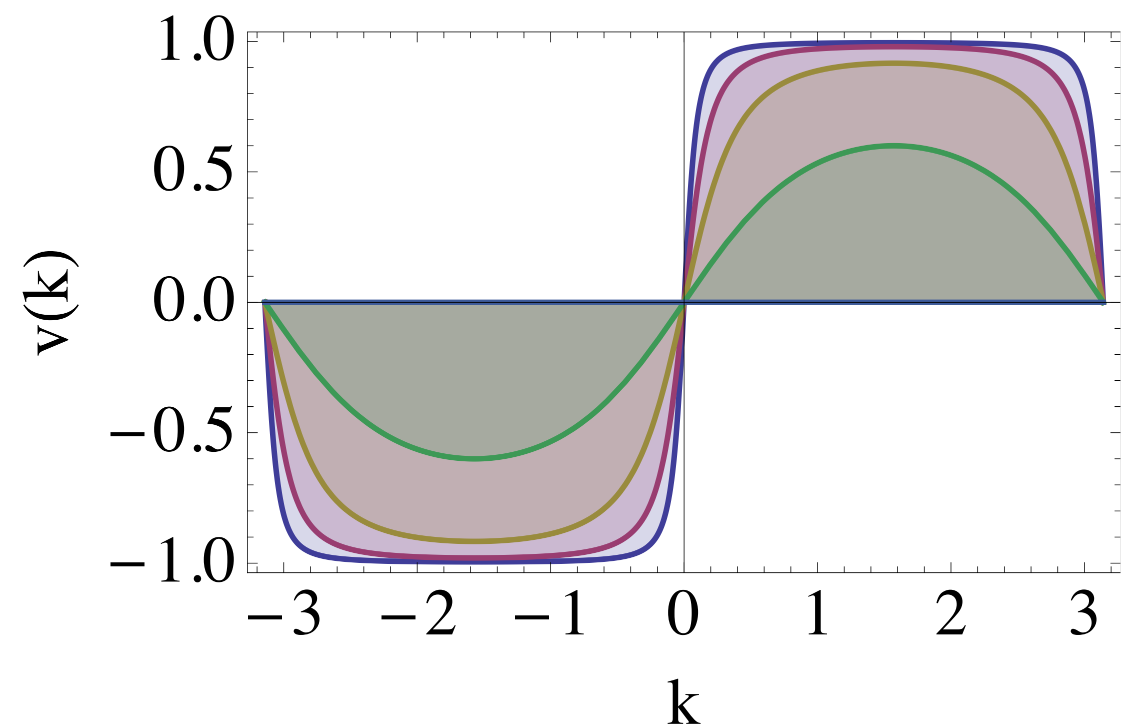
A. Bibeau-Delisle, A. Bisio, G. M. D'Ariano,
P. Perinotti, A. Tosini, EPL 109 50003 (2015)

Nonlinear Lorentz for Dirac d=1

Transformations that leave the dispersion relation invariant



$$\omega_E(k) := \pm\cos^{-1}(\sqrt{1-m^2}\cos k)$$



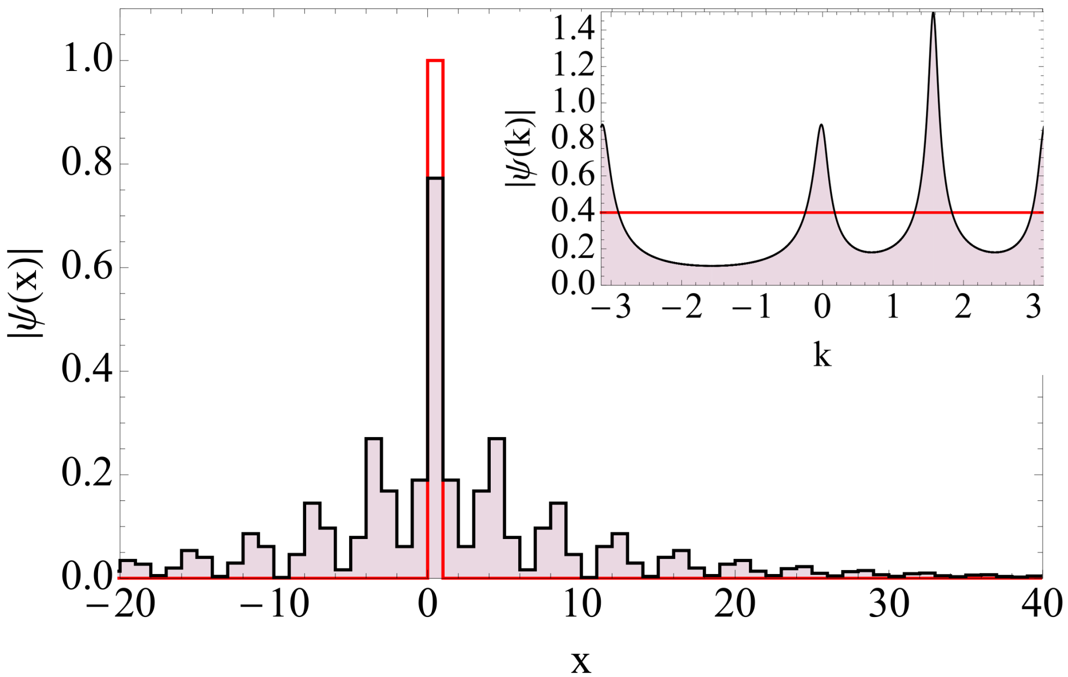
$$\omega' = \arcsin\left[\gamma\left(\sin\omega/\cos k - \beta\tan k\right)\cos k'\right]$$

$$k' = \arctan\left[\gamma\left(\tan k - \beta\sin\omega/\cos k\right)\right]$$

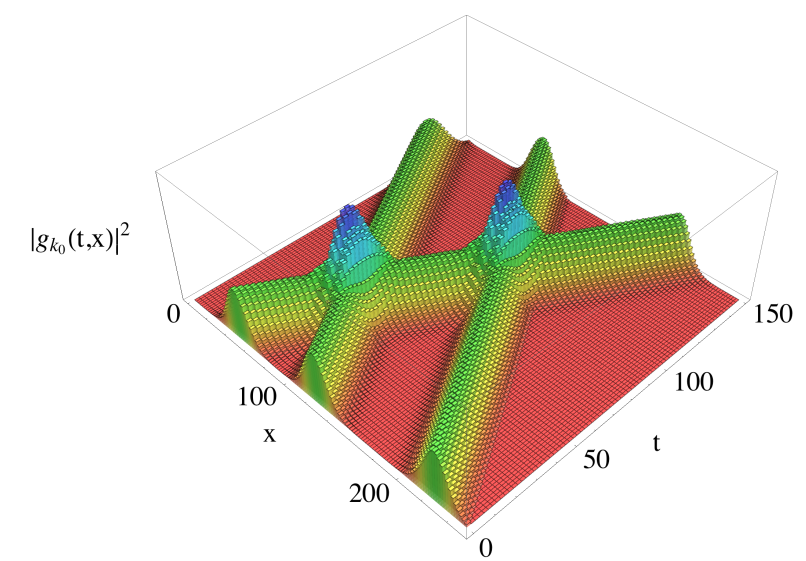
$$\gamma := (1-\beta^2)^{-1/2}$$

A. Bibeau-Delisle, A. Bisio, G. M. D'Ariano, P. Perinotti, A. Tosini, EPL 109 50003 (2015)

Planck-scale effects: Lorentz covariance distortion

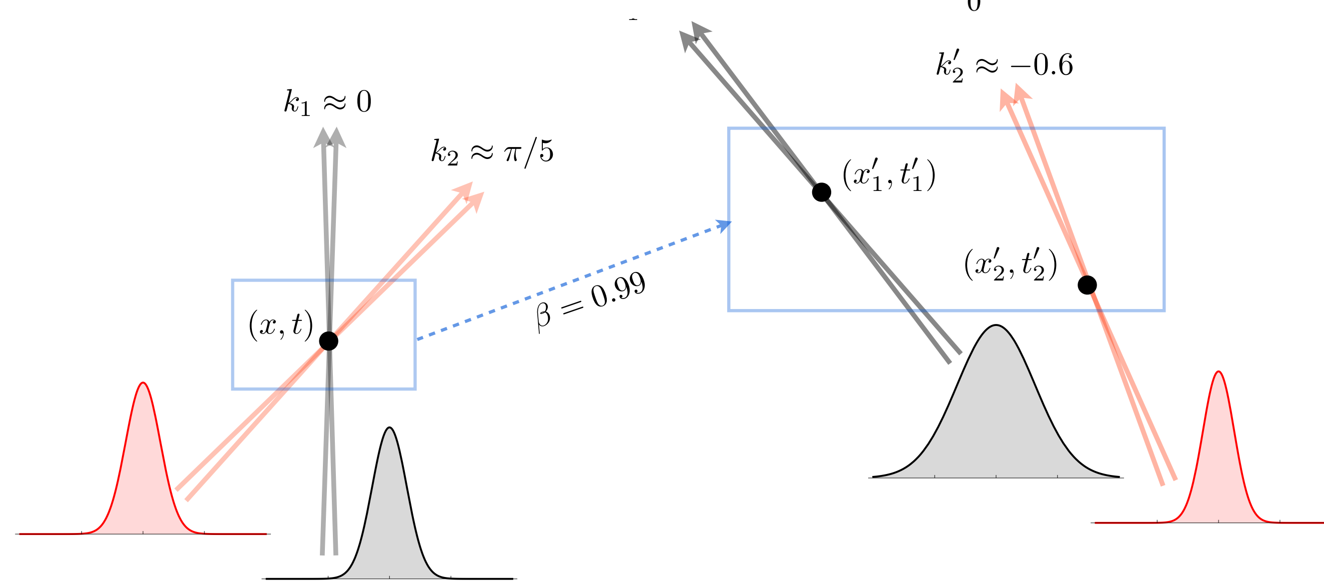


For narrow-band states we can linearize Lorentz transformations around $k=k_0$ and we get k-dependent Lorentz transformations



Delocalization under boost

$$|\psi\rangle = \int \mathrm{d}k\mu(k)\hat{g}(k)|k\rangle \xrightarrow{L^D_\beta} \int \mathrm{d}k\mu(k)\,\hat{g}(k)|k'\rangle =$$
$$= \int \mathrm{d}k\mu(k')\,\hat{g}(k(k'))|k'\rangle$$



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Time is real
space is an illusion!

QUit quantum information theory group

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SUPPORTING SCIENCE ~ INVESTING IN THE BIG QUESTIONS

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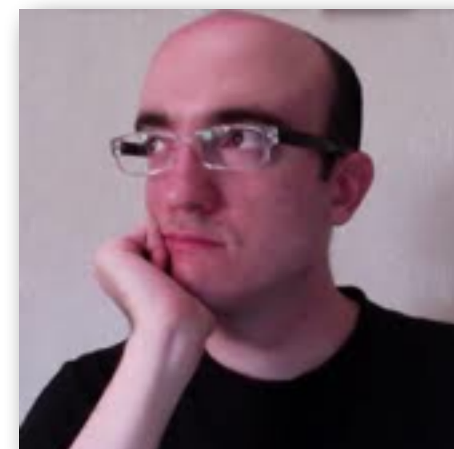

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